The Scoresby ship mirage of 1822

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ABSTRACT. A very clear mirage observed by Scoresby in the Greenland Sea shows an inverted ship floating above the horizon. This mirage can be mathematically reconstructed using a linear image diagram. Scoresby's description is here re-examined: a new set of essential assumptions is distilled from his report, and an 'exact' reproduction of the mirage is obtained to match these conditions.

Contents

Introduction

In the early 19th century, William Scoresby made numerous mirage observations in the Greenland Sea. Many of these presented fantastic appearances, such as towering coastal mountains (Scoresby 1823; Scoresby-Jackson 1861). Distinct from these, and quite unique, is one image of a ship floating inverted above the horizon. The image stands out because of its clarity and lack of distortion (Scoresby-Jackson 1861).

Rees (1988a) presented a reproduction of this image and an extract from Scoresby's description, and proposed an atmospheric temperature profile which would generate this mirage. To accomplish this he had to make certain assumptions about the observation based upon numerical values scaled from Scoresby's sketch. The purpose of the present paper is a re-examination of this mirage, based on revised assumptions about the mirage. Two alternative hypotheses are presented.

Scoresby's observation

The description of the inverted ship that Scoresby saw in 1822 is repeated here for completeness. The engraving that he published is also repeated (Fig. 1.).

> The most extraordinary effect of this state of the atmosphere, however, was the distinct inverted image of a ship in the clear sky, over the middle of a large bay or inlet, the ship itself being entirely beyond the horizon. Appearances of this kind I have before noticed, but the peculiarities of this were, the perfection of the image, and the great distance of the vessel that it represented. It was so extremely well defined, that when examined with a telescope, by Dolland, I could distinguish every sail, the general 'rig' of the ship, and its particular character; insomuch that I confidently pronounced it to be my father's ship, the *Fame*, which it afterwards proved to be, though, on comparing notes with my father, I found that our relative position at the time gave our distance from one another as very nearly thirty miles, being about seventeen miles beyond the horizon, and some leagues beyond the limit of direct vision.

A reconstruction of the mirage requires data on the dimensions of the observed ship. The image shows the ship rigged as a barque, one of the commonest rigs in the 19th century (Middendorf 1903). We will assume it to be relatively small, because Scoresby's report implies that it was privately owned. It would be smaller than the 19th century full-rigged frigates, whose length along the deck often exceeded 50 m (Landström 1961). For Nansen's ship *Fram*, built around 1890, the dimensions are (Nansen 1897):

h = mainmast height above the water line = 32 m;

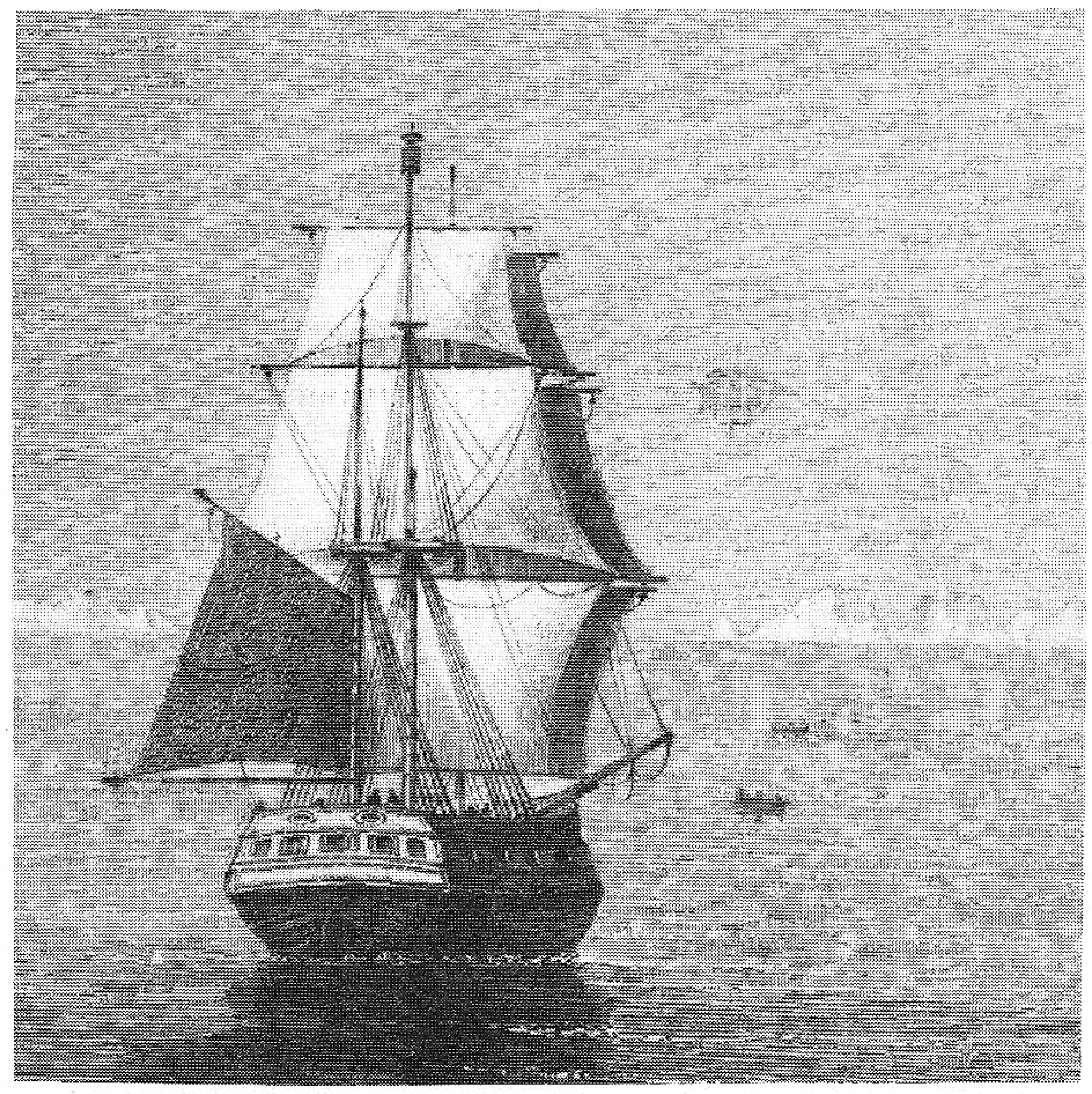

Fig. 1. Scoresby's drawing of the inverted ship mirage, as published in Scoresby-Jackson (1861).

l = length along deck (stem to stern) = 39 m;

r = ratio h/l = 0.82.

These dimensions are very similar to those of HMS *Bounty* and of James Cook's *Endeavour*, both 18th century ships (Haidle 1980). In this paper we will use h = 30 m and r = 0.82 as representing typical values. Small changes in these values will not change the basic nature of our deductions.

A fundamental feature of Scoresby's observation is the lack of distortion (with the possible exception of scaling) in the inverted image. Through his telescope he was able to identify the ship clearly. This lack of distortion requires an image diagram (ie a plot of the elevation angle of light rays at the eye against the actual height from which the rays leave the object, at a fixed distance from the observer). The image diagram, which is linear, is also called the transfer characteristic (Lehn 1978) and the transfer mapping (Tape 1985).

Rees (1990) developed a technique for the direct solution of mirages with linear image diagrams. The method requires as inputs the magnification of the image, the object distance, and the height of the observer's eye. It yields several possible solutions, in all of which the refractivity r (defined as n-1, where n is the atmospheric refractive index) varies quadratically with height. An appendix to this paper discusses the relationship between refractive index and temperature. Kropla (1988) showed that a linear image diagram and quadratic refractivity profile arise when the geometric surface defined by the path length equation has constant Gaussian curvature (see Appendix).

Image reconstruction 1

The Rees (1990) method was applied with the following assumptions:

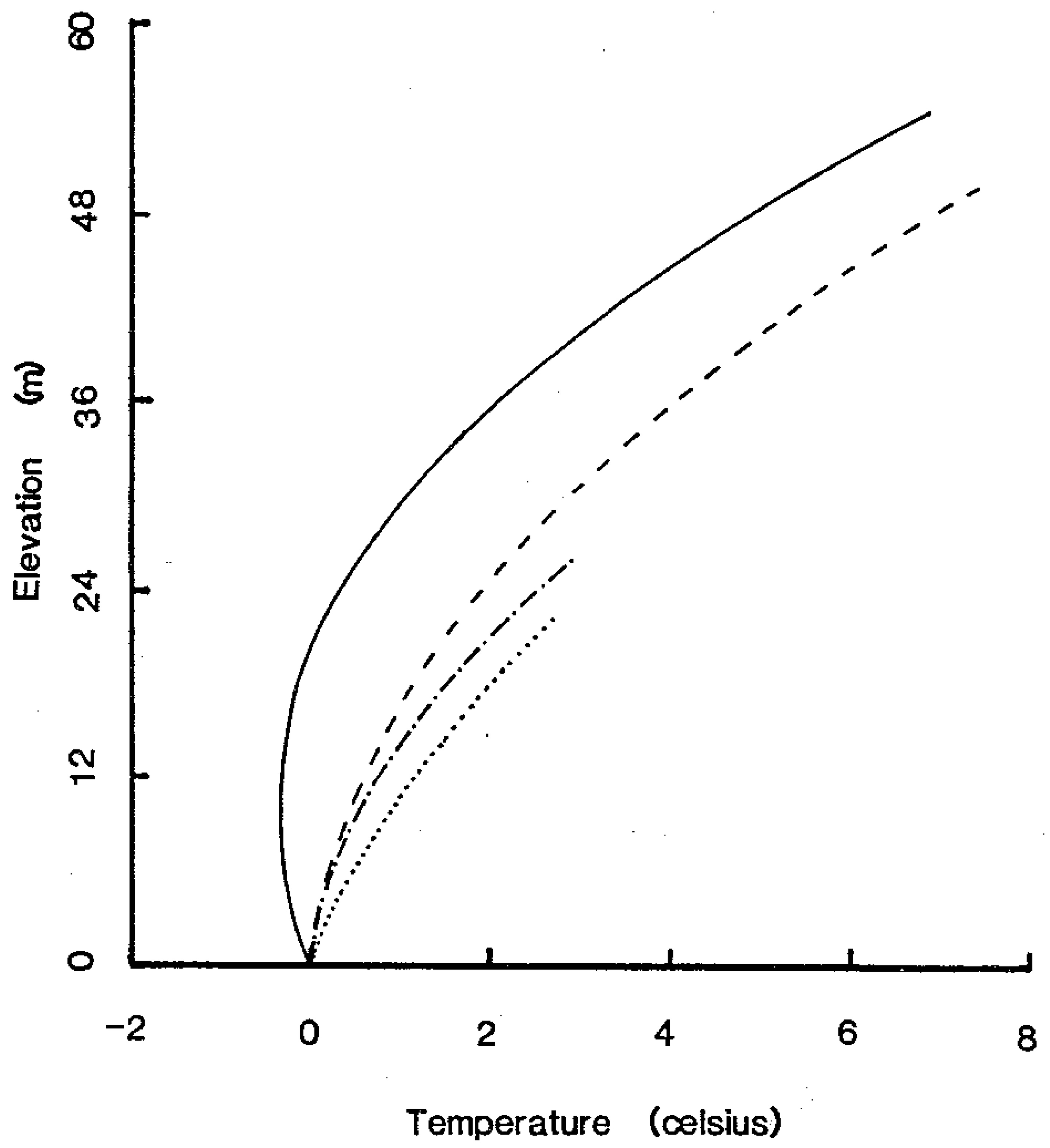

Fig. 2. Temperature profiles for reconstruction 1. The solid line represents the solution analysed in the text.

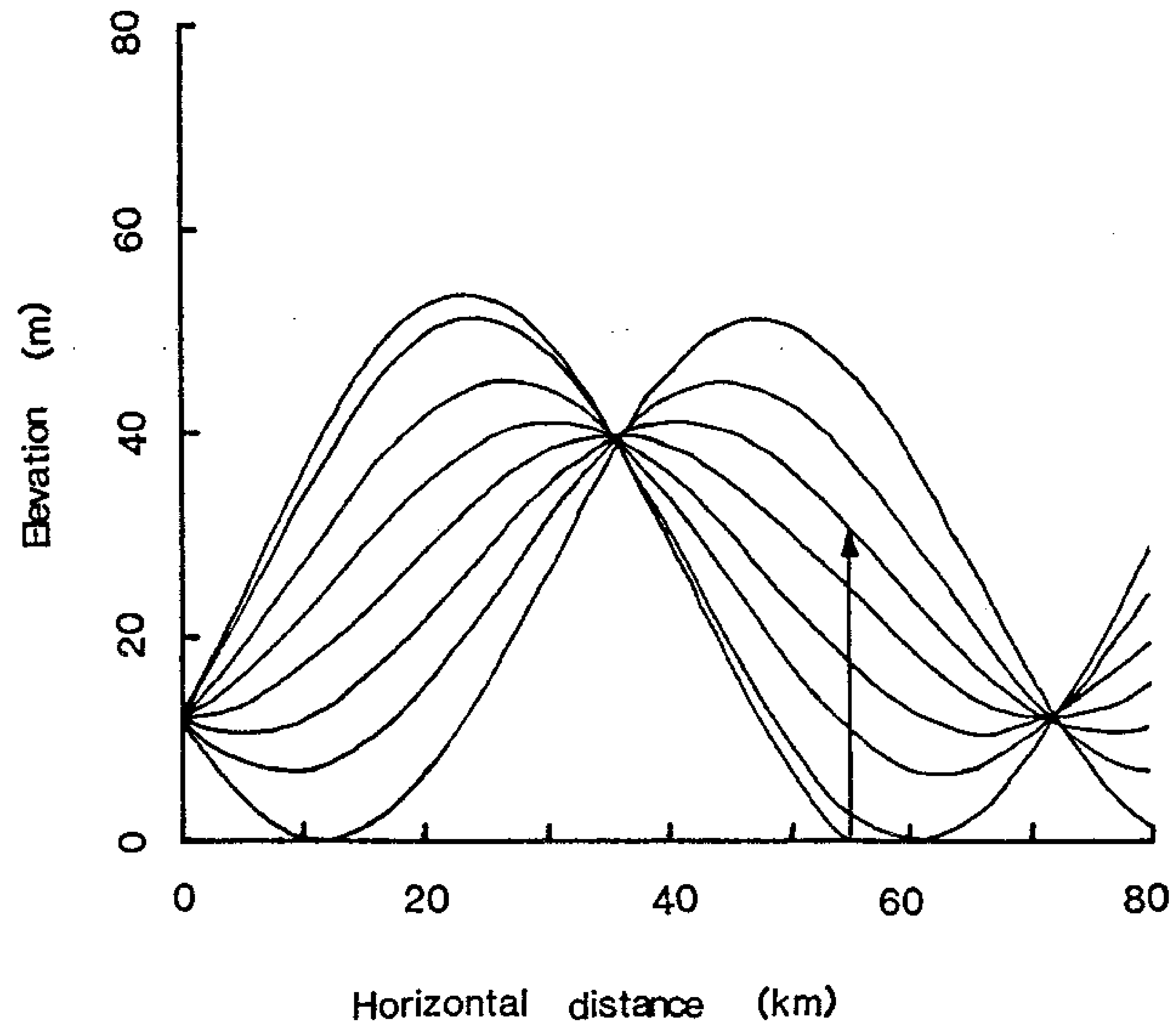

Fig. 3. Light rays for reconstruction 1. The observer's eye has an elevation of 12 m. Starting with the lowest ray, the sequence of elevation angles at the eye, in arcminutes, is -6.5, -4, -2, 0, 2, 4, 6.5, 7.25. The vertical arrow represents a ship of height 30 m at a distance of 55 km.

magnification -5

object distance 55 km (Scoresby states 'over 30 miles': one nautical mile is 1.85 km)

eye elevation 12 m (estimated from perspective of the drawing in Rees 1988b)

With these parameters there are four solutions, shown as temperature profiles in Figure 2. The one that comes closest to the observation, and incidentally extends to the greatest height in the atmosphere, has a temperature profile given by:

$$T = -0.0659\ z + 0.00321\ z^2 + 6.67 \times 10^{-6}\ z^3$$

where T is in °C and z in metres. The atmosphere is considered to be horizontally uniform, and to vary only with vertical direction, which is the usual assumption in simulating mirages.

This profile produces light rays which follow the trajectories shown in Figure 3. The ray calculation accounts for the earth's curvature; the earth is shown flat purely for graphical convenience (Lehn 1985). The vertical line at 55 km distance represents the observed ship. The local horizon has an elevation of -6.5 arcminutes and is situated 11.7 km from the observer. The ship's waterline is imaged at an elevation of +7.5 arcminutes. Rays which reach the eye at angles above +7.5 arcminutes are assumed to curve upwards, so that the temperature profile reverts to that of a more normal atmosphere above 54 m.

The calculated image, drawn to scale, is shown in Figure 4. The dark band surrounding the image of the ship's hull is an image of the sea's surface. It is produced by rays arriving at elevations between +6.5 and +7.25 arcminutes, having originated at the surface rather than from further away in the atmosphere.

Rays following paths as in Figure 3 are said to be

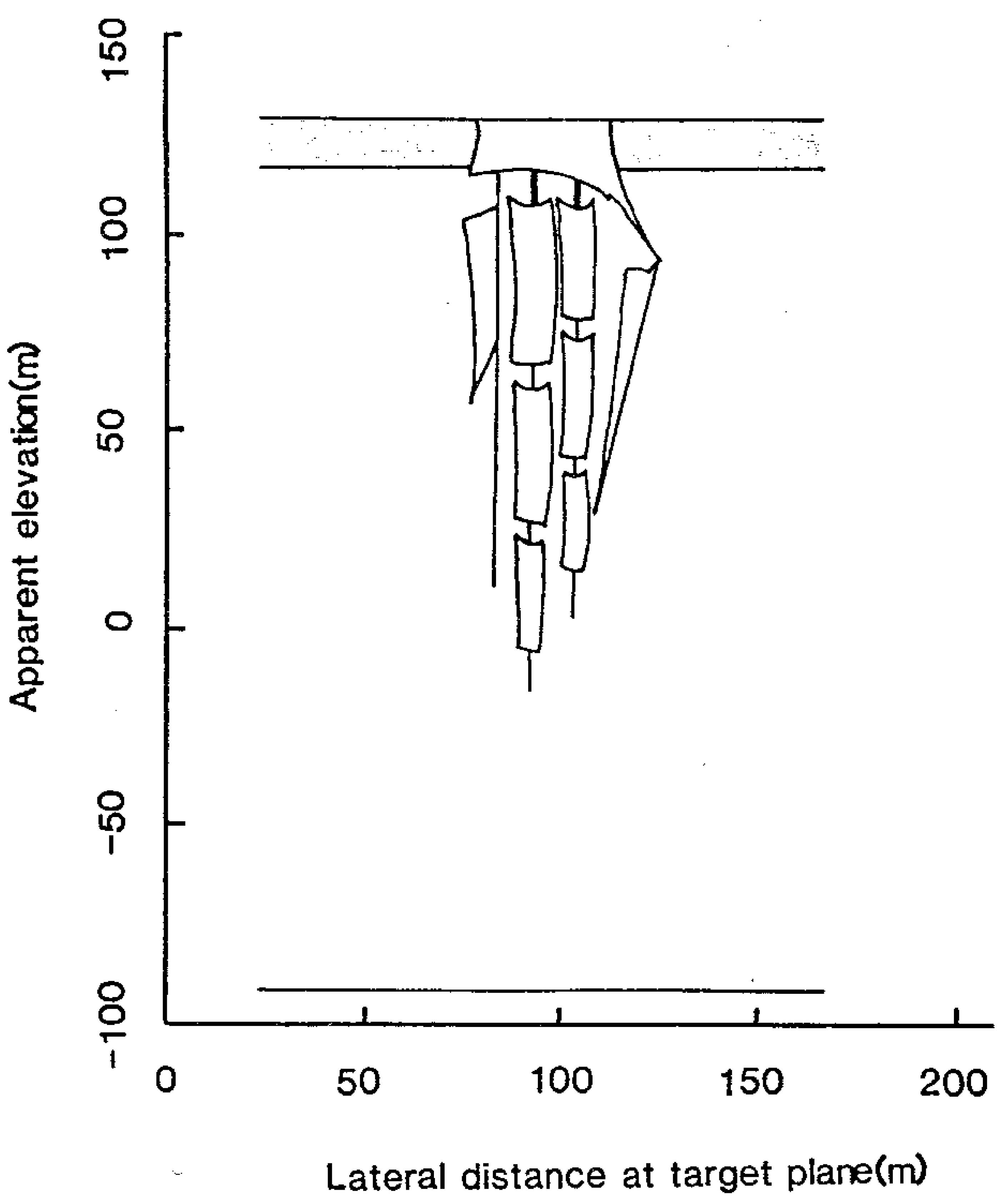

Fig. 4. Correctly scaled image seen with the temperature profile of reconstruction 1. The horizontal band on either side of the ship's hull is an image of the sea beyond the ship.

ducted. They are trapped between the earth's surface and the top of a temperature inversion, and propagate endlessly (if we ignore atmospheric extinction) in an oscillatory fashion (Lehn 1979, Rees 1988a). The periodic focusing effect is a basic property of refractive profiles characterised by constant Gaussian curvature, ie those with linear image diagrams (Rees 1990). An observer looking into such a duct is looking into a narrow horizontally-extended window within which he can see very remote objects, far beyond the normal horizon.

Figure 4 deviates from Scoresby's observation in two respects. The first, relatively minor, is the strip of sea visible behind the hull. A careful observer like Scoresby would probably have seen this had it been present. It can in any case be eliminated by minor adjustment of the atmospheric profile to arrange that the ship is situated on the edge of the duct, where the steepest ray returning from the inversion is tangent to the sea surface. The major discrepancy, however, is the magnification, which acts only in the vertical direction. At 5:1 the disproportion between horizontal and vertical dimensions is so strong that Scoresby would have found it very obvious, and he would surely have mentioned it in the text even if his engraver did not reproduce it.

Image reconstruction 2

We may apply a slightly different set of assumptions to the observation, this time not insisting on a strictly linear image diagram. It suffices that the image diagram be linear over the range of elevations occupied by the ship. Outside this region it can be quite arbitrary, except that no extraneous objects must appear in the mirage.

Again we will assume Scoresby to have been an experienced and accurate observer. To the reader with knowledge of his work in the Arctic this is very clear from the descriptions in his writings (eg the previously cited Greenland mirage, or his account of the approach to Spitzbergen (Scoresby 1969)). We thus attempt an 'exact' matching of his description of the mirage. Our assumptions are as follows:

- Distance 55 km
- Apparent height-to-width ratio of the image 1.23, as shown in Figure 1.
- No distortion of the image.
- Some magnification of the image.
- Image fills 29% of the space between the horizon and the top of the mirage.
- No image of a strip of sea exists around the image of the hull.

We reject the foreground image of Scoresby's own ship, assuming that he was on his ship when he saw and sketched the mirage, and we leave open the question of the elevation of his own eye, though it would clearly be most plausible if this could be such that he was standing on the deck. The dimensions of the ship are as assumed previously.

With these assumptions, we can immediately conclude that the magnification of the mirage is -1.23/0.82 = -1.5. In a normal atmosphere, a ship of height 30 m subtends 1.88 arcminutes at a distance of 55 km, so in the magnified image it will subtend 1.5 x 1.88 = 2.8 arcminutes. The ship's image occupies 29% of the total mirage height, so

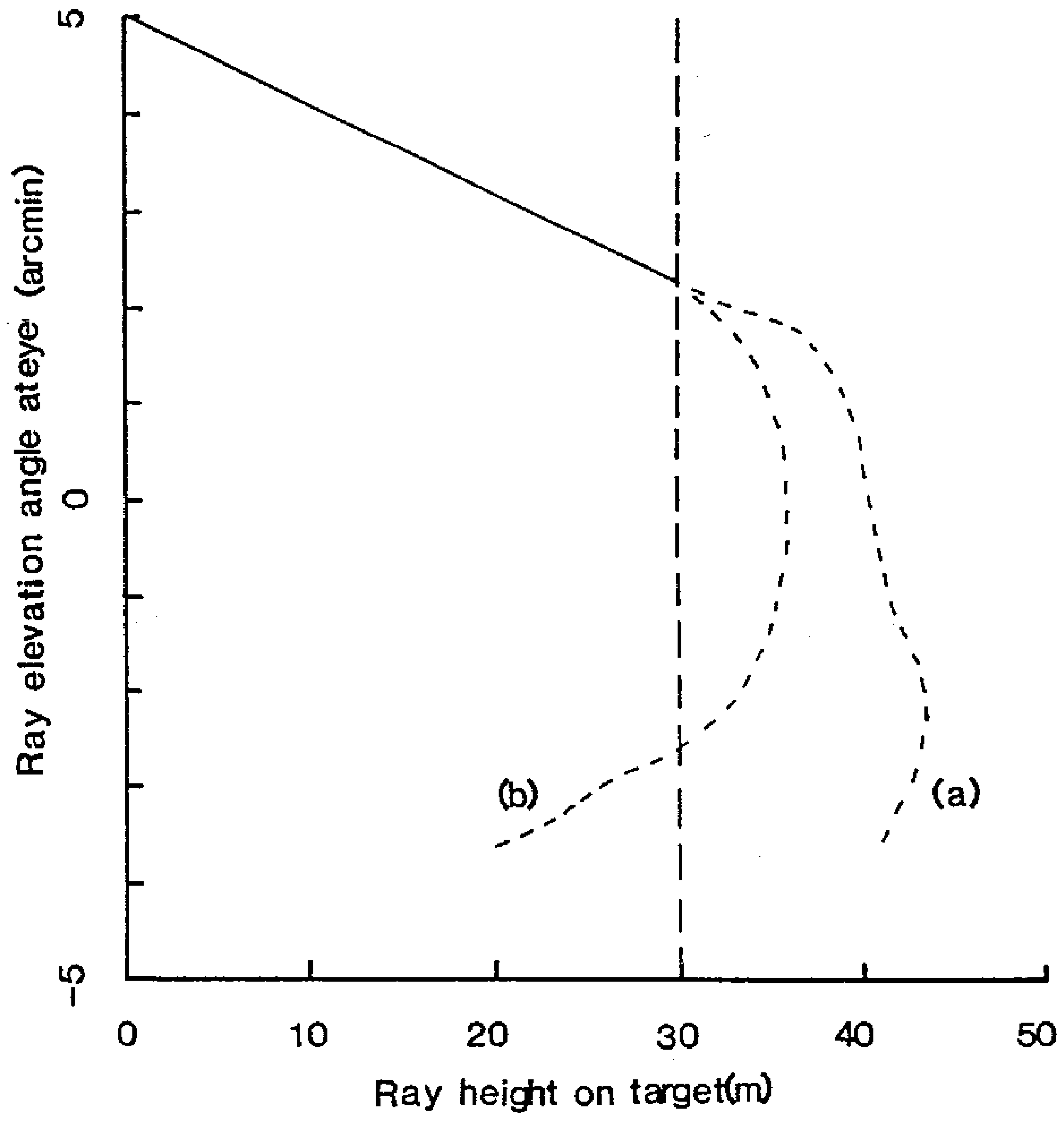

Fig. 5. The solid line represents the portion of the image diagram required for reconstruction 2. For ray elevation angles less than 2.2 arcminutes, the curve can take on any shape (a) which does not recross the height 30 m. The curve (b) is thus not permitted, since it produces an extra image of the ship near the horizon.

with very little error we can set the latter at 10 arcminutes. This dimension is typical of polar superior mirages, whose heights are commonly observed to be between 5 and 15 arcminutes. Further, if we may assume a duct model of the general form shown in Figure 3, we can see that the highest and lowest rays reaching the observer's eye from the duct must be symmetrical about the horizontal plane, so that they must be at +5 and -5 arcminutes respectively. The ray at -5 arcminutes forms the lower bound to the duct and is therefore tangent to the sea nearby. Rays which reach the eye at angles below this will form the foreground sea image.

These considerations provide a fairly complete specification of the required ray trajectories, and thus of the temperature profile. The image diagram is shown in Figure 5, and a direct synthesis process (based on Lehn 1983) was applied to generate a profile to meet these specifications. The temperature profile was first selected for the layers below eye level, to ensure that the ray reaching the eye at -5 arcminutes would not intersect the horizon. Then the profile above eye level was adjusted (by varying its steepness) to force higher rays to originate from the ship at the required heights. Again, only the rays between +2.2 and +5 arcminutes were important: other rays could go where they wished. A heuristic smoothness constraint was applied to the profile in an attempt to keep it fairly realistic.

The solution presented here is calculated for an eye height of 8.35 m. Other heights were also investigated and will be discussed later, but this value has the merit that the horizon occurs at -5 arcminutes quite naturally if the temperature gradient below the eye has the value -0.006 K/m characteristic of the standard atmosphere. We have arbitrarily but reasonably assumed that the air temperature at sea level is 0°C. The solution is relatively insensitive to this assumption.

Figure 6 shows the temperature profile found by this approach to reproduce Scoresby's drawing exactly. This solution is obviously not unique, since variations in the profile which affect the image diagram for rays below +2.2 arcminutes are permitted as long as they do not affect rays between +2.2 and +5 arcminutes. Figure 7 shows the corresponding image diagram, and Figure 8 shows the appearance of the image (cf Figure 1).

It is interesting that this profile can be approximated quite well by two segments each having parabolic form (constant Gaussian curvature). The small circles in Figure 6 show this, and the small circles in Figure 7 show the corresponding image diagram to be almost the same as the exact solution.

Some remarks on other eye heights follow. There was no difficulty in producing solutions for eye heights above 8 m; for example, 12 m was fitted easily. Below 7 m, however, we experienced a difficulty which can be exemplified by considering an eye height of 4 m. At this level, in order for the ray which reaches the eye at -5 arcminutes to be tangent to the sea, the sea-level temperature must exceed that at the eye by 0.563°C. A temperature profile can be found which reproduces the required image diagram between +2.2 and +5 arcminutes, but the image diagram follows the form of Figure 5b to produce an extra image of the ship. No realistic modification of the temperature profile below eye height was able to circumvent this problem.

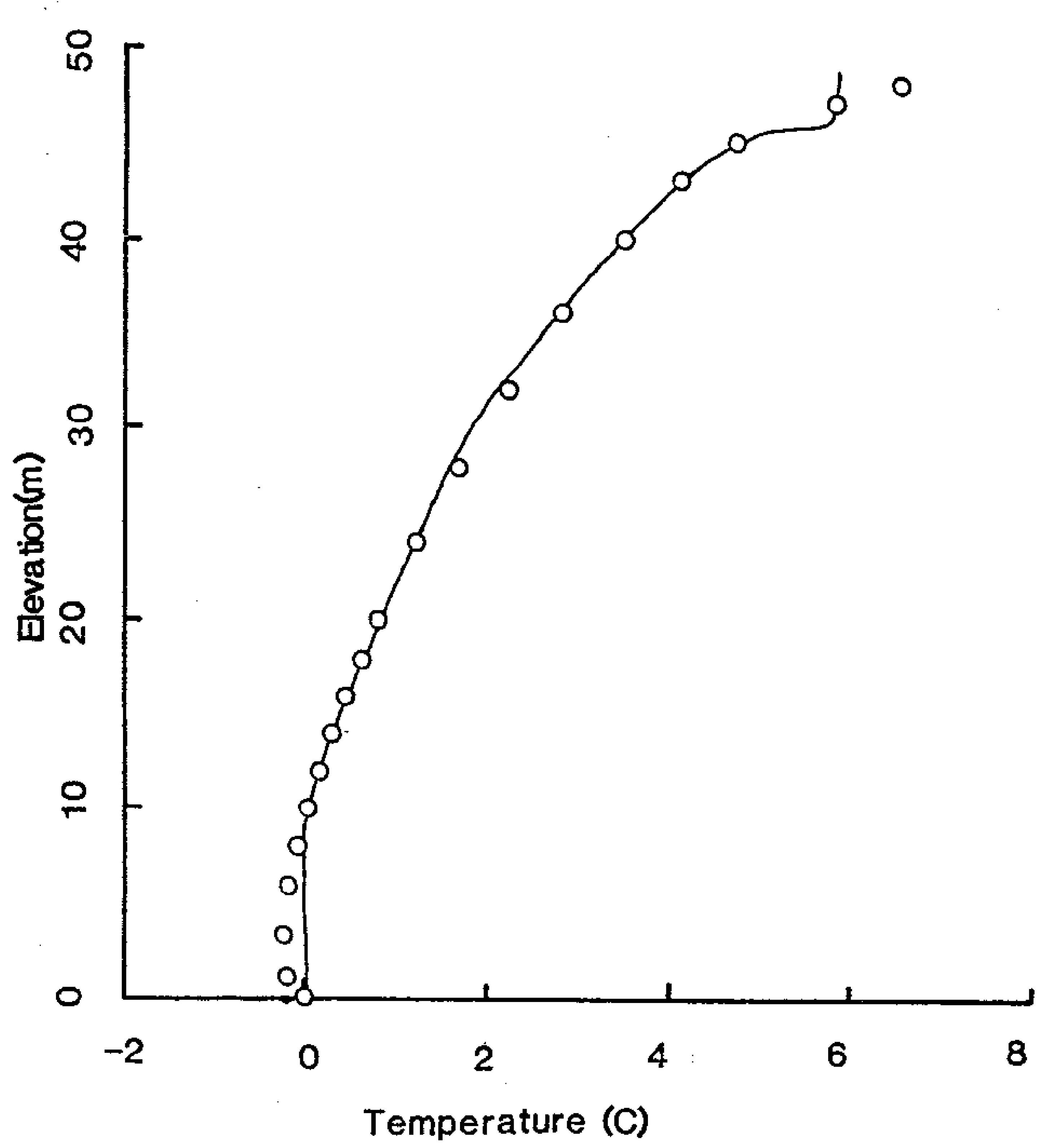

Fig. 6. Temperature profiles for reconstruction 2. The solid line is the 'exact' solution, while the small circles represent a close approximation made up of two portions with constant Gaussian curvature.

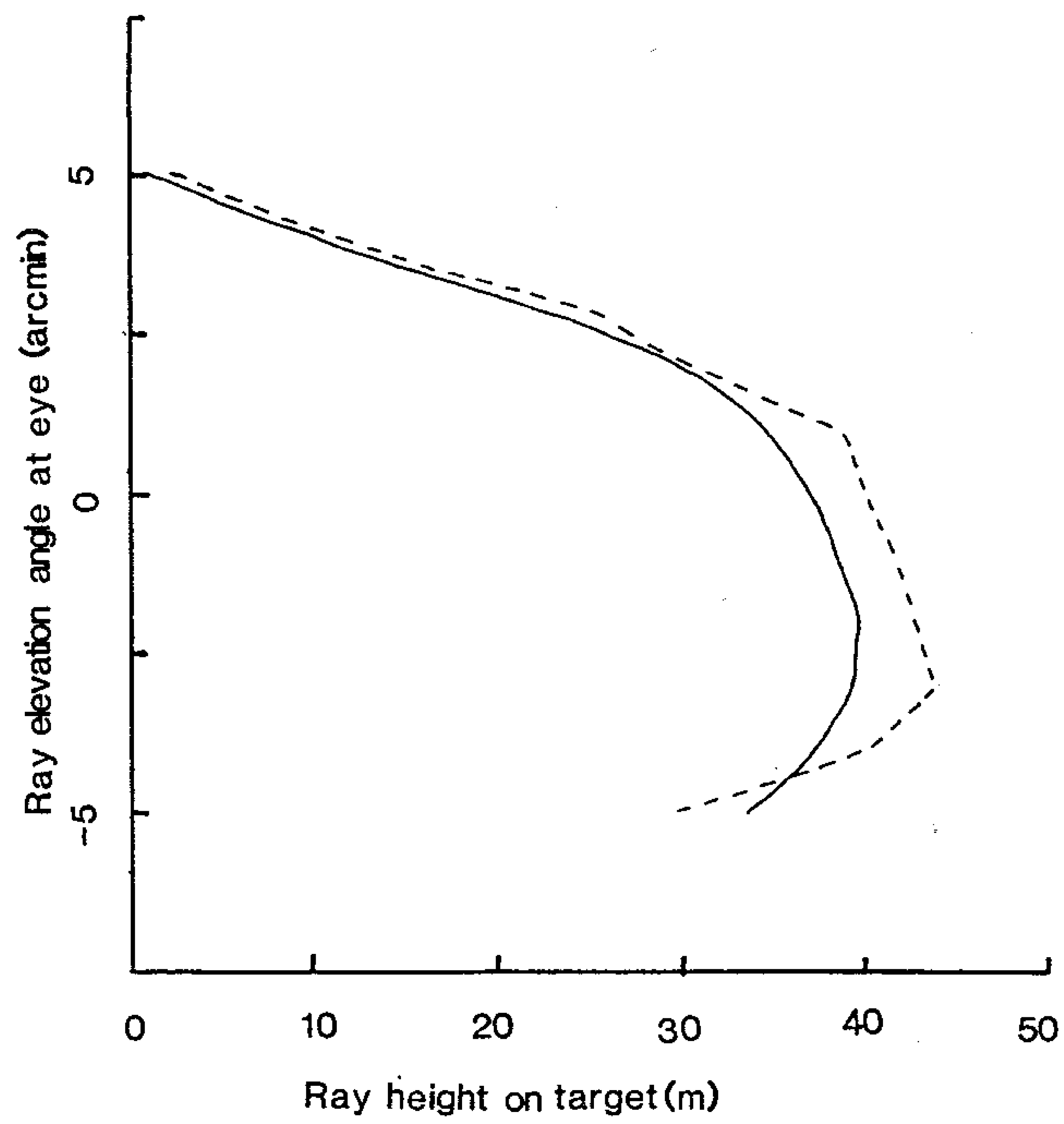

Fig. 7. Image diagrams for reconstruction 2. Solid line: 'exact' solution; broken line: approximation as two portions with constant Gaussian curvature.

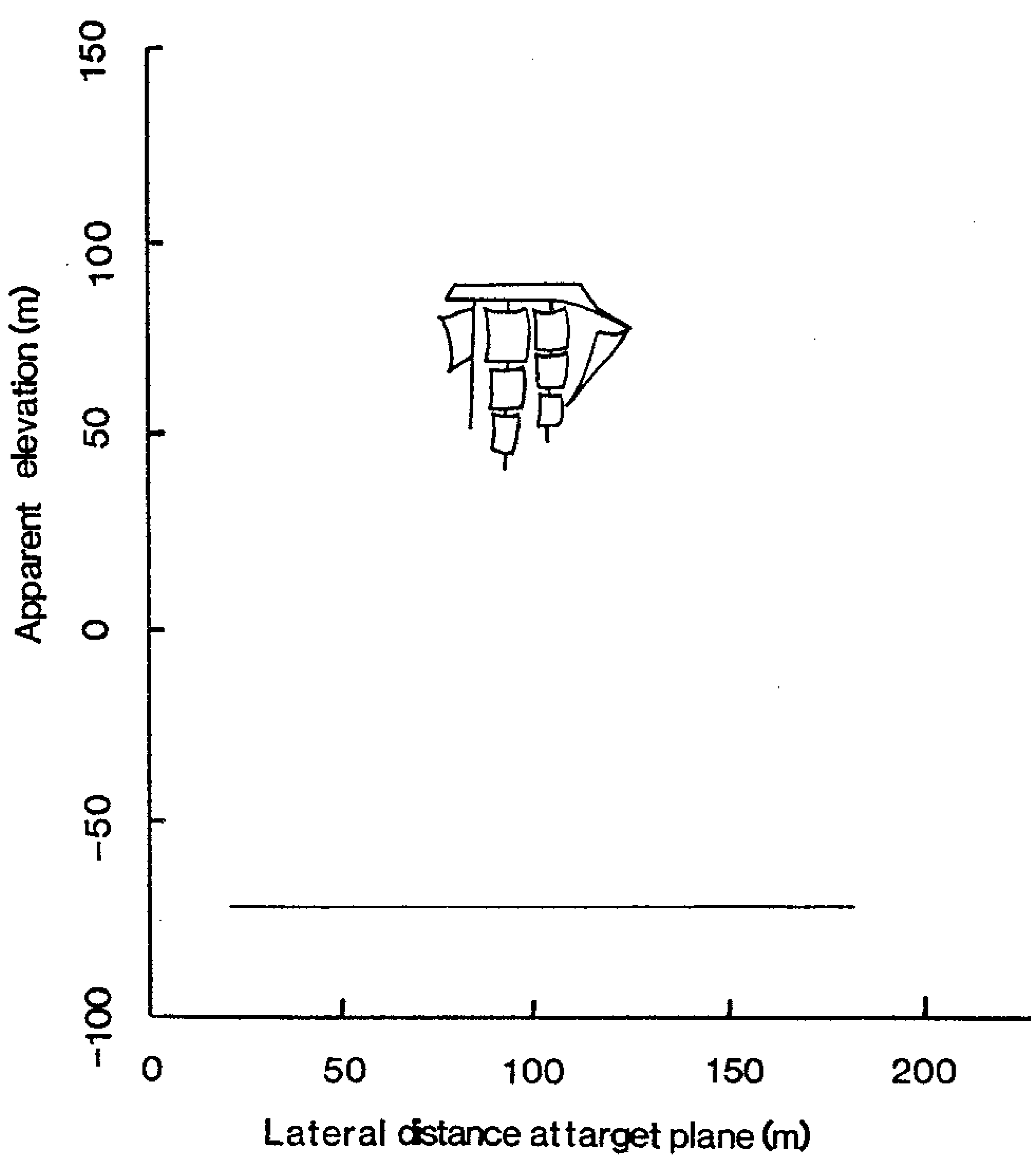

Fig. 8. Scale drawing of the image predicted by the temperature profile of reconstruction 2. Compare Fig. 1.

Conclusions

The Scoresby mirage can be reconstructed in different ways, depending on the initial assumptions made in interpreting his observation. Common to the reconstructions is the concept of the linear image diagram with constant Gaussian curvature. It may consist of a single segment as in reconstruction 1, or of several segments as in reconstruction 2. Both of these methods work because of the undistorted form of the image, and both show that the mirage can be accounted for by a temperature inversion about 50 m high in which the temperature rises by about 7°C. Reconstruction 2 satisfies all of the vital properties of the observed image and is entirely consistent with the known properties of polar superior mirages, and we believe that it is very close to the true temperature profile which existed when Scoresby made his observation in 1822.

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Appendix. Relation between refractive index and temperature

The refractive index n of air depends upon its density ρ according to the equation:

$$n = 1 + \varepsilon\rho$$

and the density varies with pressure p and absolute temperature T according to:

$$\rho = \beta p/T.$$

The constant ε is given by $a_m/2\varepsilon_0 m$ where a_m is the polarisability of the 'air molecule' and m is its mass, so it has a value of $2.26 \times 10^{-4}\ m^3 kg^{-1}$. The constant β is given by m/k where k is Boltzmann's constant, so it has a value of $3.48 \times 10^{-3}\ kg\ m^{-3}\ K\ Pa^{-1}$.

If we consider an atmosphere which is laterally homogeneous, so that pressure and temperature vary only with vertical direction z, we have:

$$\frac{dp}{dz} = -g\rho = -\frac{g\beta p}{T(z)}$$

where g is the acceleration due to gravity. This can be integrated to give:

$$p(z) = p_0 \exp\left[-g\beta\int_0^z \frac{dz}{T(z)}\right]$$

where p_0 is the atmospheric pressure at z = 0. Thus the refractivity r can be written:

$$r(z) = n-1 = \frac{\varepsilon\beta p_0}{T(z)} \exp\left[-g\beta\int_0^z \frac{dz}{T(z)}\right]$$

While this appears complex, it can be approximated quite accurately by much simpler forms if the range of height z is small (say, limited to 50 or 100 m). The exponent is then small, and if the temperature T in the integral is assumed to be constant T_a we can write an approximate expression thus:

$$r = n-1 \approx \frac{\varepsilon\beta p_0}{T(z)}\left[1-\frac{g\beta z}{T_a}\right]$$

For $T_a = 273$ K and z = 50 m the bracket has a value of 0.994 which can for many purposes be considered to be unity. Then, using $p_0 = 1.0133 \times 10^5$ Pa, the refractivity becomes:

$$r = \frac{0.07967\ K}{T(z)}$$

Gaussian curvature

In a medium whose refractive index is a function of position, the optical path length L between two points A and B is defined as:

$$L = \int_A^B n\, ds$$

where ds represents arc length along the ray. We can restrict ds to the xz plane since rays propagating under our assumption of horizontal stratification remain in the vertical plane.

The differential form dL = n ds for the optical path defines a two-dimensional surface. The Gaussian curvature k_g of this surface at any point is defined as the product of the two principal (maximum and minimum) curvatures. If k_g takes on the same value everywhere on the surface, then it can be shown (Kropla 1988) that a quadratic refractivity profile arises. If we restrict the height range to a few tens of metres, this profile implies a quadratic temperature profile of the form:

$$T(z) = T_0 + \frac{\alpha n_0 T_0^2 z}{0.07967\ K}\left[\gamma + \alpha z/2\right]$$

where $n_0 = 1+0.07967$ K/T_0 is the refractive index at z=0, $a = n_0(k_g)^{1/2}$, T_0 is the absolute temperature at z=0, and g adjusts the position of the parabola's vertex.

Thus for the temperature profile of reconstruction 1 (ignoring the cubic term) we have $k_g = 6.86 \times 10^{-9}$ m^{-2} and $g = -8.5 \times 10^{-4}$. For reconstruction 2, the atmosphere can be approximated by two segments of constant k_g, the values being 4.2×10^{-9} in the lower part and 12×10^{-8} in the upper part.