Gerrit de Veer's true and perfect description of the Novaya Zemlya effect, 24–27 January 1597

Siebren Y. van der Werf, Günther P. Können, Waldemar H. Lehn, Frits Steenhuisen, and Wayne P. S. Davidson

The first recordings of the Novaya Zemlya (NZ) effect were made during Willem Barents' third Arctic expedition. Ray-tracing analyses of the three key observations, on 24–27 January 1597, show that all the reported details can be explained by adopting one common and realistic type of temperature inversion. In particular, the Moon–Jupiter conjunction could have been visible over the central mountain ridge of the island. We show that the NZ effect distorts the relative positions of Jupiter and the Moon in such a way that the looked-for fingerprint of the conjunction occurred almost 2 h after the true conjunction. The quoted direction for the apparent Moon–Jupiter conjunction is then found to be accurate to within 1°. This delay of the apparent conjunction largely explains the error of 29° in their longitude determination. The truthfulness of these observations, debated for four centuries, now appears to be beyond doubt. © 2003 Optical Society of America

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1. Historical Notes

A. First Recordings of the Novaya Zemlya Effect

The Novaya Zemlya (NZ) effect is a strong arctic mirage, through which celestial bodies may become visible while geometrically well below the horizon. In his book *The True and Perfect Description of Three Voyages, so Strange and Woonderfull, That the Like Hath Neuer Been Heard of before*,[1–5] describing three voyages launched by the Dutch to find a northeast passage to China, Gerrit de Veer gives the first recordings of this phenomenon. The observations were made at 76° 15′.4 N 68° 18′.6 E, near the cabin, Het Behouden Huijs (House of Safety), in which the crew members of Willem Barents' expedition survived the polar winter on the east coast of Novaya Zemlya (Russian: New Land).

Their claim to have witnessed, after the polar winter, the Sun's return approximately 2 weeks before expected, understandably met with disbelief from their contemporaries and triggered heated discussion among scientists, such as Kepler and Maestlin. Today De Veer's account is generally believed to be trustworthy by natural scientists but after 400 years is still considered to be at least superficial in circles of historians.

We shall review the historical debates and the reasons why, despite later confirmation of the effect's existence, doubts have remained. Having established the ray-tracing techiques, necessary for analyzing the propagation of light rays in a nonstandard atmosphere, which includes horizontal temperature and pressure gradients,[6] we attempt an analysis of the observations that De Veer describes.

These are the key observations:

1. On 24 January 1597 three men, among them Gerrit de Veer and the captain Jacob Heemskerck, saw a glimpse of the Sun while its center was geometrically still 5° 26′ below the horizon. The return of the Sun was not expected until 8 February, and a check on the day counting was deemed necessary.

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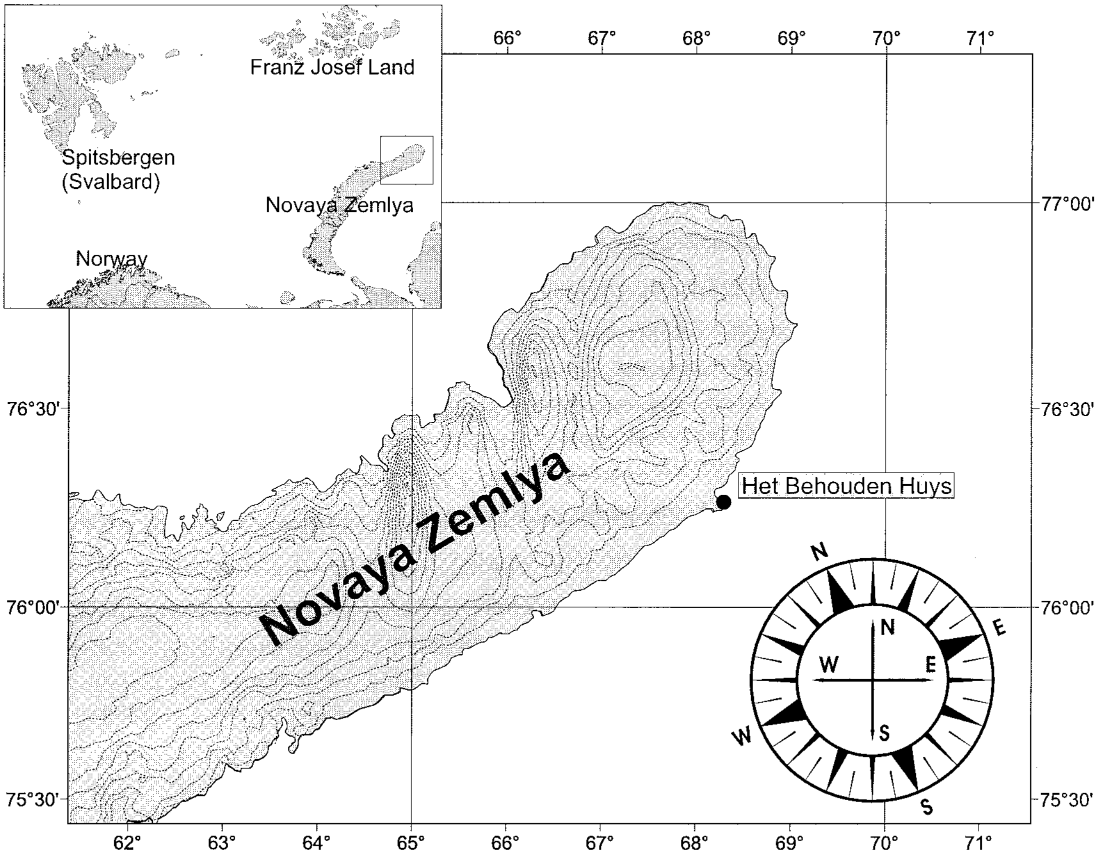

Fig. 1. Northern part of Novaya Zemlya, indicating the location of Het Behouden Huijs, where the crew of Willem Barents survived the winter. Height contours are given in steps of 100 m. The inset shows the location of Novaya Zemlya on a larger scale. Compasses are shown in the lower right-hand corner: The inner compass indicates the true bearings. The outer compass corresponds to the magnetic common compass, referred to in the text.

2. According to the astronomical tables of Iosephus Scala[7] a conjunction in ecliptic longitude between Jupiter and the Moon was to take place on the following night, on 25 January at 01:00 Venetian solar time (00:24 UT). In the words of De Veer,

> We looked constantly at the two planets [and saw] that they gradually approached each other until the Moon and Jupiter stood just one above the other, both in the sign of Taurus, and this at six o'clock in the morning. At that time Jupiter and the Moon were conjunct, in [the direction] north by east [i.e., one point or 11° 15′ East] on the compass at our house and the south of our compass was SSW, there was the true south, the Moon being eight days old.

3. A light haze in the south prevented them from seeing the Sun again on the two days that followed, but it reappeared on 27 January and was seen "in its full roundness, [its lower limb] just free of the horizon." Its center was then geometrically still below the horizon by 4° 41′.

Figure 1 shows a modern map of Novaya Zemlya on which Het Behouden Huijs is indicated. Drawn-in compasses show the Earth's magnetic variation, which at that time was 2 points West.

B. Later Observations, Explanations

Around 1900, observations by polar explorers confirmed the existence of the NZ effect. Fridtjof Nansen[8] described an observation made at 80° 01′ N on 16 February 1894, when the Sun's true (i.e., geometrical) altitude was −2° 22′. Shackleton,[9] while trapped in the ice of the Weddell Sea, saw the Sun reappear twice on 8 May 1915, while its true altitude was −2° 37′. Liljequist[10] observed the NZ effect on 1 July 1951 at Maudheim station, Antarctica, when the Sun was 4° 18′ below the horizon.

Already in 1604 Johannes Kepler, in his book on astronomical optics,[11–13] discussed the NZ observations and gave an explanation that in many respects was correct. In his time the atmosphere was thought to be a sphere of constant density, with a sharp transition to the ether outside at an estimated height of 4 km, but the possibility that both the density and the height of the atmosphere might vary with geographical location was recognized. The most likely explanation for the Sun's early appearance, Kepler proposed, is provided by multiple reflections at the boundary between the atmosphere and the ether, analogous to the way in which light can enter in a glass plate by refraction and propagate inside it by multiple reflection. In this way it is possible that instead of the Sun its reflected image could have been seen: ut pro Sole idolum eius in Noua Zembla videri potuerit.

The first basically correct optical explanation of the NZ effect must probably be attributed to Baills, who in 1875 wrote

> Would this not rather be a phenomenon of total reflection, in complete analogy, theoretically speaking, to the formation of a rainbow, where here the whole Earth plays the same role as a single water droplet? The total reflection zone would roughly be determined by the separation of the upper part of the atmosphere which is constantly heated by the Sun as opposed to the lower part, which is only heated in summer. (Ref. 14)

In 1956 Visser[15] showed convincingly that the NZ effect is caused by multiple total reflection at a temperature inversion layer. In 1979 Lehn[16] made a ray-tracing analysis of Liljequist's observation, based on the recorded temperature profile, and was able to reproduce the observed image of the Sun.

C. Criticism

Doubts about the original observations have been lingering until today. Shortly after their return, De Veer and Heemskerck were interrogated by Robbert Robbertsz, an influential specialist in navigation and former teacher of Heemskerck. During these meetings they were unable to give a satisfactory account of their calendar keeping but maintained that they had not been mistaken in the dates. Robbertsz, though not convinced at all, let the matter rest at first. But as the NZ observations continued to intrigue scientists, the cartographer Willem Jansz Blaeu, in 1627, asked Robbertsz to write an account of the discussions that he had had with De Veer and Heemskerck, 30 years earlier. This account, written in a letter to Willem Jansz, was published in the famous Grand Atlas of his son, Joan Blaeu.[17] In his letter Robbertsz suggests that a proper day counting would have been impossible under the harsh circumstances in which the crew had to survive. He explains De Veer's unwillingness to admit this as a wish to have an interesting story in the first place, which De Veer did not dare to turn back after the scientific community had jumped upon it.

In Beke's annotated version of De Veer's book,[5] Robbertsz' letter is included, in French. It can also be found, in Dutch, in an annotated version by l'Honoré Naber,[3] which is of a later date than Beke's. Whereas Beke, in his comments, defends De Veer where possible, l'Honoré Naber shares Robbertsz' conclusions.

Robbertsz' letter has influenced the opinion on the truthfulness of De Veer's book more than any other text. We therefore include it in its entirety in Appendix A, in our own translation. Until present, De Veer's journal has been considered rather inaccurate in circles of historians.

Maybe even more remarkable than the widely discussed early appearance of the Sun is the reported observation of a conjunction between Jupiter and the Moon: At the time of this conjunction, Jupiter was not only geometrically below the horizon but also behind the central mountain ridge of the island. This aspect, not to be found in Robbertsz' letter, is discussed in detail by Beke and l'Honoré Naber.

De Veer writes that the conjunction was seen at approximately 6 p.m. local time. Since it was predicted at 1 o'clock Venetian time, they concluded that their longitude was 75° East of Venice. In reality the difference in longitude between the two places is only 46°. This discrepancy has never been explained.

D. Julian or Gregorian Calendar?

l'Honoré Naber suggests that during the time of the observations the winterers lived by the old-style Julian calendar instead of the new-style Gregorian calendar, which had been introduced in 1582. This idea is also mentioned by Beke. The difference was then 10 days, and 24 January old style would have been 3 February new style. This is a possibility that needs serious investigation, since the Sun's reappearance would then have been hardly premature. On the other hand, Jupiter and the Moon would have been 10 days away from their conjunction, making its reported observation an evident falsification.

We believe, however, that the idea of the use of the Julian calendar can be refuted: The large majority of solar declinations that De Veer quotes in connection with observations for latitude appears to have been taken at face value from De Medina's tabulations[18,19] and from those of Barents himself.[20] Conversion from the Julian to the Gregorian day counting has been done in all these cases as shown by Van der Werf.[21]

We add here another, more direct argument: Occasionally De Veer mentions that a certain date is a Sunday, for example, on the journal dates 11 August, 1 September, and 15 September 1596. On the Gregorian calendar this is correct; on the Julian calendar these would have been Wednesdays.

Of particular interest is the fact that on 16 February they celebrated Vastelavont (Fasten's Eve). Not only was this a Sunday in the Gregorian calendar; it also was the Sunday that marked the beginning of the 50 days before Easter. Since it was the last day that the eating of meat was allowed, it was quite common, according to the Catholic Encyclopedia, to celebrate this Sunday with an extensive meal. This Sunday eve is therefore known as Domenica carnevala, or in German as Herrenfest. There is another tradition, the most common today, of celebrating Fasten's Eve on the Tuesday that precedes Ash Wednesday. This has led to the erroneous translation into English of Vastelavont as Shrove Tuesday.[2,5]

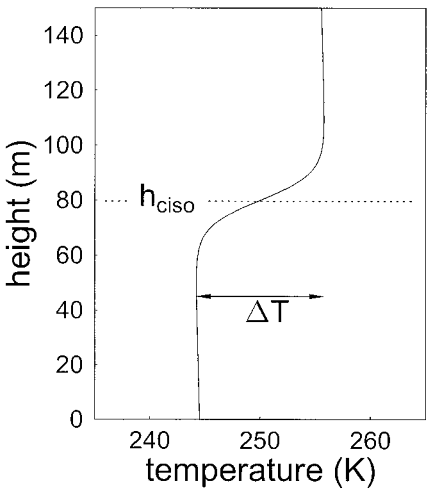

Fig. 2. Lower part of the parameterized temperature profile. h_{ciso} denotes the height of the central isotherm; ΔT, the temperature jump.

We conclude that the Gregorian calendar has been used consistently, also during the winter.

2. Ray-Tracing Analyses

A. Methods

In this section we present ray-tracing analyses of the three key observations: the first observation of the Sun on 24 January 1597; the second on 27 January 1597, when it appeared "in its full roundness, just free of the horizon"; and the observation of the conjunction of Jupiter and the Moon in the early morning of 25 January. For all observations we use the same temperature inversion.

Our scheme is described in more detail in two separate papers,[6,22] and extensions to these are summarized in this paper in Appendix B. The scheme uses the backward-tracing method as described by Lehn,[16,23] in which rays are followed from the eye of the observer to the celestial body from which they originate. We generalize this method by allowing the temperature profile to depend not only on height above the Earth's surface, h, but also on the horizontal distance, x, along it.

Exact ray tracing can be done if the refractive index, $n(h, x)$ is known. The latter is obtained from the temperature profile, and for our present analysis this temperature profile should exhibit a temperature inversion. This is conveniently parametrized by a simple analytical form, borrowed from the theory of the electron gas, where it is known as the Fermi distribution (Fig. 2). The profile is determined by four parameters: T_{ciso}, the temperature of a central isotherm; $h_{ciso}(x)$, its height; $\Delta T(x)$, the temperature difference across the inversion; and the width, or diffuseness-parameter $a(x)$, not indicated in Fig. 2. This last parameter acts as a scaling factor around h_{ciso} and determines the steepness of the inversion: 90% of the jump takes place within a width $\Delta h \simeq 6a$. Further details of the method are deferred to Appendix B.

If an inversion extends over an indefinite horizontal distance without any change in its parameters, it cannot duct light that enters from above. Away from the observer the inversion must become weaker to let the rays (when traced backwards) escape into space towards the Sun. This can be achieved by allowing the temperature jump, ΔT, to decrease gradually with x, or by letting the width parameter, a, increase with x. For the ray-tracing calculations both methods are nearly equivalent. In our present study we have chosen to keep ΔT constant and let a increase.

B. Observations of the Sun

The 1597 observations of the Sun were made towards the south over the frozen Kara Sea at an eye height of 14 m above sea level. We choose the height of the central isotherm, h_{ciso}, to be 80 m, its temperature, T_{ciso}, 250 K and the atmospheric pressure at sea level 1040 hPa. ΔT is kept constant at 12 K. The width parameter, a, is fixed at 5 m from the observer's position ($x = 0$) till $x = 200$ km and is then allowed to increase gradually beyond this distance (Fig. 3A). The inversion is overhead for the observers and is classified as a superior mirage.[6]

It should be mentioned here that the choice of inversion parameters is in no way unique: All that is needed is a sufficiently strong inversion that ensures an oscillatory ray pattern over the required horizontal distance.[6] The inversion that we choose here resembles that of Liljequist's observation in 1951.[10,16]

Light rays are traced up to a height of 85 km. We denote by ALT a light ray's true altitude relative to the true horizontal at the observer's position and by β_0 its apparent altitude as seen by the observer. Their functional relationship, which is the transformation curve between ALT and β_0, is shown in Fig. 3B.

Images of the Sun as they would have appeared to the observers at Het Behouden Huijs are shown in Fig. 4, for various solar altitudes. On 24 January 1597, at noon, the Sun's true altitude was $-5° 26'$, in between images three and four (top row). The NZ effect could have been visible from ~30 min before till 30 min past noon, where it would just be seen as in the first (top left-hand) image. On 27 January, assuming the identical inversion, a flattened image of the Sun, apparently free of the horizon, could have been visible for most of the time between 50 min before till 50 min past noon (images 11–15), except for a short time around noon (last image).

These images fit De Veer's description. One gets a good impression of what they might have seen from Fig. 5, which shows a recent observation of the NZ effect, on 15 November 2001 at Resolute Bay. A strong temperature inversion, extending up to 300 m in height, is seen in the first frame (a). It does not duct the Sun's light to the observer's position and therefore looks like a dark band. Only a diffuse lighter spot indicates the direction of the Sun. De

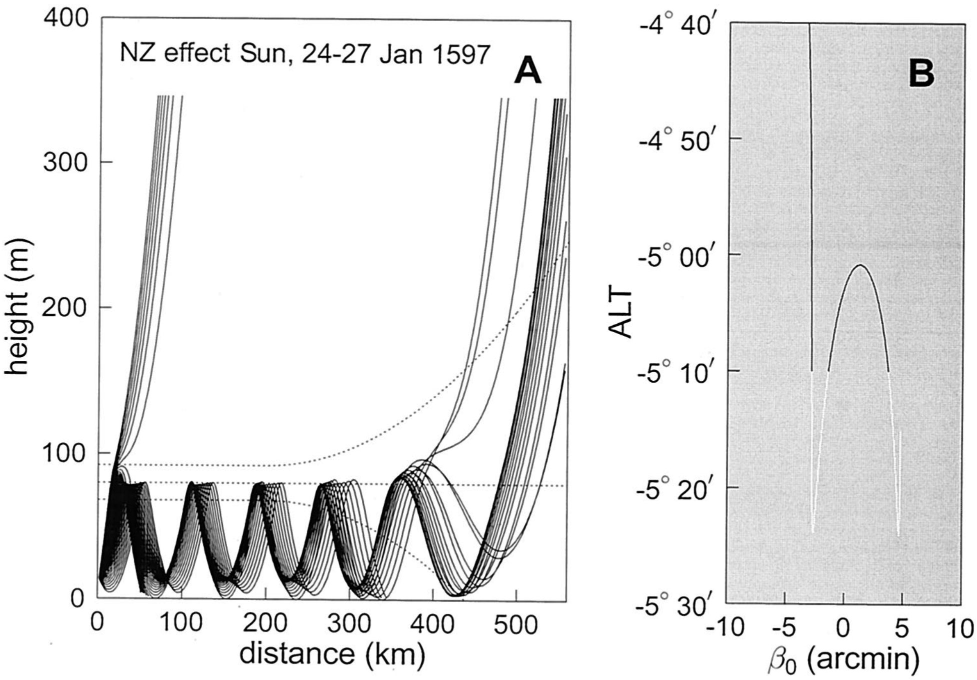

Fig. 3. A, Backward ray-tracing from the observer's position (left) for the Sun (solid curves). Isotherms are shown as dashed curves for 245, 250 (central isotherm), and 255 K. B, The transformation curve between the apparent altitude for the observer, β_0, and the geometrical altitude, ALT, of the celestial source that emits the ray. The image of the Sun, as seen by the observer, is the point-to-point mapping on the β_0 axis of all ALT values subtended by the solar disk. Horizontal angles are unaffected by this transformation. On 24 January, at local noon, the Sun's center had a geometrical altitude of $-5° 26'$ at Het Behouden Huijs. The part of the transformation curve that matches the Sun is indicated in white.

Veer noted this darker band and took it for a haze that prevented them from seeing the Sun on 25 and 26 January 1597. Frame (b) of Fig. 5, taken 36 min later than frame (a), shows that now some of the Sun's rays, within a small range of altitudes, reach the observer directly by ducting through the inversion. A glimpse of the Sun is seen and fits De Veer's description of 24 January 1597.

De Veer's phrasing on 27 January that the Sun was seen "in its full roundness, [its lower limb] just free of the horizon" is first of all meant to emphasize this latter aspect. A strong flattening of the image would not have been alarming. Indeed, it may be shown that the apparent vertical compression of the setting Sun depends strongly on the temperature gradient at the observer's height. At standard atmospheric conditions, $T_0 = 15$ °C and $P_0 = 1013.25$ hPa, this compression is[6]

$$\frac{\text{vert.diam.}}{\text{hor.diam.}} = 0.79 - 6.13\left(\frac{dT}{dh}\right)_0, \qquad (1)$$

which gives a ratio 0.83 for the standard temperature gradient of -0.0065 °C/m. For a not uncommon positive gradient of 0.05 °C/m, the compression is already 0.48; i.e., the vertical diameter appears compressed by more than a factor 2. As sailors, Barents and his crew members were used to the variability of this deformation and would see nothing alarming in it. In fact, the solar flattening is so common to ev-

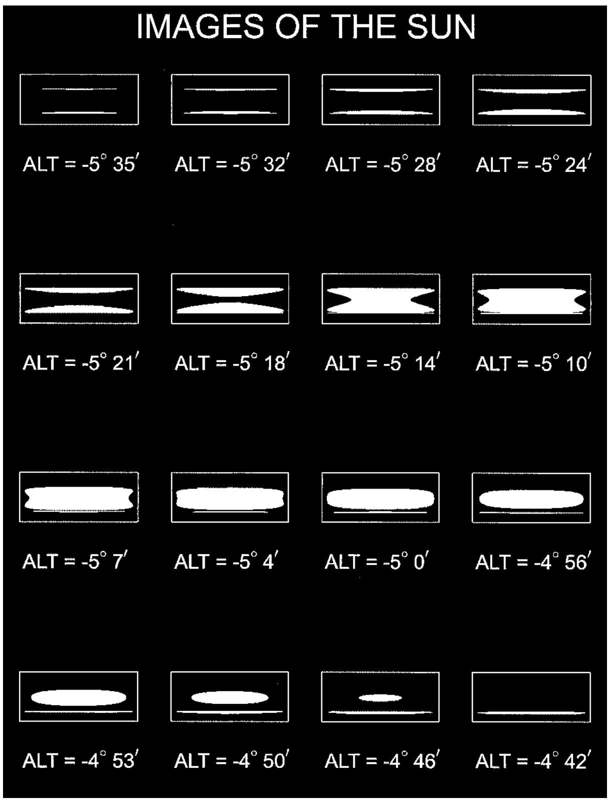

Fig. 4. Images of the Sun, as seen at Het Behouden Huijs, calculated from the β_0 versus ALT transformation curve in Fig. 3B for different solar altitudes. The boxes are 35 × 15 arc min, their bottoms corresponding to the apparent horizon.

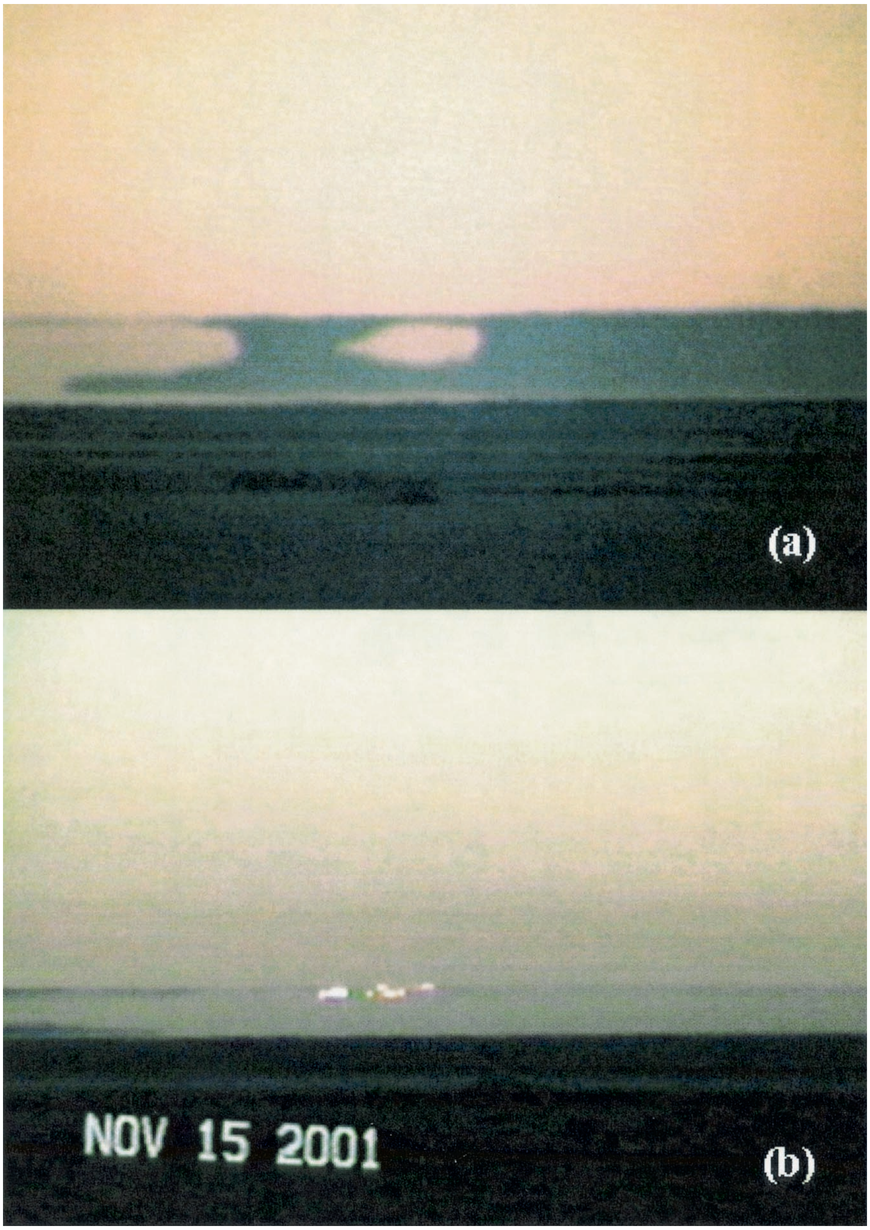

Fig. 5. NZ effect, seen at Resolute Bay (74° 44′ N, 94° 57′ W) on 15 November 2001. Observer's height, 100 mm above sea level. Ground temperature, −27 °C; ground pressure, 1015.5 hPa. Average temperature gradient, 0.069 °C/m up to ~200 m, leveling off to 0.0 °C/m near 300 m. (a) 18:05 UT, local noon. Sun's true altitude, −3° 22′; true bearing, 180° 1′. Photo by Wayne Davidson. (b) 18:41 UT. Sun's true altitude, −3° 34′; true bearing 188° 40′. From a video recording by Julie Crowther.

eryone that the perceived near roundness of the setting Sun is a well-known optical illusion.[24]

C. Moon–Jupiter Conjunction

We now turn to the conjunction between Jupiter and the Moon. Using modern programs on celestial mechanics,[25,26] one finds that the conjunction in ecliptic longitude took place at 00:14 UT (00:50 Venetian solar time, 10 min before the time given by Scala[7]) with azimuthal directions of 347° 28′ for the Moon and 345° 52′ for Jupiter. The Moon's true altitude was 46′, and its light rays are only slightly affected by atmospheric refraction, resulting in an apparent altitude of ~1° 20′. Jupiter's true altitude was, however, −2° 02′, and moreover its light must have passed over the central highland of Novaya Zemlya, in order to have been visible at Het Behouden Huijs.

The terrain is rather featureless and can therefore be considered to act as a temperature shield. Hence the inversion is assumed to have followed the landscape's contours in a smoothed fashion. In our calculation this is modeled by making the height of the central isotherm, h_{ciso}, depend on distance, x, such that, also over the hill, it follows a smooth curve ~80 m above the terrain (Fig. 6). We kept $T_{\text{ciso}} = 250$ K and $\Delta T = 12$ K as before. The width parameter a was, also as before, taken 5 m from the observers' position till the top of the highland ($x = 52$ km) and

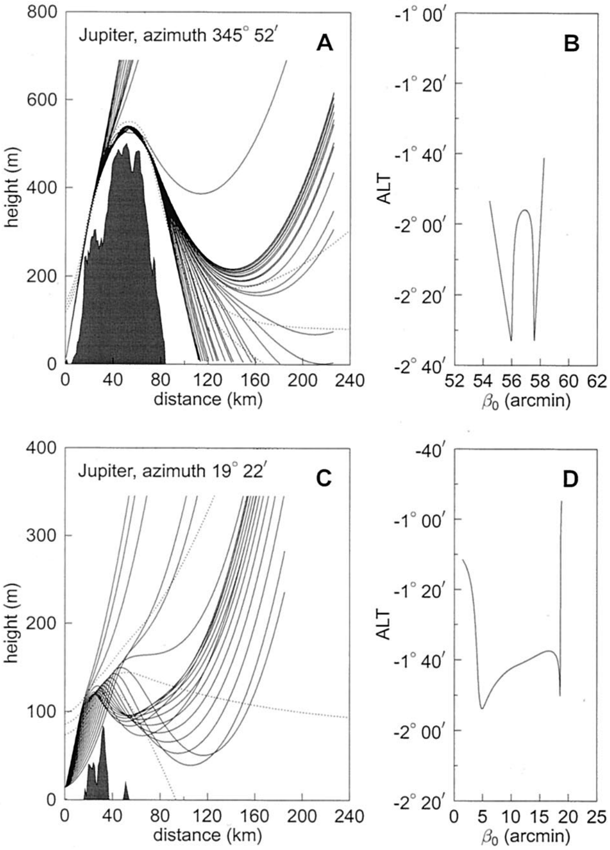

Fig. 6. Ray tracings and β_0 versus ALT transformation curves for Jupiter at the times of the ecliptic conjunction (A, B) and the azimuthal conjunction (C, D). The isotherms (dashed) are for 245, 250, and 255 K. The distances over which the rays are followed are chosen 225 and 185 km, respectively. Jupiter's geometrical altitudes at Het Behouden Huijs are ALT = −2° 1.5′ (A, B), respectively, −1° 39.9′ (C, D). The transformation curves (B, D) are narrow and though Jupiter's images were multiple; they would have appeared as single to the naked eye.

then increased monotonically. This transition at the hilltop and the stronger increase of a than in the case of the Sun can be justified by the increased vertical air mixing that is due to the up-slope component of the wind, which was reported to have been westerly.

Again, the choice of the inversion parameters is by no means unique. Our wish to reproduce with identical parameters all three NZ observations, for the Sun on 24 and 27 January and here, on the 25th, for Jupiter, is self-imposed.

In Figure 6 ray tracings are shown into the direction 345° 52′, where the ecliptic conjunction took place at 00:14 UT, and towards 19° 22′, where the azimuthal conjunction occurred at 02:30 UT. For all intermediate directions the same inversion is found capable of conducting Jupiter's light and making it pass smoothly over the mountain ridge.

Figure 7 shows the view from Het Behouden Huijs at six different times from 00:14 UT, the time of the ecliptic conjunction, till 03:15 UT. The Moon's shadow edge is closely perpendicular to the ecliptic.

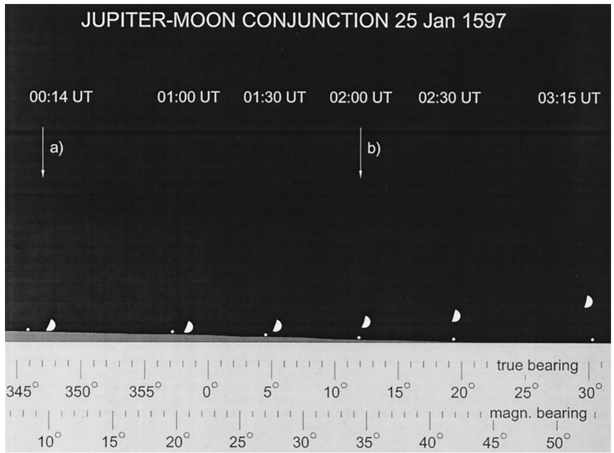

Fig. 7. View from Het Behouden Huijs during the early morning of 25 January 1597. Apparent positions of Jupiter and the Moon; the slight flattening of the latter and also the appearance of the mountain ridge were obtained by ray-tracing calculations for the same temperature inversion. The Moon is shown at twice its real size. The ecliptic conjunction occurred at 00:14 UT, arrow (a), but only around 02:00 UT, arrow (b), could the looked-for alignment be seen. The scales at the bottom give true bearings and magnetic bearings on the ship's compass.

The best practical criterion by which the moment of the ecliptic conjunction could have been identified was the alignment of Jupiter with this nearly straight edge of the Moon, which was close to its first quarter. However, because of the NZ effect, Jupiter's image was lifted much more than that of the Moon. By no criterion can the arrangement at 00:14 UT, shown as the leftmost situation in Fig. 7, be identified with a conjunction of whichever kind. The looked-for alignment would have been seen only at 02:00 UT at 12° East from North (Fig. 7, fourth situation).

3. Discussion

A. Compass Reading

De Veer gives the direction in which the conjunction was seen as North by East, which is one point (11° 15′) East of North. Beke[5] and later l'Honoré Naber[3,4] have assumed that the true direction must have been one point West of North: That is where the ecliptic conjunction took place in reality and at the same time is the direction that, in the presence of the established variation of two points West, would have read North by East on a magnetic compass.

Ironically, it is the coincidental agreement between these two facts that has obscured, for so long, the simple fact that De Veer himself gives the solution: On 8 February he writes "we saw the Sun rise in SSE and set in SSW, well understood on the compass at our house that we had made of lead and which we had adjusted to the proper meridian, else, by our common compass it differed by at least two points." There were *two* compasses. The compass rose, made of lead, is shown in Fig. 8. It was found by the Norwegian captain Elling Carlsen, who discovered the

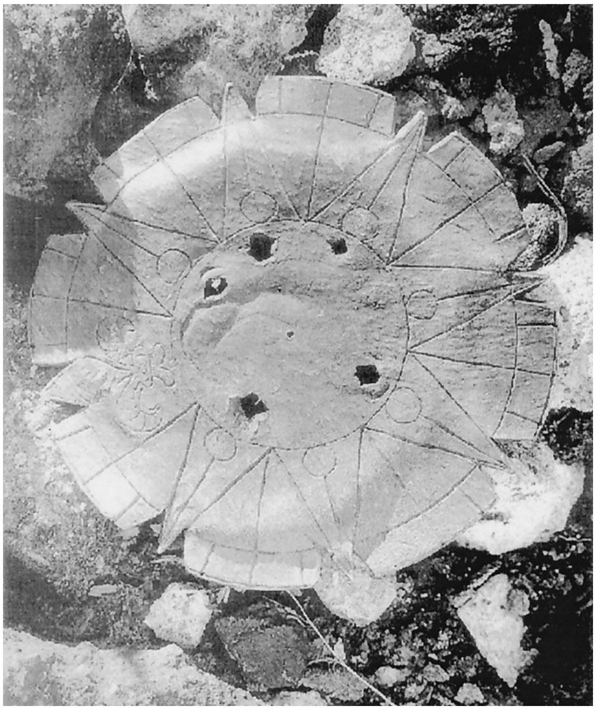

Fig. 8. Leaden compass rose that at Het Behouden Huijs had been adjusted to read true bearings. It is in the collection of the Rijksmuseum, Amsterdam.

remains of Het Behouden Huijs in 1871, and it is now in the collection of the Rijksmuseum of Amsterdam.[27] When De Veer speaks of "the compass at our house," he means this leaden compass. The "common compass" that he mentions is the ship's compass, which had been taken ashore.

De Veer's quote of North by East (11° 15′ East) is therefore a true bearing, and it agrees within 1° of our calculated result.

B. Determination of Longitude

De Veer places the conjunction at approximately 6 o'clock, local time, in the morning, which agrees well with our calculated time of 02:00 UT for the apparent conjunction: At Het Behouden Huijs, local mean time is ahead of UT by 4:33 h, and when we take into account the equation of time of −13 min, this corresponds to a local solar time of 06:20.

The conjunction was found in Scala's book to have occurred at 01:00 Venetian time, and since their solar time appeared to be 5 h ahead of this, it put them at 75° east of Venice. However, the NZ effect had given a distorted image of the relative positions of the Moon and Jupiter and had delayed the apparent conjunction by almost 2 h. Correcting for this delay, which in our calculation is 1 h, 46 min, equivalent to 26°.5, reduces the longitude to 48°.5 east of Venice. This compares well with the real difference in longitude, which is 46°.

Although there is a wide range of parameters that could produce the NZ effect, the outcome of this delay in apparent conjunction is independent of the particular parameter choice: When visible, Jupiter is seen just over the hill, and the Moon's altitude is high enough that its image is hardly affected by the inversion.

4. Summary and Conclusions

We have demonstrated that the three key observations of the Novaya Zemlya (NZ) effect on 24–27 January 1597, as described by Gerrit de Veer, can be explained by a single common type of temperature inversion. Details such as the bare visibility of the Sun on 24 January and its roundish image on the 27th, when its lower limb was seen just above the horizon, are reproduced naturally.

In arctic regions the NZ effect is quite common. The Inuit, for example, are used to the fact that the setting Sun can come in many shapes, whether seen over sea or over land.

The NZ effect can occur even where the view is over a smooth highland, as we demonstrate in this paper. De Veer's observation of Jupiter across the central mountain ridge of Novaya Zemlya remains to this day the only documented observation of this kind.

Along the northern horizon Jupiter could have been visible by the same temperature inversion that we used to simulate the Sun's images. The accompanying changes in the position of Jupiter's image relative to that of the Moon must have delayed the occurrence of the apparent conjunction by almost 2 h, thereby shifting this event from North by West to North by East. Taking this delay into account, the longitude determination at Het Behouden Huys proves to be accurate within 2°.5. This latter conclusion is independent of the detailed choice of parameters.

We conclude that De Veer's description of these "strange and woonderfull" events appears to be "true and perfect" indeed.

Appendix A: Robbert Robbertsz' Letter

The following is our translation of Robbertsz' letter including the introductory text by Joan Blaeu[17]:

. . . therefore I have included the aforementioned letter by Robbert Robbertsz le Canu (a man, knowledgeable in the art of navigation and who, as Master, instructed others therein), written to my late father Willem Jansz. Blaeu; so that the truth-loving Reader would know the true account of the dispute that the mentioned Master had with the persons, who have done and described this voyage themselves; which letter follows:

My dear friend Willem Jansz.,

Because you have asked me that I should put into writing what I remember about the discussions that I had with Jacob Heemskerck, Gerrit de Veer, Jan Cornelisz. Rijp and others of my pupils, who in the year 1596 set sail and returned in the year 1597, not having achieved anything of what was their mission, namely to find a way to the kingdoms of Cathay and China, and who came to see me in the year 1597 in the month of November to tell me about their wonderful *experiences, that, apart from other remarkable things, they lost the Sun on 4 November of the year 1596 and saw the Sun again on 24 January of the year 1597 at the same latitude of 76 degrees, where they had constructed their house on Nova Zembla, and whereof they say that all scholars will have much to deliberate about, and because you have indicated to me that learned mathematicians all over Europe have been moved and stirred by this, I will therefore tell you in short the discussions that I had with Jacob Heemskerck, Gerrit de Veer and others of my pupils, who have joined on this voyage. The discussions that I had with them are the following: Because it had been always day without darkness for them, for longer than ten weeks, and because during that same period of time the skies had not been clear enough to count the revolutions of the Sun, I asked them how they knew that it had been accurately on the 4th of November that they lost the Sun; because at that time the Sun was more than 15 degrees south of the equator; they answered me that they had always had their clocks and sandglasses ready, so that they had been certain to have the time right. I asked them then whether they had ever found their clocks standing, or their sandglasses empty: also I asked them how old the Moon was at the time when they lost the Sun. This they could not tell me, which made me believe that it had not been 4 November. But suppose, I said, that you would have been right and that it was really 4 November, and that during the summer you have not miscounted by a single day, how can you be sure that during the winter, when it was night for more than eleven weeks, you have not miscounted or simply missed a day when you were taking shelter in your house against the severe cold and the snow-drift or because of thunder-storms, not even daring to stick your heads outside for many days, and could see nor Sun, nor Moon, nor stars. Gerrit de Veer answered me and said that they could see the Pole Star through their chimney and so could keep track of the revolutions of the Guards* [Kochab and Pherkad in the Little Bear] *around the Pole; besides they had their clocks and their sandglasses, which they carefully had looked after all days* (*as he, Gerrit de Veer, put it*). *I left the matter there, being convinced in no way, as they had been very much occupied defending themselves against the bears in the summer, as they said, and during the winter often with trapping foxes, so that, in my opinion, they could not always have found the time for a proper observation of the celestial bodies, nor for the necessary care of their clocks and sandglasses, but that they often must have found their clocks frozen and their sandglasses empty. Do you think then, Mr. Robbert, asked Jacob Heemskerck upon these and similar arguments, that we have been very much off in our time-keeping? This is not only my belief, I answered, but my strong conviction, that the uncertainty in the time was so big that you could not be sure whether it was the end of January or the beginning of February: for, although I asked them where or in which direction on the compass they had seen the Moon, the planets and the stars on the 24th of January* (*on which day they said to have seen the Sun*)*, either at six o'clock in the afternoon, at midnight or at six o'clock the next morning, or at any other time and whether they had taken their heights, they could not answer any of my questions, because they had not made such observations at that time. Hence, I concluded that they could well have been off in their time-keeping by ten or eleven days, or more. The day after they came again to see me and could now tell me where the Moon had been on 24 January 1597. But I answered them: This you must have looked up and calculated from some clever ephemerides or almanacs: but yesterday, when I asked you this question, you could not give me an answer.*

Gerrit de Veer, who is the author of the Navigation to the North, had many more, equally unfounded, disputes with me, which I intended to write down here, but then, finding them unnecesary, have scratched out, because he remained of the same unbending opinion and has put these matters in a different letter type, fol. 34 and 35, in order for it to stand out, as you can see in his book, printed by Cornelis Claesz. op 't Water in Amsterdam in the year 1598, wherein he writes that he wants to give account of these matters. But I remember well the account that Gerrit de Veer has given to Martinus Everardus Bruggensius, of Leyden, who had requested such account of his writings; for he himself came to me with that letter and gave it to me to read, asking me what would be best to do. I answered him that there would be no better advice than to confess guilt and to admit that he and his companions could easily have been off by a few days in the long summer and could well have overslept a few days during the long winter-night, because of the great cold, snow-drive and bad weather. But no, he had not published his Journal to later correct it, but has, without proof or reason, maintained its correctness till the end of his days. And he, Gerrit de Veer, has managed to fill in his Journal the 56 days of the period between 24 January and 21 March, in which he writes that the Sun climbed from their horizon to only 14 degrees above their horizon, fol. 39, which should have been more than 19 degrees in that same time of 56 days; from this I come to the conclusion that Gerrit de Veer has squeezed in 13 or 14 days too many between 24 January and 21 March: on which days he has described (*to support his opinion*) *weather and wind, but has given no declinations: whence I stay by my conclusion, that during the long and cold winter-night of eleven weeks, they must have overslept a few days and had the date wrong, so that it could have been 6 or 7 February, when, due to their prolonged sleep, they thought it was only 24 January; which days he then put back in between 24 January and 21 March, in order to triumph with their observations and so to abuse all learned scholars and make them dispute the adventures of Gerrit de Veer. I leave it to anyone to believe of these matters what he likes; but I believe that Gerrit de Veer is like the sacristan, whose clock was off by at least an hour from what the Sun did show: and when some wise people asked him about this, his an-*

swer was: the Sun may lie, but my clock does not. Likewise, I understand that Gerrit de Veer would rather blame the Sun, the Moon and the stars, than to ever in his life admit that he could have been wrong in his reckonings or could have made any mistake in counting the days. This is, in short, my answer to your request; for, I have never believed and can not believe it today, that one could lose the Sun below the horizon on 4 November, whichever the latitude, when she is more than 15 degrees below the equator, and, being at the same latitude, see the Sun reappear above the horizon, when she is more than 19 degrees south of it; and to find her again only 14 degrees above the horizon on 21 March, as Gerrit de Veer writes in his Journal, fol. 39. It is contrary to nature and reason, in my view: therefore I believe that they have missed some revolutions of the Sun during the long daylight of the summer and have overslept in the long winter-night and were mistaken in the time and have not been able to keep their Journal as those who can, morning and evening, distinguish day and night every 24 hours, enabling them to accurately count their days, which these seafarers of the North could not have done in this way, and which one may also forgive them.

With this I end, with a wish for God's grace. Anno 1627, 15 September.

Appendix B: Mathematical Details

The curvature, $1/r$, of a light ray in the atmosphere is proportional to the gradient of the logarithm of the refractive index, n. When this depends not only on height, h, but also on x, the distance along the Earth's surface, it reads as follows[6]:

$$\frac{1}{r} = \frac{1}{n(h, x)} \left[\cos(\beta) \frac{\partial n(h, x)}{\partial h} - \sin(\beta) \frac{\partial n(h, x)}{\partial x} \right], \tag{B1}$$

where r is the light ray's radius and β is the tilt of the ray relative to the local horizontal. The curve is concave relative to the Earth's center when $r < 0$, convex when $r > 0$.

In polar coordinates (R, ϕ) any curve obeys[6,22]

$$\frac{dR}{d\phi} = R \tan(\beta), \tag{B2}$$

$$\frac{d\beta}{d\phi} = 1 + \frac{1}{r} \frac{R}{\cos(\beta)}. \tag{B3}$$

Here β is the complement of the angle between the position vector and the curve in the point (R, ϕ). In our case, the origin is the center of the Earth and the curve is the path of the light ray. β is therefore the tilt angle of the ray, relative to the local horizontal, just as defined above.

Equations (B2) and (B3) form a system of two coupled first-order differential equations for R and β with ϕ as independent variable. It is amenable in this form to numerical integration, e.g., by the fourth-order Runge–Kutta method.

The radius of curvature, $r = r(R, \beta, \phi)$, follows Eq. (B1) from the index of refraction, which is given by[6,16]

$$n(h, x) = 1 + \frac{AP(0, x)}{T(h, x)} \exp\left[-B \int_0^h \frac{g(h')}{g(0)} \frac{dh'}{T(h', x)} \right]. \tag{B4}$$

Here $T(h, x)$ is the temperature profile, $P(0, x)$ the atmospheric pressure at sea level, and $g(h)$ the gravitational acceleration at height h. $A = 7.87686$ 10^{-5} °C/hPa for yellow light with a wavelength of 580 nm, and $B = 3.4177$ 10^{-2} °C/m.

For the temperature profile we adopt the modified US1976 standard atmosphere, MUSA76,[6,22] to which a temperature inversion is added. In the troposphere this temperature profile reads as

$$T(h, x) = T_{\text{ciso}} - 0.0065[h - h_{\text{ciso}}(x)] - \Delta T(x) + \frac{\Delta T(x)}{1 + \exp\{-[h - h_{\text{ciso}}(x)]/a(x)\}}. \tag{B5}$$

Here, with reference to Fig. 2, T_{ciso} is the temperature of the central isotherm and $h_{\text{ciso}}(x)$ its height. $\Delta T(x)$ is the temperature jump across the inversion, and the diffuseness parameter $a(x)$ determines the width of the jump. Heights are expressed in meters. The explicit dependence of h_{ciso} on x enables one to make the inversion follow the height profile of the terrain. This feature is used in our analysis of the Moon–Jupiter conjunction.

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