A Purely Pandiagonal 4*4 Square and the Myers-Briggs Type Table

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The most widely used inventory of normal personality is the Myers-Briggs Type Indicator. It consists of 16 types based on preferences of four pairs of dichotomous (or bipolar) variables which grew out of the work of Katherine (Cook) Briggs and her daughter Isabel (Briggs) Myers, and is closely related to Carl Jung's *Theory of Psychological Types.* The first attitudinal preference is for introversion (I) or extraversion (E); the second preference is for the perceiving pair of functions of consciousness, sensing (S) and intuition (N is used since "I" has already been taken for introversion); the third preference is for the judging pair of functions of consciousness, thinking (T) and feeling (F); with the final attitudinal preference being between judging (J) and perceiving (P). At some point before the publication of *Gifts Differing* (I. Briggs Myers *with* P. B. Myers, CPP Books, 1980, 1993) in the development of the MBTI as a practical tool, the 16 types were arranged very cleverly in a 4*4 matrix, known as the Type Table, as shown in Figure 1.

ISTJ	ISFJ	INFJ	INTJ
ISTP	ISFP	INFP	INTP
ESTP	ESFP	ENFP	ENTP
ESTJ	ESFJ	ENFJ	ENTJ

Figure 1.

The arrangement is: left half for S; right half for N; middle two columns for F; outer columns for T; top half for "I"; bottom half for E; middle two rows for P; outer rows for J. An interior element has the property that horizontal or vertical movements change just one of the letters, one diagonal step changes two letters, and two diagonal steps change all four. The edge (and corner) elements need to be continued periodically to the opposite edges by rolling the matrix into a horizontal or vertical cylinder, or by doing both to form a torus (bagel or donut with a hole).

I became aware of a possible connection with magic squares after finding a paper cut-out "square" torus in *Mathematical Curiosities I* (G. Jenkins and A. Wild, Tarquin Publications, 1980) decorated as a 4*4 diabolic (pandiagonal) square. Seeking more information on magic squares, I found *The Second Scientific American Book of Mathematical Puzzles and Diversions* by Martin Gardner (1961) which illustrates the smooth torus for a 4*4 magic square and gave a description which directed my attention to B. Rosser and R. J. Walker's

1938 paper (*Bulletin of the American Mathematical Society*, 44:6, pp. 416-420, June 1938) which noted the connection with a torus and (in a footnote) that H. S. M. Coxeter had considered the four dimensional cross polytope connection, counting 384 pandiagonal magic squares. C. Bunio, E. Emberly and P. Loly (*Physics in Canada*, 53, front cover and p. 53) have published another variation of the square torus. The four pairs of letter choices in the MBTI Type Table may be thought of as the endpoints of four Cartesian axes; i.e., the 4D cross polytope. We remark that the Type Table compresses 4D to 2D; i.e., the horizontal direction codes the end points for the two dimensions of S/N and T/F, while the vertical direction codes the end points for I/E and J/P.

Because the psychological connection would not appear to benefit from a numerical labelling of the 16 types (compare with the nine numbered types of the enneagram system), I only recently checked out an idea that emerged from reading a detailed psychological discussion of the well known 3*3 Chinese magic square of the integers 1 to 9 (M.-L. von Franz, *Number and Time*, Northwestern University Press, 1974). The context there was closely related to the binary character of the broken (yin) lines and full (yang) lines of the trigrams associated with this magic square.

The idea then is to assign a binary zero or one to each pair of dimensions, and then to render the four resulting digits into a single integer; e.g., taking I=0, E=1; N=0, S=1; F=0, T=1; and P=0, J=1, translates INTJ into 0011. If a general four letter code is then converted to a single integer by the following algorithm:

$A * 2\hat{\ }3 + B * 2\hat{\ }2 + C * 2\hat{\ }1 + D * 2\hat{\ }0 + 1$

With the "1" added to start the counting at "1" rather than zero, we find $0 * 8 + 0 * 4 + 1 * 2 + 1 * 1 + 1 = 4$ for the corresponding integer. I fully expected to find either a magic square (all rows and columns adding to give the magic constant of 34), or even a pandiagonal (diabolic) magic square (with the addition of all split diagonals having the same sum). The complete translation of the Type Table into the sequence 1 to16 according to the given algorithm is as shown in Figure 2:

8	6	2	4
7	5	1	3
15	13	9	11
16	14	10	12

Figure 2.

To my surprise I had found a 4*4 square with only diagonal and split diagonal ("a purely pandiagonal square", if we omit the rows and columns of pandiagonal magic squares) magic sums, meaning that none of the rows or columns have the magic constant. Checking the up and down diagonals and split diagonals gives:

$$34 = 8 + 5 + 9 + 12 = 6 + 1 + 11 + 16 = 2 + 3 + 15 + 14 = 4 + 7 + 13 + 10$$

$$= 4 + 1 + 13 + 16 = 2 + 5 + 15 + 12 = 6 + 7 + 11 + 10 = 8 + 3 + 9 + 14$$

What about the generation of other members of this family? Clearly interchanging "I" and E causes no more than a vertical flip, and similarly for the horizontal flip due to interchanging S and N. Interchanging J and P swaps the middle rows with the outer ones and preserves the properties listed, as does a swap of F and T for interchanging the middle and outer columns. Thus we expect 4 rather than 16 members of the family on that basis. One can run the translation algorithm in 24 ways through placing one of the four letters first in four ways, times the three choices for the second letter, and times a choice of two for the third letter. Gideon Garland has helped the author check that the properties are maintained on application of the interchanges of the MBTI letters with the given algorithm. All in all, one expects one expects a factor of 4 from the letter exchanges and a factor of 24 from the algorithm, resulting in

96 members of this purely pandiagonal family, which is less than the number of 384 for the pandiagonal magic squares (those that also have the row and column magic sums).

We end with the thought that the compression of two dimensions into one dimension for these 2D squares can be extended to 4*4*4 cubes which encode six dimensions of bipolar variables. Is this cube split diagonal?