

# Bright superior mirages

Waldemar H. Lehn

Superior mirages of unusual brightness are occasionally observed. Two such cases, photographed over the frozen surface of Lake Winnipeg, Canada, are documented. Visually, these mirages appear as featureless bright barriers far out on the lake. They are just images of the lake ice, yet the luminance in one case was 2.5 times (in the other, 1.7 times) the luminance of the ice surface in front of the mirage. The mirage itself can be modeled by means of a conduction inversion, but a proper explanation of the brightness is not yet available. © 2003 Optical Society of America

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## 1. Introduction

I have observed several instances of unusually bright superior mirages that occurred over a frozen lake on a warm spring day. The effect appears as a distant but bright featureless barrier resting upon the horizon. To my knowledge, there is only one previous mention of this phenomenon in the literature, a brief nontechnical discussion by Fraser and Mach,<sup>1</sup> and no adequate explanation exists. In this paper I will describe and illustrate two observations and identify the kind of mirage that can cause them. Suggestions will be made about the underlying phenomena; however, the exact explanation for the anomalous brightness will have to await further research.

## 2. Observations

The two observations that will be analyzed here were both made on Lake Winnipeg, Manitoba, Canada. The map in Fig. 1 shows the locations of the observation points and the lines of sight. Figure 2 shows a brilliant superior mirage photographed from the east side of the lake on 30 April 1978. On this day the sky was completely free of clouds, the wind was calm, and the air temperature over land was  $\sim 15^\circ\text{C}$ . The surface of the lake was entirely ice covered, and the air appeared to be extremely clear. The trees visible on the left-hand side of the picture identify Grand Marais Point, which lies 2.5 km from the cam-

era at a bearing angle of  $249^\circ$ . To the right-hand side of the point, where the mirage is seen, the distance to the far shore is 26.4 km. The camera was loaded with Kodachrome 64 film and equipped with a 560-mm telephoto lens. Its elevation was 2 m above the lake ice.

The phenomenon began at 4:15 p.m. Central Daylight Time. It lasted until 5:25 p.m., when the photograph of Fig. 2 was taken. During this interval the bearing and elevation of the Sun varied from  $(239^\circ, 41^\circ)$  to  $(256^\circ, 31^\circ)$ . Thus the line of sight, at bearing  $249^\circ$ , was within  $7^\circ$  of being exactly toward the Sun. The phenomenon was visible over a considerable lateral extent; at 4:34 p.m., when it extended to a bearing of  $262^\circ$ , the line of sight deviated from the bearing of the Sun by  $19^\circ$ . The elevation of the top of the mirage was measured by theodolite to be 6 arc min. The whole mirage strip subtends 6 arc min vertically. If the base of the bright strip is considered to portray the local horizon, then it follows that this horizon is somewhat elevated; i.e., it is at an elevation of 0 arc min, whereas the expected horizon for a 2-m camera elevation would be at  $-2.5$  arc min.

Figure 3 shows a bright mirage observed on the west side of Lake Winnipeg on 17 April 1980. The wooded point on the left is Drunken Point, on the west shore just north of Arnes, 3.7 km from the camera. Beyond the point, at a bearing angle of  $33.5^\circ$ , lies Hecla Island, 28 km away (see map, Fig. 1). Weather conditions were almost identical to those of the first observation. The sky was completely clear, the wind was calm, the air temperature over land was  $\sim 20^\circ\text{C}$ , and the lake was entirely covered with ice. The phenomenon, which began at 2:58 p.m. Central Standard Time and lasted until 6:47 p.m., was visible over a wide range of bearings, from  $33.5^\circ$  to  $107^\circ$ . During this event the bearing and elevation of the

W. Lehn (lehn@ee.umanitoba.ca) is with the Department of Electrical and Computer Engineering, University of Manitoba, Winnipeg R3T 5V6, Canada.

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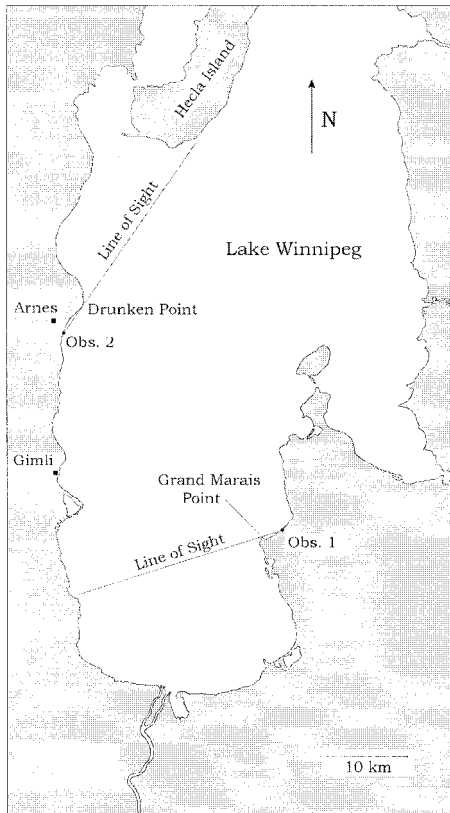


Fig. 1. Lake Winnipeg, South Basin. The first observation looks past Grand Marais Point toward the far shore, which is 26.4 km from the camera. The second observation looks past Drunken Point to Hecla Island, 28 km away.

Sun varied from  $(231^\circ, 40^\circ)$  to  $(281^\circ, 5^\circ)$ . At the time of the photograph (3:22 p.m.) the Sun was at  $(237^\circ, 37^\circ)$ ; thus the line of sight was almost directly away from the Sun. Specifically, the angle between the line of sight and the bearing of the Sun was  $157^\circ$ .

The photograph was taken with a 1270-mm lens using Kodachrome 64 film, from a camera elevation of 2 m. The elevation of the top of the mirage was measured to be 6 arc min, and the base of the bright band was at an elevation of 0 arc min. Again, if this



Fig. 2. Bright mirage at Grand Marais Point.



Fig. 3. Bright mirage at Drunken Point. The noticeable fall-off of brightness with radial distance from the center of the image is due to vignetting. The lens, a Celestron 5-in. telescope attached to a camera with a T adapter, is seriously adversely affected by this flaw. The effect is visible in every photograph made with this lens.

is considered to be the local horizon, then it is elevated above the expected value of  $-2.5$  arc min.

The brightness of the mirage was visually and photographically striking. Its luminance was estimated from scans of the original color slides. Comparison with scans of calibrated gray scales permitted calculation of the absolute density on the slides. Published characteristic curves for Kodachrome 64 film<sup>2</sup> were then used to find the relative luminance between the mirage and the regions above and below it. The luminance of the mirage strip was roughly twice the luminance of the ice surface immediately below it. Specifically, the ratio was 2.5 at Grand Marais and 1.7 at Drunken Point.

### 3. Discussion of the Observations

Both observations took place on warm still days over a frozen lake. The consequent minimal advection and turbulent mixing suggest that warm air slowly drifting over the ice will be cooled from below by conduction. The resulting temperature distribution, called a conduction inversion,<sup>3</sup> thus becomes the basis for modeling these mirages. The temperature profile of a conduction inversion has its lowest temperature as well as its steepest gradient at the ice surface. The temperature increases with elevation, and its gradient decreases; both vary monotonically. The resulting density distribution has its maximum at the ice surface, and density decreases with increasing elevation. This is a stable configuration that can maintain itself over long periods of time. More cooling from below just makes it more stable.

Light rays traveling below the inversion are curved toward the region of higher refractive index (higher density, lower temperature). Since the curvature of the rays increases with temperature gradient, the rays nearest the surface are curved downward the most strongly. When a ray that is concave down-

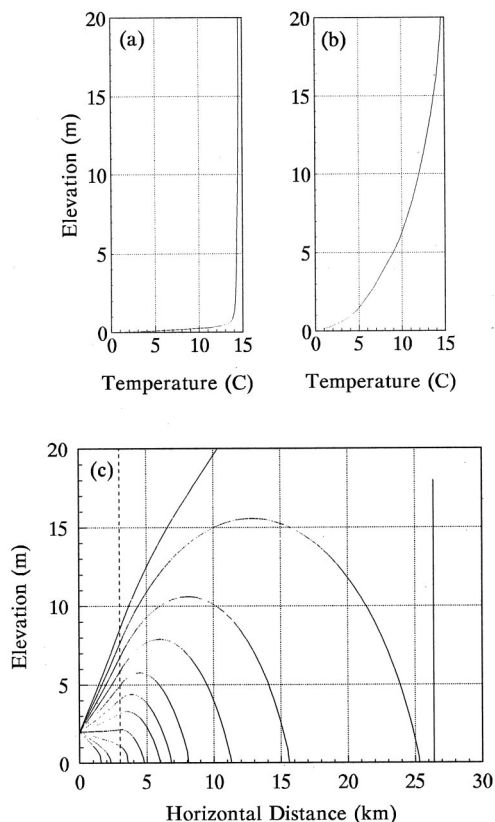


Fig. 4. Model for the Grand Marais image. (a) Temperature profile used for the first 3 km, where the line of sight is close to the shore; (b) temperature profile over the open lake beyond 3 km; (c) light rays traced backward from the camera. The vertical line at 26.4 km represents the far shore. The elevation angles of the rays span the range  $[-3^\circ, 7^\circ]$ , at  $1^\circ$  intervals.

ward enters the eye, the observer sees the source of the ray as elevated above its true position. The mirage is consequently an image of the lake ice lifted into an apparent vertical position.

Examination of the map and the images reveals that both observations possess certain similarities. The foregrounds, terminated by points of land, have a normal appearance, with a slightly elevated horizon. The line of sight for this region is parallel to the shore and rather close to it. The main refraction occurs beyond the points, where the line of sight passes over a large basin of lake ice. The situation is well modeled by an atmosphere consisting of two distinct regions, with different thermal histories. In the foreground region, where there is little refraction, the air is considered to have the same temperature as over the land, with a shallow layer of conduction cooling at its base to produce the slight horizon lifting. In the background region, where the air is far from land, the conduction inversion is taken to be well developed and deep, to produce the main refraction observed.

From the above considerations, the mirage at Grand Marais Point can be represented by the model of Fig. 4 (a model for Drunken Point would be similar). The temperature profiles taken for the fore-

ground and background regions are shown in Figs. 4(a) and 4(b), respectively, and the corresponding ray paths in Fig. 4(c). The transition between regions should be gradual and continuous, but the abrupt transition used in the model makes calculation easier while making little difference to the ray paths. In the background region the rays all strike the ice surface rather quickly. The eye sees these points imaged upward in an apparent vertical wall. This ray-tracing model easily explains the geometric appearance of the mirage, but it does not go far enough to explain the brightness.

In two decades of mirage observations I have found that the bright mirage is not often seen. The events reported here are the only ones for which I can make a good case for a deep stable conduction inversion of wide extent. Most of my other observations have involved inversions of the advection type, in which the temperature jump occurs in a fairly narrow zone well elevated above the ice. Such inversions often produce optical ducts, from which nearly horizontal light rays cannot escape upward (just as in the conduction inversion), but with the important distinction that the ducted rays do not strike the ice. The observer thus does not see the bright ice surface but rather images of distant objects such as the far shore of the lake.

#### 4. Considerations for Future Model Construction

Several potential explanations are discussed below. Most of them give rise to objections that must be addressed if the corresponding model is to be considered seriously.

In a mirage the atmosphere acts as a distorting lens. It redistributes light rays in an angular sense, concentrating them in some directions and spreading them out in others. It might then be thought that the brightness of a mirage could deviate from that of the original object.<sup>1</sup> However, the redistribution does not affect the brightness of the image. For any lens that is ideal and lossless, the photometric brightness of the image is the same as that of the object.<sup>4</sup> This has also been demonstrated for mirages<sup>5</sup>; i.e., a Lambertian object has the same perceived brightness no matter from which angle it is viewed, independent of whether the rays are straight or curved.

The whiteness of the mirage is reminiscent of brilliantly illuminated clouds or fog. Both the sky above and the lake ice just below the mirage have distinctly blue tones. The whiteness suggests a large optical thickness of nonabsorbing scatterers. It would not be unreasonable to suggest the existence of a concentration of microscopic water droplets trapped within the inversion layer, owing to strong surface melting and insolation. Yet the air everywhere else appeared to be extremely clear. On other occasions I have indeed observed ice fog within inversion layers; when the fog dissipated, a mirage was visible within the layer. But in those cases there was no unusual brightness to the fog. The strongest argument against the fog hypothesis is the apparent transparency of the inversion layer, as evidenced in Fig. 5.





Fig. 5. Dark mirage at Grand Marais.

This image, taken 65 min before the image of Fig. 2, shows a distinct dark band, which I interpret as an image of the dark forest on the far shore. To show such contrast over a 26-km range, a very clear atmosphere is required. It should be noted that such an image cannot be obtained from a conduction inversion; the temperature profile clearly changed in the interval between the two observations.

It could also be suggested that the effect depends on directional preference for scattering (the scattering phase function) of either the ice surface or the molecules or droplets within the inversion. What argues against this is the fact that the effect appears to be entirely independent of the angle between the bearing of the Sun and the direction of view. For example, in Fig. 2 the view is into the Sun, whereas in Fig. 3 it is away from the Sun.

If we accept that it is the ice surface itself whose image is anomalously bright, then we must conclude that the inversion is of the conduction (not advection) type. Further, as suggested by an unnamed reviewer, the ice surface cannot be considered Lambertian.

For the mirage at Grand Marais, where the direction of view is toward the Sun, it has been suggested that the rough, highly reflective and possibly wet ice produces a strong glitter, just like the effect seen over slightly rough water below the Sun. Figure 2 indeed shows a few such points in the near foreground. At large distances such glittering points of light would not be resolved individually, but rather they would blend together and add significantly to the luminance of the mirage.

Some of the observational evidence raises questions about such a model. For example, the ice surface between the camera and the point shows no sign of glitter up to a distance of 2.5 km (with the exception of a few points in the near foreground). There is no reason to assume that the ice surface is significantly different farther out and hence no reason to

expect a drastic increase in glitter at the larger distances.

The directions of view also raise questions, because the bright mirage was recorded as far as  $19^\circ$  from the bearing of the Sun. Such an angle is likely too large for the glitter effect. Finally, an entirely different model would be needed to account for the same phenomenon when seen looking away from the Sun.

A model that will be further investigated proposes that the extreme brightness is caused by the capture of light flux below the inversion. A small fraction of the light incident upon the ice cannot escape upward through the inversion, because the light rays are bent back downward. This light can do nothing but illuminate other parts of the ice surface. The corresponding rays have shallow elevation angles and strike the surface with grazing incidence. If the ice is not Lambertian, but has instead a reflectance near unity for such incidence, it is conceivable that its luminance could be enhanced when viewed from these same shallow elevation angles.

## 5. Conclusion

Full observational data for two unusually bright superior mirages have been presented. Two conclusions can be drawn with some certainty from the data. First, the temperature inversion that produces the mirage must be of the conduction type, in order to image the lake ice into the vertical barrier that is seen. Second, the ice surface itself cannot be modeled as Lambertian; rather, enhanced reflectance at grazing incidence must be included in any model. Several possible physical models are briefly discussed and discarded on the basis of observational evidence. However, a quantitative theory that correctly explains the phenomenon, by providing calculated brightnesses that agree with observation, is not yet available.

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