Skerrylike mirages and the discovery of Greenland

Waldemar H. Lehn

The Norse discovery of Greenland is associated with the sighting of low barren islands called Gunnbjörn's Skerries, which have never been satisfactorily identified. Here the historical references that connect the skerries to Greenland are reviewed. A mirage of the Greenland coast, arising specifically from optical ducting under a sharp temperature inversion, is used to explain the vision of skerries seen by the Norse mariners. Images from both ducting and uniform inversions are calculated. Under the assumption of a clean Rayleigh atmosphere, sufficient visibility remains to see the skerry image at a distance of 220 km. There is significant circumstantial evidence to indicate that the Norse were familiar with the skerrylike mirage and that they used it to discover new lands. © 2000 Optical Society of America

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1. Introduction

The Norse discovery of Greenland is intimately linked with the concept of Gunnbjörn's Skerries, a controversial group of islands between Iceland and Greenland. In this paper I will briefly review the historical record of the discovery and then show that the skerries can be interpreted as a specific form of mirage. The idea that mirages may have been involved is mentioned in passing by the historians Jones[1] and Gjerset,[2] but this idea is not scientifically pursued. The following discussion supports the hypothesis that Gunnbjörn's Skerries are a mirage of the mountainous Greenland coast and constructs several possible optical models and demonstrations. Images that the Norse mariners may have seen are calculated and presented.

2. Historical Background

The expansion of the Norse across the North Atlantic, and their discovery of America, followed a pattern of minimum open-sea distance. Over the course of 200 years the Norse moved from the Faeroes to Iceland and Greenland, and finally to Vinland (Newfoundland, Canada). Along this route the longest single stretch of open sea is merely 430 km, between the Faeroe Islands and Iceland. Greenland and Iceland are even closer together, the minimum separation across the Denmark Strait being 285 km.

Much of Norse history is drawn from medieval Icelandic sagas, of which fourteenth-century copies are preserved. Two sagas in particular (the Grænlendinga Saga and Eirik's Saga, originally written in the twelfth and thirteenth centuries, respectively) describe the discovery and colonization of Greenland and Vinland. The sagas are exhaustively analyzed in many studies. Important fundamental references are books by Jones,[1] Nansen,[3] Gad,[4] and Seaver.[5]

The first sighting of Greenland by a European is attributed to Gunnbjörn, near the end of the ninth century. His was the second ship to sail around northwestern Iceland. Gunnbjörn passed far out to sea, being windblown to the west. Although he did not lose sight of Iceland's mountains, he reported seeing skerries and a glacier in the west.[6]

The Old Norse word for skerries means barren rocky islands of low profile, with little to recommend them for habitation. The historical record makes a point of not using the word *holm*, which is reserved for land or large habitable islands. In northwestern Iceland the story of the skerries remained alive for a century, before the skerries became a destination point for the next expedition of the ever land-hungry Norse. Although this could be attributed to the excellent oral tradition of the Icelanders, there may have been the occasional corroborative sighting in this westernmost region of Iceland. The Denmark Strait is at its narrowest here, and the possibility of seeing skerrylike images from coastal waters would be greatest in this area.

The written record places the next European sight-

The author is with the Department of Electrical and Computer Engineering, University of Manitoba, Winnipeg R2T 5V6, Canada. His e-mail address is lehn@ee.umanitoba.ca.

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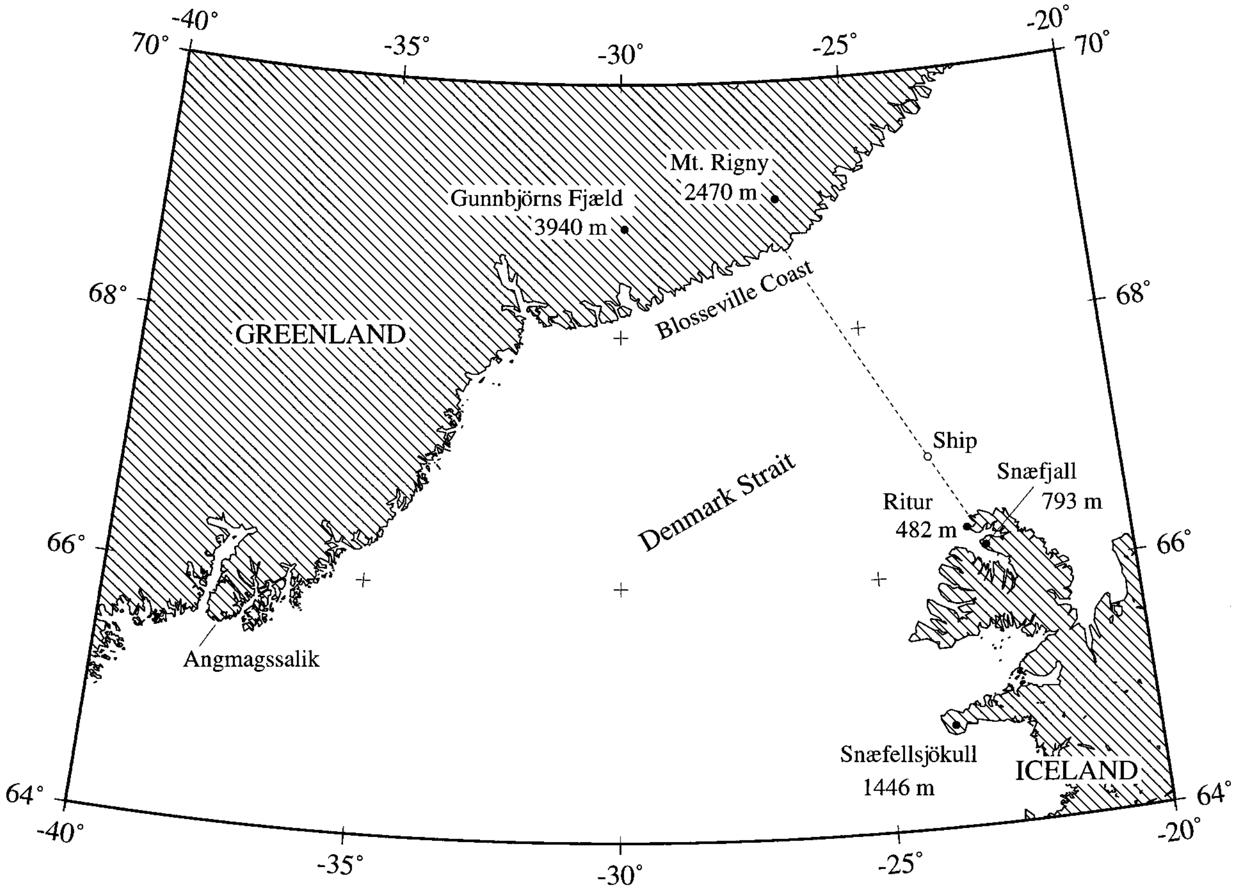

Fig. 1. The Denmark Strait separates Iceland and Greenland. The dotted line indicates the shortest distance between them, 285 km.

ing late in the tenth century, when Snæbjörn and Hrolf led an expedition of 24 people to find the skerries that Gunnbjörn had seen.[7] Although Gunnbjörn had given no precise direction as to their location, local tradition had now positioned them as lying northwest from Isefjord. The report simply states that "they set out in search of Gunnbjarnarsker and found land."[8] After a harsh winter the survivors returned to Iceland, no doubt with reports about the very inhospitable land that they had found.

In spite of this recorded expedition the credit for the discovery of Greenland has always been given to Eirik the Red,[9] who according to the sagas decided to seek out the skerries in 982. Since both Snæbjörn and Eirik lived in northwest Iceland, it is difficult to imagine that Eirik had not heard of Snæbjörn's voyage, even though the sagas do not acknowledge this.

The problem with all three of these stories is that the skerries do not exist. There are no islands in this part of the Denmark Strait (Fig. 1). Many historians have attempted to identify the skerries, without success. The definitive paper to date (Holm,[7] 1918) places the skerries at Angmagssalik, right on the coast of eastern Greenland. His explanation remains unsatisfactory, because these islands do not look like skerries and because anyone seeing the islands could not avoid being awestruck by the fantastic Greenland coast looming over them. So what was actually happening? Snæbjörn and Eirik set out to find skerries and discovered land instead. Did they expect this? The rest of their stories suggest yes. Snæbjörn's heading was northwest—the shortest path to the Blosseville Coast[10] (see Fig. 2)[11] of Greenland. Eirik might well have taken the same route; the exact direction is not given, but Tornøe[12] argues that it would be northwest. Norse scholars in the first half of the twentieth century did not accept the northwest course to Greenland, because in modern times the path has been blocked by ice throughout the year. The concept that the climate may have been different in the tenth century has been accepted only in the past few decades; this era is now called the Medieval Warm Period.[13] In a warmer climate such a course would be very practical: It minimizes the time the ship spends out of sight of land, consistent with the Norse tradition of landmark sailing.[14,15]

Once Greenland is discovered, the name Gunnbjörn's Skerries is dropped without comment, for more than 300 years.[16] It is as though the skerries have simply been identified as the land across the Denmark Strait (Greenland). When the skerries reappear in the histories in 1350, they do so as an inaccessible landmark.[17] They reappear because they are occasionally seen, from land or from coasting fishing boats, but the interpretation that they repre-

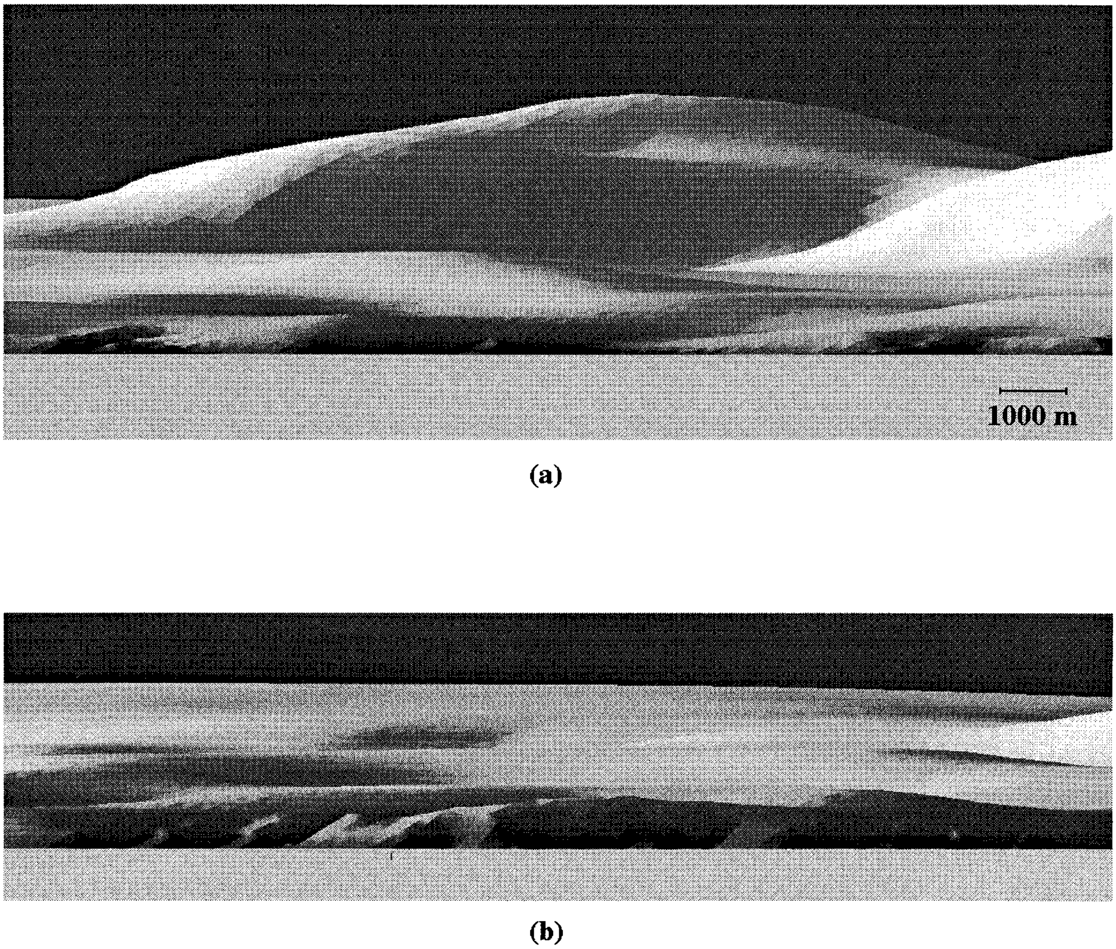

Fig. 2. Images of the Greenland coast. These perspective views are calculated from GTOPO30 digital elevation data with Generic Mapping Tools software, which considers the Earth to be flat and the observer at infinite distance. (a) The region around Gunnbjörns Fjæld (3940 m), the highest peak in Greenland. (b) Portion of Greenland nearest to Iceland. The tick mark identifies the nearest point, at a distance of 285 km.

sent distant land has been forgotten. There are two reasons for this. First, the Icelanders had lost their seafaring experience and their ability to interpret skerry sightings as an indicator of distant land,[18] because they lacked the natural resources to build ocean-going ships.[19] Second, by this time the climate had cooled,[20] and the sea ice that came down the Denmark Strait prevented all other (foreign) sailors from steering toward the Blosseville Coast on the shortest path to Greenland and thus learning the truth about the skerries.

In 1625 Björn Jónsson recorded contemporary accounts that claimed sightings of the skerries. One of these,[21] a statement contained in "many reliable and true narratives," is quite precise: The Gunnbjörn Islands exist, and when the air is clearest they can be seen from a mountain called Ritur in the Adalvík district of northwestern Iceland. The story from Ritur is easily believed under the mirage hypothesis, because this mountain is located close to the minimum distance to Greenland.

The attitude toward the skerries in the tenth century, and the change in this attitude when they reappear in the fourteenth, can be explained in a consistent fashion, if all the skerry sightings are considered to be caused by a specific type of mirage of the Greenland coast. Images in which high coastlines look like skerries are conveyed by optical ducts.

3. Optical Explanation

In modern times there have been ships in the Denmark Strait, at a latitude of ~68°, whose crews actually saw land on both sides of the strait simultaneously.[12] It takes only a small amount of atmospheric refraction to accomplish this. Thus intervisibility is certainly possible. But the discovery of Greenland has always been associated with skerries, not intervisibility.

Polar mirages occur with such frequency that they have been recorded in nearly every polar explorer's journal. There are well documented cases in which a mirage has been active at just the right time to have a profound effect. One example occurred during Sir John Ross's search for the Northwest Passage.[22] As his ship entered Lancaster Sound north of Baffin Island, he found his path completely blocked by a

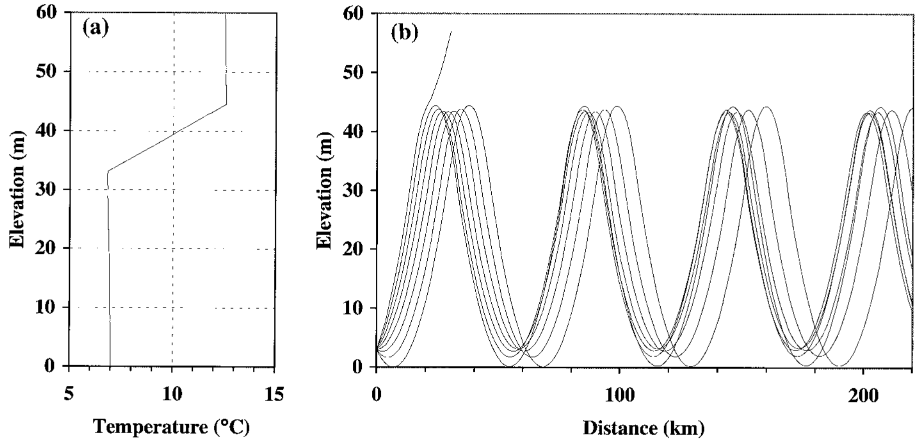

Fig. 3. (a) Profile of a temperature inversion that creates an optical duct. (b) Light rays under a full duct that extends all the way to Greenland. The observer's eye has an elevation of 3 m at the origin of the distance scale. At the observer the ray elevation angles span the range [−3′, +4′] in 1′ steps. At 220 km the rays intersect the Greenland coast, which rises almost vertically from the sea. The plot is curvature corrected so that the Earth shows as a flat plane. Straight rays then appear to curve upward.

mountain range, which he entered on his map as the Croker Mountains. He then sailed out of the sound, abandoning his search. On subsequent voyages the mountains were identified as a mirage.

Under the right conditions skerrylike apparitions can occur. The present objective is to investigate several models, to determine which is capable of producing the right image. Gunnbjörn's ship will be considered as sailing 65 km off the Iceland coast, as it crosses the shortest line between Iceland and Greenland. Its position is shown on the map (Fig. 1). Iceland is in full view, as Gunnbjörn's story claims: Tornøe[12] proposes that the landmark that Gunnbjörn saw at the same time, Snæfells glacier, be interpreted as Snæfjall in the Adalvík area. The coastal cliffs, 480 m high, with the 793-m Snæfjall behind them, subtend 10 arc min vertically on the horizon. The distance to Greenland is 220 km. In a normal atmosphere, none of the Greenland mountains would be visible. Just how these mountains could appear to be skerries is discussed in Subsections 3.A–3.C.

A. Full Duct over the Whole Distance

A sharp low-level temperature inversion creates an optical duct that traps light rays and transmits them around the curve of the Earth.[23] Ducts of length exceeding 200 km have been observed on a number of occasions, by Nansen,[24] by Shackleton,[25] and by Liljequist.[26] If the inversion exists over the whole 220-km line of sight from the ship to Greenland, then light reflecting from the Greenland coast would be captured and pass to the eye of the observer. An example of a ducting temperature profile and the associated light rays is shown in Fig. 3. Note that the light rays are traced backward, as if emanating from the observer's eye. The inversion has a strength of 5.5 °C, and its elevation is 33 m above sea level. There is no special reason for choosing this exact temperature profile—practically any fairly sharp low-level inversion will do, for example, one of strength 8 °C at elevation 50 m. Note that the rays change direction and cross one another many times. This process produces a complex image, in which one source point can be imaged many times at different apparent elevations; target features would be completely mixed up as to relative elevation and not be recognizable. The mirage is computed from the source image of Fig. 2(b) by means of a mapping (transfer characteristic) extracted from the ray data. The mapping is a graph of apparent versus actual ray elevation at the target distance of 220 km, where apparent elevation is calculated for each ray by projection of the ray tangent at the eye out to the target plane. The horizontal scan lines of the source image are then relocated, as dictated by the mapping, to the new elevations that they occupy in the mirage.[27] Because the duct traps all the rays below a ceiling of 45 m, the entire image (Fig. 4) is made up of objects on the Greenland coast located within 45 m of sea level. The vertical size is ~5 arc min, which is easily visible to the unaided eye. The image has a characteristic flattopped appearance common to all ducted images. Therefore it would look like low relatively flat islands—skerries. The issue of visibility over such great distances is discussed in Section 5.

B. Partial Duct

The inversion need not extend all the way from the ship to Greenland. A partial duct is quite capable of producing the flattened and highly distorted skerry image. Consider a similar inversion as in the first case, but let it end some distance from the observer. Over the remaining distance use the standard atmosphere with a lapse rate of 0.006 °C/m. When the transition distance is experimentally varied, suitable skerrylike images are found for distances greater than 120 km. The result is not particularly sensitive to the exact value of this distance. It is also not sensitive to the abruptness of the transition from one atmosphere to the other; a gradual transition pro-

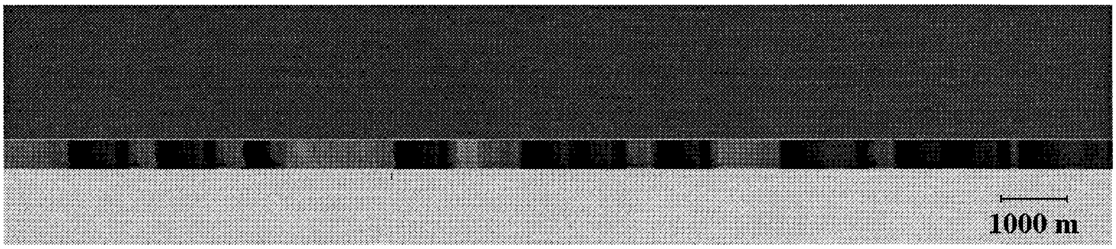

Fig. 4. Mirage of the nearest part of Greenland, calculated from the image of Fig. 2(b) and the rays of Fig. 3(b). The height of the mirage is 6 arc min, a size easily perceived by the human eye. This image is not compensated for loss of contrast due to atmospheric scattering.

duces basically the same result. It does not even matter much whether the inversion undergoes changes after ~50 km from the observer; the observer will still see the skerry image if the rays intersect the Greenland coast at all. Figure 5 shows the inversion and the ray paths, as well as a profile of the Greenland landscape along the line of shortest distance. The rays strike the coast much higher than in the previous case, but the observer sees only a mirage of the same nature and extent as before (Fig. 6).

The isothermal layers of the atmospheric duct need not be exactly parallel to the Earth's surface. In other words, the duct could be slightly inclined, the inversion increasing in elevation with increasing distance from the observer. Such slopes can occur as a result of advection processes.[28] A partial duct was tested with a slope of 5 arc min and a transition distance of 100 km. Results indicated no advantages for this more physically complex atmosphere. Further study was therefore not pursued.

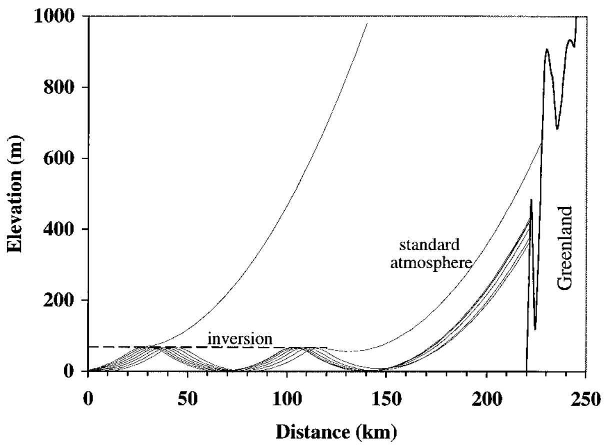

Fig. 5. Light rays for a partial duct of length 120 km. The inversion has a strength of 9 °C and a gradient of 0.6°/m, centered on an elevation of 60 m. Observer position and ray angles are the same as for Fig. 3(b). The standard atmosphere has a lapse rate of 0.006°/m.

C. Uniform Atmospheric Refraction

Nonducting atmospheres can also make Greenland visible. In an atmosphere with a uniform temperature gradient, rays of light follow paths of constant curvature. If the surface temperature is 0 °C and the temperature increases with elevation at the rate of 0.112 °C/m, the rays have the same curvature as the Earth's surface. Level rays within this atmosphere propagate parallel to the Earth's surface, and inclined rays gain elevation linearly with distance. Only when the rays exit this atmosphere do they begin to gain elevation with the square of distance. If such an atmosphere occupied, say, the first 120 km of the line of sight from the observer to Greenland, it would lift the mountain peaks of Greenland into view. Icelanders are familiar with this effect; their word for it is *hillingar*.[29] This is the kind of mirage to which Jones[1] refers. Figure 7 shows the geometry and the ray paths for an inversion of depth 120 m and 13 °C temperature span. Again the landscape profile is also shown. The lowest three rays are the only ones that stay entirely within the inversion for the first 120 km; they produce an undistorted image of approximately unit magnification, 2 arc min high. All the higher rays escape upward out of the inversion; they contribute to an image of much reduced magnification. So the observer would see a thin horizontal strip of the coast, topped by a highly com-

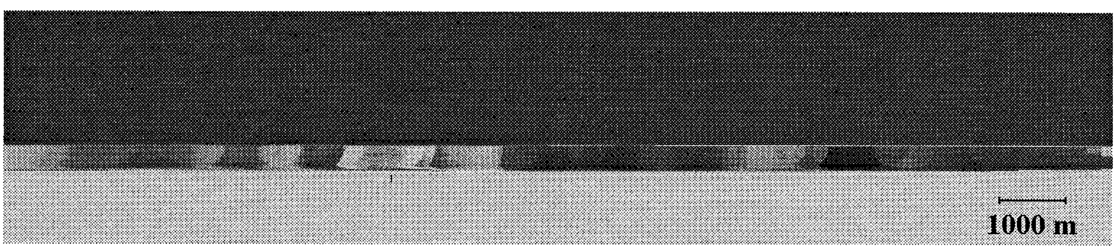

Fig. 6. Mirage of the nearest part of Greenland, as seen with the partial duct. The image is different from that of Fig. 4, because the rays intersect the landscape at higher elevations, but the mirage has the same height of 6 arc min. The image is not compensated for loss of contrast.

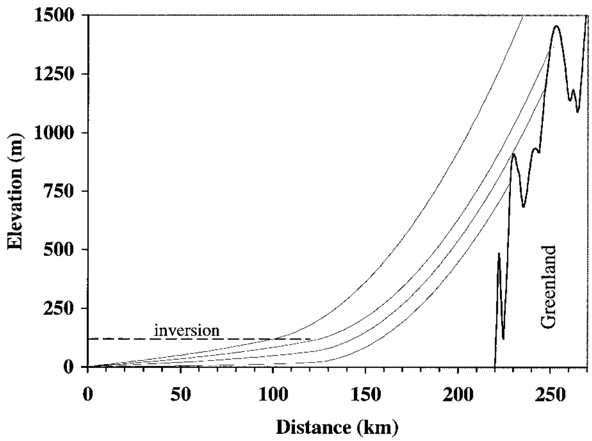

Fig. 7. Inversion with no ducting. The inversion takes the form of a uniform temperature gradient of 0.112°/m, going from 0 °C at sea level to 13.44° at an elevation of 120 m. Beyond 120 km the atmosphere reverts to the standard atmosphere with lapse rate 0.006°/m. Ray angles at the observer are 0–4 arc min, in steps of 1 arc min.

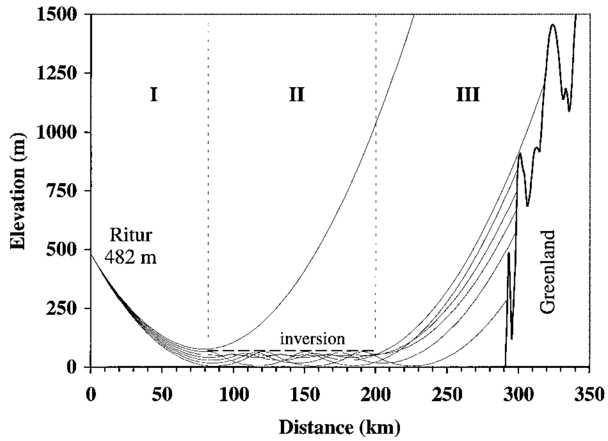

Fig. 8. Rays from Ritur in Iceland to the nearest part of Greenland. The inversion of Region II, which is the same as the inversion in Fig. 5, is flanked by two regions (I and III) of standard atmosphere. The rays at the observer's eye have elevation angles from −38 to −35 arc min, in steps of 0.5 arc min.

pressed version of the mountaintops. In his study on seeing Greenland with the aid of refraction, Búason[30] limits himself to this kind of atmosphere and concludes that Greenland would not be seen. This mirage is certainly harder to see than a ducted image. But with its compressed top the image would again look somewhat like skerries. The major difficulty with this model is the physical requirement on the atmosphere; a deep inversion of uniform gradient must exist over a long distance. It appears unrealistic to expect this.

4. View from Iceland

The story of Gunnbjörn's Skerries, and its interpretation as land, stayed alive from Gunnbjörn's to Eirik's time. Quite possibly it was reinforced by the occasional ducted image from Greenland. Similar sightings after 1350, from sea and land, may have revived old memories. The seventeenth-century annals by Björn Jónsson,[21] wherein he reports that the skerries can be seen from Ritur, will be briefly examined here.

Ritur, of elevation 482 m, lies within a chain of coastal cliffs. The mountain is 291 km away from the Greenland coast, only 6 km more than the shortest distance. Under the right conditions it is theoretically possible to see a skerrylike image of Greenland from Ritur. Several atmospheric structures suggest themselves, all of them involving optical ducts.

One consists of an inversion just above the observer (say ~500 m above sea level). If this inversion extends 200 km or more outward from the observer to Greenland, a typical flattopped ducted image of the Greenland coast arises. However, there will be one flaw in the image: The skerries do not rest on the horizon. There is a significant gap below the image, in which the sky is imaged, so that the image appears to float above the horizon. This is not an acceptable model.

A model that often works for long-range mirages[31] is based on a low-level inversion in mid-channel. An example with specific numerical values is shown in Fig. 8. Light rays from the coastal highlands of Greenland are captured by the duct, channeled around the curve of the Earth, and released on the Iceland side to reach the observer on Ritur. The skerry image is very small, subtending 2.5 arc min vertically, but based on the author's experience in the Arctic, even 2-arc min objects on the horizon are obvious to the naked eye.

Thus the legend is corroborated, even though the understanding as "harbinger of land" has been lost.

5. Visibility

The question of visibility is significant when the viewing distance exceeds 200 km. It is generally accepted that a small object whose brightness differs from that of the surrounding area by less than 2% is not visible to the human eye. This result arises from white-light experiments; however, Wyszecki and Stiles[32] state that the contrast limit remains near this value across the visible spectrum. Loss of contrast is caused by molecular and aerosol scattering. Whereas the aerosol component can vary widely, extremely small values have been measured in the North Atlantic. In medieval times one could expect even cleaner air than today; for this reason aerosol scattering is ignored and only molecular (Rayleigh) scattering is represented in the calculated values that follow. Details are discussed in Appendix A.

The cases will be discussed in the order in which they appeared above. The visibility of two basic objects is considered: a black object (areas of heavy shadow or dark exposed ground, say, 100 times less luminous than the horizon sky) and a bright white object (sunlight reflecting off glacier ice, say, three times brighter than the horizon sky). The horizon sky itself is considered to be white, normalized to unit luminance. Koschmieder's theory, with a transmission factor averaged over visible wavelengths, gives the luminance of the objects relative to the horizon sky without regard to color (see Appendix A). The theory is applied here in spectral form, to the three standard wavelengths that the *Handbook of Optics*[33] uses to characterize the extinction of air: 632.8 nm (red), 514.5 nm (yellowish green), and 488 nm (bluish green). The black object has spectral luminance R, 0.78; G, 0.97; B, 0.99; where R, G, and B are red, green, and blue. There is sufficient contrast to see this, in the red and (barely) in the green; because of the deficit in red, the black object appears to be a pale bluish color, darker than the horizon. The bright object should be easy to see, with luminance R, 1.45; G, 1.06; B, 1.03. It is significantly brighter than the horizon, and contrast in the red and the green is quite reasonable. The excess red gives this object a yellowish cast. In reality a Rayleigh sky has a pale blue color at the horizon. This makes the bright object easier to see, by virtue of improved color contrast, and the black object slightly harder to see. These deductions are consistent with personal experience; I have seen and photographed a glacier at a distance of 240 km in the Alps.

The next two cases (partial duct and uniform refraction) will have slightly improved visibility, because the light rays pass through higher elevations for a portion of their paths. The thinner air imposes less extinction and carries an image of higher contrast to the observer. The improvement is however small, and the numerical results are not presented.

The view from Iceland has the greatest limitation. For the most conservative case, in which the entire light path is at sea level, the RGB values for a viewing distance of 292 km are R, 0.86; G, 0.99; B, 1.00 (black object) and R, 1.29; G, 1.02; B, 1.01 (for a white object 3 times brighter than the horizon sky), again with the horizon brightness normalized to unity. The bright object has the better contrast, but both should be visible.

6. Conclusions

Gunnbjörn's Skerries are a mirage image of Greenland's Blosseville Coast. The mechanism is an optical duct whose exact nature need not be identified, because it has been shown above that a number of different atmospheric situations can produce the skerry image. In fact combinations of these atmospheres are possible. Images vary reasonably smoothly if the atmosphere is permitted to evolve from one of these forms to another. Many more situations could be tested. The main point is that the optical duct model explains the historical observations and overcomes the difficulties of vertical compression inherent in *hillingar*-type mirages. The partial duct model appears to be the most reasonable, because it places the minimum constraints on the atmospheric structure.

Further, one can interpret Eirik's comment about Gunnbjörn's Skerries as a search for the land represented by the skerries. Then the discovery of Greenland was initiated by a mirage, seen by Norse navigators who knew how to interpret its effects.

Appendix A

Extinction of light in the atmosphere depends on scattering and absorption due to air molecules and aerosols. For visible light molecular scattering is significant, but absorption is negligible. Aerosol effects are generally much stronger; however, on the basis of extremely low values measured in the North Atlantic,[34] aerosol scattering and absorption will be neglected. Any aerosol scattering higher than the measured minima would seriously degrade the required visibility.

In the following discussion wavelength λ and distance d are measured in centimeters. For an optical path in air, the Rayleigh model of molecular scattering[35] gives the extinction coefficient at wavelength λ,

$$\alpha_\lambda = f\,\frac{8\pi^3}{3}\,\frac{(n_s{}^2 - 1)^2}{N_s{}^2\lambda^4}\int_0^d N\mathrm{d}x,$$

where N_s is the number of air molecules per cubic centimeter at STP; N is the number of air molecules per cubic centimeter at temperature T and pressure p along the optical path; and $f = 1.061$, a constant.

n_s, the refractive index of air at STP and wavelength λ, is calculated with the Cauchy formula,[36]

$$n_s - 1 = A[1 + (B/\lambda^2)],$$

where $A = 28.79 \times 10^{-5}$ and $B = 5.67 \times 10^{-11}$. At the temperature 15 °C and standard sea-level pressure, these equations exactly reproduce the molecular scattering coefficient data for the wavelengths of 488, 514.5, and 632.8 nm given in the *Handbook of Optics*[33] (0.0190, 0.0153, and 0.00658 km^{-1}, respectively).

At constant temperature and pressure the extinction coefficient is equal to the product of distance with the molecular scattering coefficient:

$$\alpha_\lambda = d \times \sigma_m.$$

The transmission factor, which gives the fraction of the original light intensity that reaches distance d, is

$$q_\lambda = \exp(-\alpha_\lambda).$$

The calculations that are relevant here are done for the three colors listed above.

Koschmieder's visibility model[37] gives the luminance of a small black object as

$$B = B_h(1 - q_\lambda),$$

where B_h is the luminance of the horizon. For a bright object of luminance B_0 the result is

$$B = B_0 q_\lambda + B_h(1 - q_\lambda).$$

Irmgard I. Schroeder assisted in preparing the historical portion of this paper.

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7. G. Holm, "Gunbjørns-Skær og Korsøer," Medd. Groenl. **56,** 291–308 (1918), p. 292. Holm quotes from *Grønlands historiske Mindesmærker* (The Historical Records of Greenland) (Copenhagen 1838 ff), Vol. 1, p. 105.
8. Ref. 1; p. 129.
9. Ref. 1; 165.
10. The Blosseville Coast consists of very high mountains (the Watkins Range), the highest of which, Gunnbjörns Fjæld, exceeds 3900 m.
11. All geographical information is taken from the global digital elevation model GTOPO30. This public-domain digital elevation model, which covers the globe at a resolution of 30 arc sec, is obtainable at http://edcwww.cr.usgs.gov/landdaac/gtopo30/gtopo30.html. The map and the perspective images of Fig. 2 are produced with Generic Mapping Tools Version 3.2 (available from http://www.soest.hawaii.edu/gmt). See also P. Wessel and W. H. F. Smith, "New, improved version of the Generic Mapping Tools released," Eos Trans. Am. Geophys. Union **79,** 579 (1998).
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15. Mt. Rigny, a mountain of elevation 2470 m, is located 35 km inland from the Greenland coast, nearly along the shortest line of sight from Iceland to Greenland. Its distance from Snæfjall (793 m) in Iceland is 357 km. In the standard atmosphere the horizon distance d (km) from either peak is approximately $d = 3.9\sqrt{h}$, where h is the elevation in meters. For an observer whose eye elevation is 3 m, a ray tangent to the horizon would meet the tips of Snæfjall and Mt. Rigny at distances 117 km and 201 km, respectively. The gap in intervisibility is thus 39 km. However, the distance to be sailed without a comfortable view of land (say, 5 arc min high) is somewhat longer, 62 km.
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