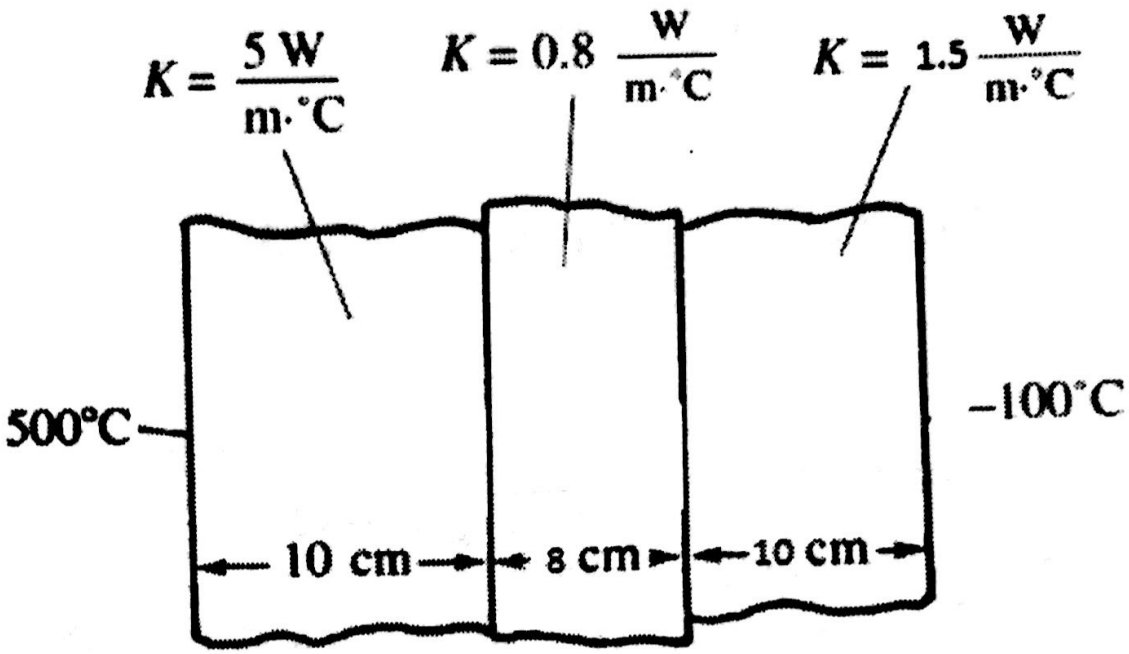
Solving Equations (2-5) of Equation (2)

$t_2 = 58.3°C, t_3 = 66.7°C$

$t_4 = 75°C, t_5 = 83.3°C$

13.6



Area = A (Can use unit A)

$$[k^{(1)}] = \frac{A(5)}{(0.1\text{m})}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 50 & -50 \\ -50 & 50 \end{bmatrix}\frac{\text{W}}{°\text{C}}$$

$$[k^{(2)}] = \frac{A(0.8)}{(0.08\text{m})}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix}$$

$$[k^{(3)}] = \frac{A(1.5)}{(0.10\text{m})}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 15 & -15 \\ -15 & 15 \end{bmatrix}$$

$$\{f\}'s = 0$$

Assemble global equations with $t_1 = 500°C$ and $t_4 = 100°C$

$$\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = A \begin{bmatrix} 50 & -50 & 0 & 0 \\ & 50+10 & -10 & 0 \\ & & 10+15 & -15 \\ \text{Symmetry} & & & 15 \end{bmatrix} \begin{Bmatrix} t_1 = 500\,°C \\ t_2 \\ t_3 \\ t_4 = 100\,°C \end{Bmatrix}$$

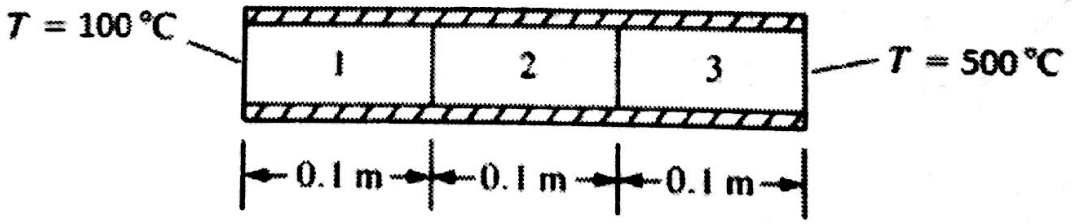
Solving the 2nd and 3rd equations above

$$t_2 = 457.1°C,\ t_3 = 242.86°C$$

$$q^{(3)} = -K_{xx}\frac{(100 - 242.86)°C}{0.10\,\text{m}}$$

$$q^{(3)} = 2145\ \frac{\text{W}}{\text{m}^2}$$

13.7



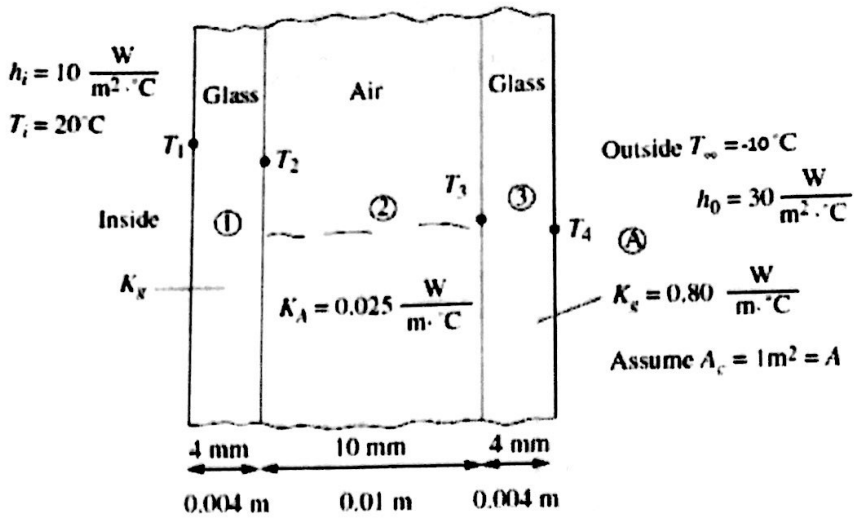
$$\frac{AK_{xx}^{(1)}}{L} = \frac{(0.1\text{m}^2)5}{0.1\text{m}} = 5$$

$$[k^{(1)}] = \frac{AK_{xx}^{(1)}}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 5\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[k^{(2)}] = 10\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad [k^{(3)}] = 15\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Assemble global equations

$$\begin{bmatrix} 5 & -5 & 0 & \cdots \end{bmatrix}$$



Find T_1 T_2, T_3, T_4, Q (heat transfer through the double pane) (use $A = 1\ cm^2$)

$$[k^{(1)}] = \frac{AK_g}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + h_i A\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} = \frac{1m^2\ 0.80\frac{W}{m\cdot K}}{0.004\,m}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + 10\frac{W}{m^2\cdot °C}(1m^2)\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$$

$$= \begin{bmatrix}210 & -200\\ -200 & 200\end{bmatrix}\frac{W}{K}$$

$$[k^{(2)}] = \frac{AK_A}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \frac{1m^2\ 0.025\frac{W}{m\cdot K}}{0.01\,m}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \begin{bmatrix}2.5 & -2.5\\ -2.5 & 2.5\end{bmatrix}\frac{W}{K}$$

$$[k^{(3)}] = \frac{AK_g}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + h_0 A\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix} = \frac{1m^2\ 0.80\frac{W}{m\cdot K}}{0.004\,m}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + 30\frac{W}{m^2\cdot °C}(1m^2)\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}200 & -200\\ -200 & 230\end{bmatrix}$$

$$\{f^{(1)}\} = h_i T_\infty A \begin{Bmatrix}1\\ 0\end{Bmatrix} = 10\frac{W}{m^2\cdot K}(693\ K)(1m^2)\begin{Bmatrix}1\\ 0\end{Bmatrix} = 2630\ W\begin{Bmatrix}1\\ 0\end{Bmatrix}$$

$$\{f^{(2)}\} = \begin{Bmatrix}0\\ 0\end{Bmatrix}$$

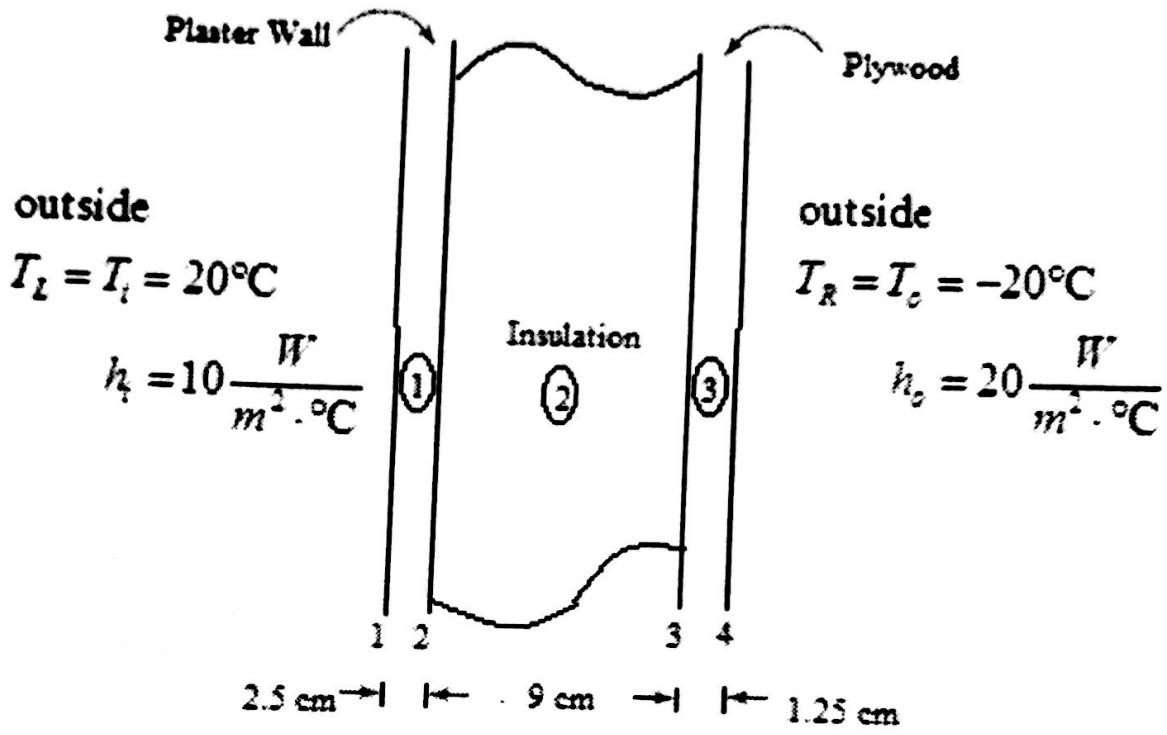
$$\{f^{(3)}\} = h_0 T_\infty A \begin{Bmatrix}0\\ 1\end{Bmatrix} = 30\frac{W}{m^2\cdot K}(263\ K)(1m^2)\begin{Bmatrix}0\\ 1\end{Bmatrix} = 7990\ W\begin{Bmatrix}0\\ 1\end{Bmatrix}$$

$$\{F\} = [K]\,[T]$$

$$\begin{Bmatrix}F_1 = 2630\,W\\ F_2 = 0\\ F_3 = 0\\ F_4 = 7990\ W\end{Bmatrix} = \begin{bmatrix}210 & -200 & 6 & 0\\ -200 & 200+2.5 & -2.5 & 0\\ 0 & -2.5 & 2.5+200 & -200\\ 0 & 0 & -200 & 230\end{bmatrix}\begin{Bmatrix}T_1\\ T_2\\ T_3\\ T_4\end{Bmatrix}$$

$$T_1 = 287.5\ K = 14.5°C$$
$$T_2 = 287.3\ K = 14.2°C$$
$$T_3 = 265.1\ K = -7.88°C$$
$$T_4 = 264.8\ K = -8.15°C$$

13.10



$$K_{xx}^{(1)} = 0.20 \frac{W}{m \cdot °C} \qquad K_{xx}^{(2)} = 0.038 \frac{W}{m \cdot °C} \qquad K_{xx}^{(3)} = 0.12 \frac{W}{m \cdot °C}$$

$$K = \frac{AK_{xx}}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hpL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} + K_{hend}$$

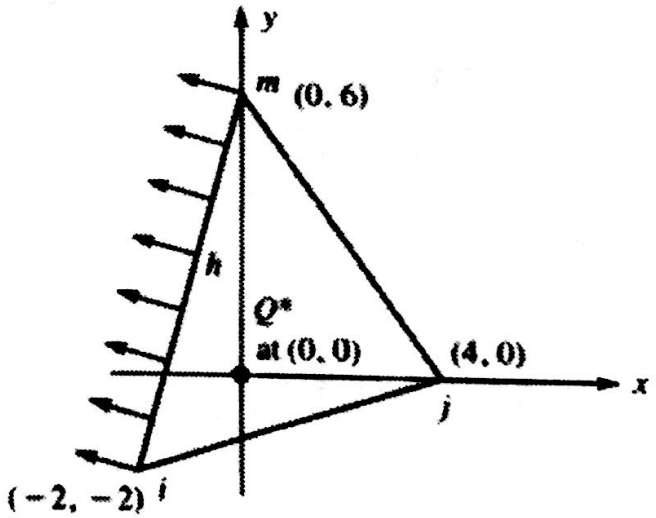
Assumptions Determine temperatures at inner/outer

$$\therefore \quad \{f_Q\} = 300\,(1)\begin{Bmatrix}0.333\\0.333\\0.333\end{Bmatrix} = \begin{Bmatrix}100\\100\\100\end{Bmatrix}\ \frac{\text{Btu}}{\text{h}}$$

$$\{f_h\} = \frac{hT_\infty L_{ij} t}{2}\begin{Bmatrix}1\\1\\0\end{Bmatrix} = \frac{(20)(70)(4.123)1}{2}\begin{Bmatrix}1\\1\\0\end{Bmatrix}$$

$$\{f_h\} = \begin{Bmatrix}2886\\2886\\0\end{Bmatrix} \quad \therefore \quad \{f\} = \begin{Bmatrix}100+2886\\100+2886\\100+0\end{Bmatrix} = \begin{Bmatrix}2986\\2986\\100\end{Bmatrix}\frac{\text{Btu}}{\text{h}}$$

13.20



$$[k] = t\,A\,[B]^T\,[D]\,[B] + \frac{h\,L_{im}}{6}\begin{bmatrix}2 & 0 & 1\\0 & 0 & 0\\1 & 0 & 2\end{bmatrix}$$

$$2A = \begin{vmatrix}1 & -2 & -2\\1 & 4 & 0\\1 & 0 & 6\end{vmatrix} = 44$$

$$L_{im} = 8.246 \text{ m}$$

$$\beta_i = -6, \beta_j = 8, \beta_m = -2$$

$$\gamma_i = -4, \gamma_j = -2, \gamma_m = 6$$

$$N_i = \frac{1}{44}[24 + (-6)(0) + 0] = 0.545$$

$$N_j = \frac{1}{44}[12 + 8(0) + 0] = 0.278$$

$$N_m = \frac{1}{44}[8 + (-2)(0) + 0] = 0.181$$

By (1)

$$[K] = \frac{1}{4(22)}\begin{bmatrix} -6 & -4 \\ 8 & -2 \\ -2 & 6 \end{bmatrix}\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}\begin{bmatrix} -6 & 8 & -2 \\ -4 & -2 & 6 \end{bmatrix}$$

$$+ \frac{20(8.246)}{6}\begin{bmatrix} 2 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

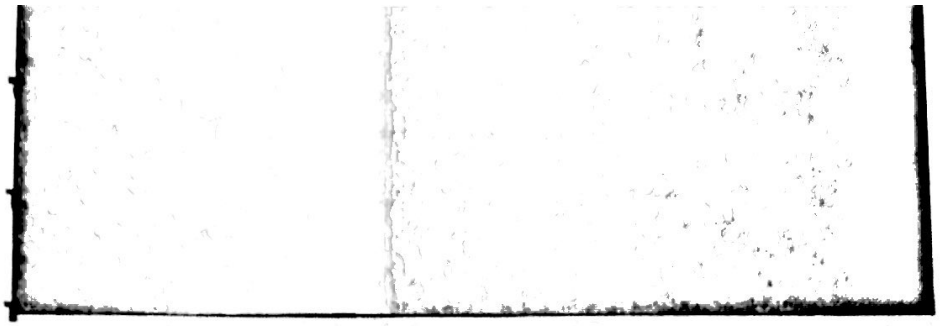
$$[k] = \begin{bmatrix} 60.88 & -4.55 & 26.12 \\ & 7.73 & -3.18 \\ \text{Symmetry} & & 59.52 \end{bmatrix} \frac{\text{W}}{^\circ\text{C}}$$

$$\{f\} = \int_v [N]^T|_{(0,0)}\, Q^*\, dV + \int_{S_3} hT_\infty [N]^T_{\text{Along } S_3}\, ds$$

$$= Q^* \times \begin{Bmatrix} N_i \\ N_j \\ N_m \end{Bmatrix}_{(0,0)} + \frac{hT_\infty L_{im}}{2}\begin{Bmatrix} 1 \\ 0 \\ 1 \end{Bmatrix}$$

$$= 200\begin{Bmatrix} 0.545 \\ 0.273 \\ 0.183 \end{Bmatrix} + \frac{(20)(15)(8.246)}{2}\begin{Bmatrix} 1 \\ 0 \\ 1 \end{Bmatrix}$$

$$\{f\} = \begin{Bmatrix} 1346 \\ 54.6 \\ 1273 \end{Bmatrix} \text{W}$$



13.36