1 16.142 Supplementary Notes [mnp142]

5 Feb. 2001

1.1 Introductory Lecture

We read through the course outline and discussed how this course moves on from the topics in **16.142 "New Physics"**, given first in Fall 1997. A similar approach is anticipated. One chapter was omitted from 16.142, ch. 6 in 1998 and 1999, on Temperature, Heat and the Second Law of Thermodynamics, which now becomes essential for the present course, e.g. the cooling of the Big Bang to the current background temperature of 2.73 K. Another aspect concerns the "direction (or arrow) of time". As Prigogine has reminded us in a recent issue of Science [ ], most of physics is time reversible, and yet it is clear, for example, that the effects of an explosion cannot be reversed.

1.2 Interference and Diffraction

We will add some material at this point for understanding the resolving power or "seeing" ability of optical instruments such as the telescope and microscope, because these topics will have a strong bearing on how we get information on the physical world. This will include X-ray diffraction for the determination of the structure of large molecules that have been crystallized.

For chapter 13 on materials, a useful resource is Rodney Cotterrill's *The Cambridge Guide to the Material World*. Then we take a look at nuclear physics in ch. 14, elementary particles in ch. 15

December 23rd, 1897 marked the Centenary of the discovery of the electron by J. J. Thomson, and we also have the 50th anniversary of the invention of the transistor in 1947. Time magazine selected Andrew Groves (Andras Grof) of Intel as Man-of-the Year for 1997, because of the continuing impact of the microprocessor.

The four elements of the Greeks, earth, air, fire and water have by now been superseded by the electron, proton, neutron and photon - the last being massless (ch. 16). {neutrino)

The definitive information about the atomic particles comes from gas discharge experiments in which the beams are deflected by electric and magnetic fields. The invention of a pump for producing low pressures around 1700 by Guericke and improvements in glass blowing for scientific experiments were in place before batteries were available. The early discharge experiments, as well as electrolysis ones, were done with static electricity from Leyden jars.

In 1886 Goldstein had observed "canal" rays coming out the back of a perforated cathode. Wien (1898, 1907, 1910, ...) found that these canal rays had values of e/m much smaller than for electrons. Thomson (1907, 1909, ...) developed the "parabola" method for studying positive rays, including the ions of monatomic (both without the neutralizing electron - the proton, AND with an extra electron which made it a negative ray) and diatomic hydrogen, as well as oxygen, carbon, neon, mercury, and many other ionized molecules. It should still be possible to find a more precise date for the proton. Rutherford in 1899 is credited with the discovery of alpha and beta (electron) rays in radioactive atoms. Bequerel in 1896 discovered radioactivity by its effect on photographic film, and Pierre and Marie Curie in 1898 discovered radium and polonium as the sources of this radioactivity. The neutron was not discovered until 1932 by Chadwick who analyzed Bothe's 1931 experiment.

Talked about the microprocessor, the first being Intel's 4004, which have many transistors. The latter act as diodes with control valves or taps, much the same way that kitchen or bathroom faucets work to control the main flow.

Connections came to mind with several projects last term: Kristin and Derek for the crystal radio; Michael for Computers; Justin for a DC power supply; Nik for electric motors (Tesla); Christine for the aurora.

In connection with the Second Law of Thermodynamics, which concerns the decay of physical systems from more ordered to less ordered states, the **Game of Life** was raised as a model for the thinking about how systems of atoms or molecules might grow, die, reproduce, move, ... We all played the game on a large sheet with a square grid of cells and "checkers" of two colors. A new cell is born in the next generation if exactly 3 of its 8 surrounding cells is occupied in the present generation. A cell dies if it has one or fewer, and if it has more than 3. [See P. W. Adkins, *The Second Law of Thermodynamics*, or John L. Casti, *COMPLEXification: Explaining a Paradoxical World Through the Science of Surprise.*] Life is an example of cellular automata which have been devised in one, two and three dimensions.

Chaos is a related topic and at the end a question reminded PDL of the recent video "*Colors of Infinity*" hosted by Arthur C. Clarke (2001, ...) which had a good discussion of chaos, fractals and iterated maps (more later on $z^{'} = z^2 + c$, ...)

1.3 Discussed ch. 6.

Temperature conversion: $°\mathrm{F} - 32/(212 - 32) = °\mathrm{C}/(100 - 0)$ gives formula in text.

Absolute zero on the Celsius scale is $-273.16\,°\mathrm{C}$.

The absolute Kelvin scale is then measured so that $0\,°\mathrm{C}$ is $273.16\,\mathrm{K}$ (Note: no "degree" symbol is used in this scale).

Heat capacity.

Latent heat of freezing ... ("icing fruit and flowers to combat frost in Florida")

Triple point - ice, water, vapor

efficiency of engines

[heat pump]

Arrow of time (Ilya Prigogine, Science,

Thursday, Jan. 15, 1998

Finished ch. 6.

[PDL: remember to talk about Boltzmann!]

1.4 Second half of lecture:

Discussed interference and diffraction (see ch.8, p.), handed out 4 pages from PDL's **16.353 Notes**, and watched diffraction experiments on a 1978 video made in the TV labs in the basement of the Armes Building (the video is in the Science Library, and Elmer Hywarren and Peter Loly published a corresponding article in Wireless World, later finding it requested for an applied physics textbook (Physics 2000, Longmans) in New Zealand).

2-sources: the path difference is $d \sin(\theta)$ and constructive maxima occur when this is equal to n , i.e.

$d \sin(\theta) = n\lambda$, where $n = 0, \pm 1, \pm 2, ...$

N-sources: the principal maxima are still given by the same equation, but smaller secondary maxima appear between them. A diffraction grating may have 10,000 lines per inch, and produce a very strong separation between different wavelengths (colors).

ONE wide source (single slit or aperture): the model for the large objective lens in telescopes, binoculars, microscopes, ... Here the MINIMA are given by

$a \sin(\theta) = n\lambda$, where $n = \pm 1, \pm 2, ...,$

where a is the diameter or width of the opening. Note that $n = 0$ gives a maximum here!

X-ray diffraction from crystals (Bragg): the maxima are given by a similar result:

$2d \sin(\theta) = n\lambda$, where $n = 0, \pm 1, \pm 2, ...,$ and now d is the spacing between parallel planes of atoms.

The video showed many examples, including the Rayleigh criterion for detecting two stars or car headlights in terms of the angle between the beams and the angle between maximum and first minimum in the one-slit case.

THE BOTTOM LINE: resolution depends on the wavelength. One cannot see anything smaller than a wavelength of light, or electron waves, ...

1.5 Began ch. 13.

After a discussion of tetrahedral bonding for semiconductors, silicon, germanium and diamond, and their placement in column 4 of the periodic table, we could see how gallium arsenide they (bracket germanium) might have the same structure (it is used for the red laser diodes).

Then we got to electric properties on p. 355. Distinguish between conductors, insulators and semiconductors - we spent most time on the latter. The band structure approach to crystals explains conductivity differences in terms of energy bands in which electrons can move freely throughout the crystal, and energy gaps in which they cannot move. An insulator has a large energy gap separating filled states in the valence band from empty states in the conduction band, say 5 electron volts (eV, the energy of an electron charge raised by one volt), a semiconductor about 1 eV, which is lessened greatly by impurity states in the gap. A conductor has empty states above the highest filled level, the

Fermi level, either due to odd valence leaving it half full, or because of band overlap.

A flat 2D picture of the 4 covalent bonds in the tetrahedral structure is shown on p. 357 - remember that the solid is really three dimensional.

Skip p.358 - 362/2 for now (superconductors).

Phosphorous doped silicon gives excess $(5-4=+1)$ donor electrons for type "n" semiconductor in energy levels about $1/100eV$ below the conduction band, while aluminum doped silicon gives a deficit $(3-4=-1)$ acceptor electrons for type "n" semiconductor in energy levels about $1/100eV$ above the valence band. The latter missing electron may described as a "hole".

A diode has one side "p" and the other "n" (flip Fig. 13-10 to make direction same as 13-9 and 13-11).

Current flows easily through a diode from p to n (low resistance), and much less in the opposite direction. This is the basis for rectification. One can show this with a basic multimeter.

Solar cells employ a transparent pn diode, one layer has 15% efficiency, and a double visible-UV cell reaches 21%.

Boltzmann's factor: $\exp(-E/(k_BT)$

We gave an argument based on multiplying probabilities and adding energies for the exponential variation with energy.

$\exp(A) \times \exp(B) = \exp(A+B)$

P.S. make sure that you understand the graph of the exponential function.

The negative sign is needed to make higher energy states less probable, while temperature increases the probability of higher energies. The constant is known as Boltzmann's constant, $k_B = 1.38 \times\ 10^{-23} J/K$.

Example: for $T = 300K$ (about room temperature), the energy in "k_BT" is about $1/40\,\mathrm{eV}$, where $1\,\mathrm{eV} = 1.6 \times\ 10^{-19} J$.

[By comparison, the Fermi energy of a metal is about $50,000K$.]

Video: We spent the last 15 minutes watching the beginning of Steven Hawking's "Cosmos". In future we will set aside time each Thursday and try to finish it by the end of term.

Talked for about superconductivity in ch. 13 for 15 minutes, using transparencies from Serway's "Physics" for high temperature superconductors, e.g. YBCO - yttrium barium copper oxide.

1.6 chapter 14

Then began ch. 14 by recollecting $E = mc^2$ (see page 125-6 in 16.142, and page 451-2 later). For the atomic system of the hydrogen atom, the binding energy that holds the electron closest to the proton at a distance of half an Angstrom $(10^{-10}m)$ was $-13.6\,\mathrm{eV}$, and got less strong by dividing by n^2 for larger orbits.

For the protons and neutrons in the nucleus in a very much smaller volume of diameter

$10^{-14}m$, the binding energies are measured in millions of eV, $M\,\mathrm{eV}$.

The nucleus of heavy hydrogen, the deuteron of deuterium, consists of one proton and one neutron $(p+n)$ bound by the strong nuclear force, and has a binding energy of about $2.2M$ eV, or $1.1M$ eV per nucleon. The nucleus of tritium ($p + 2n$) is bound by $8.5M$ eV, the nucleus of He-3 by $7.7M$ eV, and the alpha particle nucleus of He-4 ($2p + 2n$) by $28M$ eV. Up-to-date masses and binding energies are available on a WWW page: **www.physics.utoronto/UPSCALE**

N.B. that list also reports the "mass excess" or $M - A$. For the neutron this has a value of $(+)8M$ eV, while for Cu-63 it is $(-)65.6M$ eV.

The proton was discovered as one of Thomson's positive rays around 1900, the neutron in 1931.

Figure 14-2 shows a plot of the atomic number (number of protons) Z v. N, the number of neutrons. Heavier nuclei tend to be neutron rich, with a range of isotopes (different values of N) for each element, the most stable usually bracketed by increasingly unstable ones.

Note the notation used for nuclei, e.g. ${}^{4}_{2}He$, with the presuperscript drawn usually over the presubscript $\left({}^{A}_{Z}X\right)$.

The important binding energy curve is not given in the text (Serway, Fig. 45.3) and will be supplied from another source. This curve rises from the low mass numbers ($A = Z + N$), through a flattish region around $A = 20 - 40$, before a steady decline to uranium. This curve helps understand how very light nuclei can be fused at very high speed (or temperature) to form more stable heavier nuclei with the release of energy (as kinetic energy of motion) due to the increased binding energy. This is the nuclear fusion process that powers the sun and the hydrogen bomb. N.B. Fig. 14-13 misprints photon for proton in the fusing of He-3's.

Similarly, uranium-235 or plutonium-239 can fission into two large fragments (Fig. 14-10) and several neutrons, also with the release of energy due to the greater binding of the fragments. Usually the fragments have a mass ratio of about 3:2, and the fission is triggered by a "slow" neutron which destabilizes the uranium-235.

Nuclear models: Bohr proposed a liquid drop model which would be as spherical as possible, except when dumbbell oscillations lead to fission. Maria Goeppert-Mayer and Jensen used a shell model (analogous to Bohr's atomic model) and won the Nobel Prize in 1963. One feature that merged were magic numbers of Z and N, namely: $2, 8, 20, 28, 50, 82, 126, ...$ While the reasoning is beyond our scope, the smaller numbers are suggestive of a dumbbell (2), crossed dumbbells (in 2D: $2+2 = 4$), the 6 corners of an octahedron which could house a dumbbell, the 20 corners of a dodecahedron, and perhaps so on? Some physicists have proposed crystal models of nuclei.

Four forces: gravity, electromagnetism, strong nuclear, and weak nuclear.

The strong force overcomes Coulomb repulsion between the protons to bind nucleons, and forms an attractive potential well (quantum box) with energy levels that are filled according to Pauli's exclusion principle by spin up and down protons, and a separate set for neutrons of slightly lower energy. Think of C-12.

Radioactivity: Becquerel in 1896 found 3 types of radiation: alpha (helium nuclei), beta (electrons), and gamma rays (photons). Marie Curie and Pierre Curie studied radioactivity of pitchblende and other ores containing radium.

Pauli in 1930 predicted that a strange new particle was emitted in beta decay, the anti-neutrino which was found in 1956. There is a great deal of interest in whether it has a small mass or no mass (like the photon). The neutrino will be seen again in later chapters.

Half life (Fig. 14-8): the curve follows from a simple argument: for a large number of as yet undecayed nuclei, $N(t)$, the number that will decay in a small time interval "dt" is proportional to the product of these factors with a coefficient characterizing the particular nucleus and given decay: $dN = N(t)dt$. This differential equation solves to produce the exponential decay curve: $N(t) = N_0 \exp(-\lambda t)$, where the half life ($\tau$) is closely related to the reciprocal of λ, which has the correct units of time. C-14 has $\tau = 5700\,\mathrm{y}$, some are much shorter, others much longer.

Decay chains: see Fig. 14-9.

1.7 Chapter 15

(continued with ch. 15 begun earlier?)

In 1931 Pauli predicted that a new particle, the neutrino, was needed to explain the energy spectrum of beta decay (this has a maximum electron energy, but also a broad range which indicates energy taken up by something else). We now know that in addition to beta decay from the surface of a nucleus with the emission of an electron and the conversion of a neutron to a proton which remains in the nucleus, the free neutron decays similarly with a half life of some 12 minutes. This may be indicated by:

Neutron decay: $n- > p + e^- + \bar{v}$, where overline is used to indicate the antineutrino.

We (now) know that the neutron has enough mass to convert into the proton and the electron and have something left over. For this reason the free proton cannot decay (think about the masses).

However in a nucleus the conversion of a proton is often expressed by

$p + \gamma - \ > \ n + e^+ + \nu$, where the e^+ is a positron. We understand this by having a photon in the nucleus decay into an electron-positron pair, with the high energy electron then colliding with a proton in the nucleus to form a neutron and a neutrino, with the positron from the "pair" escaping as positive beta emission.

1.7.1 Antiparticles

We have mentioned Einstein's $E = mc^2$ before, but now need a related equation:

$E^2 = p^2c^2 + (m_0c^2)^2$, where E is the total energy, p the momentum, c the speed of light, m_0 the rest mass. In 1928 Dirac took the square root of this to find E, and since we know that $(-E)^2$ is the same as E^2, interpreted the negative energies in the following way:

SEE SKETCH

for $p = 0$ (not moving)

$E = \pm m_0c^2$,

with one branch increasing
and the other decreasing further
as p increases (particles moving, $p = mv$).

Take all the negative states to be filled initially (solid circles), then excite one to the upper branch to become a positive energy electron, with its "absence" interpreted as a positron of positive energy. This requires a minimum energy of $2m_0c^2$ for the creation of such a pair (see above for positron emission. The rest energy of an electron is about $0.5M$ eV, and so the pair requires twice this energy. This energy is easily found in moderate to heavy nuclei since the binding energy curve measures up to $8M$ eV per nucleon. [A single proton has a mass of $1000M$ eV or $1G$ eV.]

N.B." PET" or positron emission spectroscopy is based on the annihilation of electrons in matter with positrons from a radioactive source which form two gamma rays (high energy photons) - see page 422.

Particle Zoo (page 419)

From a transparency various objects were described in terms of their constituents and the type of packing (close packed as in crystals or liquids, or not):

virus; molecule; atom (proton and electron in a planetary type of orbit); positronium is an electron and positron in a dumbbell rotation about a common centre of mass midway between the two equal masses (it has strong similarities to the hydrogen atom); nucleus (close packed protons and neutrons, e.g. the liquid drop model, crystal models, the alpha particle as a close packed tetrahedron, etc.); nucleons (neutrons and protons made of quarks which have charges $\pm 1/3e$ or $\pm 2/3e$ - see Table 15-2, page 423: e.g. proton = uud with charge $+2/3 + 2/3 - 1/3 = 1$; neutron = udd with charge $+2/3 - 1/3 - 1/3 = 0$).

Hadrons: quarks combine in pairs to give mesons, e.g. $\mu^+ = ud$ with charge $+2/3 + 1/3 = 1$; - ...; K mesons as well (TRIUMF in Vancouver means TRI Universities Meson Factory - the billion dollar Kaon factory successor was cancelled a few years ago) ; in triples for the baryons (n, p, and many others); possibly in quartets as well?

Leptons: these are the electron, mu (often confused with the mu meson hadrons, especially in early literature), tau - the mu and the tau appear to be heavy electrons; also the massless neutrinos which accompany electron, ...; and of course all their antiparticles.

Four forces: each is mediated by a boson particle, the electromagnetic photon and the conjectured graviton are massless, while the weak forces W and Z bosons, and the (indirectly observed) gluon for the strong force have mass. [see page 426]

[Ended with about 20 minutes from Cosmos (Hawking).]

(finish ch. 15; begin ch. 16)

1.8 Chapter 16

Discussed force as exchange of particles (page 426). Note that the graviton is conjectured, and the gluon only indirectly observed. [see also Lederman and Schramm]

Grand Unified Theory: see page 427/8 and the diagram. Gravity separates from the other forces at $10^{19} GeV$ (around $10^{-38} seconds$ after the Big Bang, the closest time to which is the Planck time of 10^{-43} seconds); the strong separates from the electroweak at $10^{15} GeV$ at $10^{-33} seconds$; while the weak and electromagnetic separate around $100 GeV$ or $10^{-10} seconds$ after the Big Bang.

Theories of Everything (TOE): the present hopes centre on an eleven dimensional N-brane theory (recent Scientific American article by Duff).

Note also the reference to **Theories of Organization (TOO)**.

Thursday, February 26, 1998

2 Two Frames of reference:

S is stationary frame for an observer on the ground, S' is moving to the right of S at a speed v for another person on a moving a train . Initially at time $t = 0$ the two frames coincide. Take the x-coordinate to run to the left in S; similarly $x\prime$ in S'.

N.B. we change the use of u, v on p. 437 to be consistent with page 437 onward - this is also in line with convention in many texts.

3 The Galilean transformation (GT)

is easy to write down:

$x\prime = x - vt$;

$y\prime = y$ (no change in that direction); $z\prime = z$;

$t\prime = t$ (the same time everywhere).

It follows on taking small time intervals and the corresponding changes of distance that:

the x-component of velocity of an event in S is given by

$u = x/t$, and similarly, $u\prime = x\prime/t$.

GT gives for constant v: $x\prime = x - vt$, and dividing by t gives $u\prime = u - v$, or $u = u\prime + v$ which makes sense for what the stationary observer sees happen when the person in S' throwing a ball a speed $u\prime$ to the right. [see Fig. 16-2 with the notation revised as stated.]

NOW observe that if the moving observer shines a torch to the right, the light of speed c, or about 3×10^8 m/ s, according to this velocity transformation would appear to the stationary observer to move faster than the speed of light. This does not happen (Michelson and Morley in 1880's), so we seek a better transformation.

4 The Lorentz transformation (1890)

followed from Maxwell's equations but was not properly appreciated until Einstein's Special Theory (no acceleration) of 1905. The GT equations are modified:

$x^{'} = \gamma(x - vt)$;

$y' = y$ (no change in that direction); $z' = z$;

$t' = \gamma(t - vx/c^2)$, NOW NOT the same time everywhere!

where $\gamma = \frac{1}{\sqrt{(1-(v/c)^2)}}$ and is 1 for $v = 0$ and tends to infinity for $v \to c$. [**plot graph**]

The velocity transformation is now: $u'_x = (u_x - v)/(1 - u_x v/c^2)$, or $u_x = (u'_x + v)/(1 + u'_x v/c^2)$,

Let us apply this to the light beam where we take $u'_x = c$ and find $u_x = (c + v)/(1 + cv/c^2)$ which simplifies to $v = c$ and shows the constancy of the speed of light. (See page 449)

Einstein gave a full treatment of moving events, including the variation of mass with speed, i.e. $m = m_0$, and the relation between mass and energy, namely $E = mc^2$, in which plain m is the moving (inertial) or relativistic mass, m_0. [see page 451]

Other useful results are that momentum $p = mv$ or $(m_0)v$, and $E^2 = p^2c^2 + (m_0c^2)^2$ (discussed earlier on page 11 of these Notes). Incidently, force is STILL given by the rate of change of momentum (Newton's second law), and not by a simple change to "$F = ma$".

If $m_0 = 0$, we still have $E = pc$ and for a light photon (speed c) a momentum $p = E/c = hf/c = h/\lambda$, using Planck's $E = hf$ and $c = f\lambda$.

4.1 Time Dilation:

a muon (mass 207 times the electron) has a lifetime of $2.2\mu s$ and travels $650m$ if moving near the speed of light in that time. Generated by cosmic rays at high altitude, they were observed on a mountain top in California AND also $4km$ lower at sea level! Time dilation (page 443) gives a slower decay by the factor which is 7.1 for a speed of $0.99c$, giving a "lifetime" of 2.2×7.1 or $16\mu s$ and a distance travelled of $4.7km$. (SMM/2, p. 16).

Ended with about 20 minutes from Cosmos (Hawking).

Tuesday, March 3, 1998

Note for page 445: at the bottom of the age the Lorentz factor should be written as about $1 - 2 \times 10^{-15}$, so that the last of the "9's" should be "7 or 8".

4.2 Length Contraction:

this is the shortening of a moving object by an amount given by dividing by (see page 447).

4.3 General Relativity

Consider the wave equation for a vibrating string:

The first term is the speed squared, the second is the curvature ($\mathcal{C}$), and the right-hand side the acceleration: $v^2\mathcal{C} = a$.

Using $F = ma$, or $a = F/m$, we can write $F = mv^2\mathcal{C}$, which shows how force arises from curvature - this makes sense for a curved string.

Note for page 455: Under section 3. the Doppler effect or shift IS NOT the same as the gravitational red shift.

Photon falling under gravity

Above we showed that for a light photon $p = E/c = hf/c = h/\lambda$. Then the inertial mass "m" is given by "$p = mv$" divided by "c" or $m = p/c = hf/c^2$. There is no rest mass, and the inertial mass depends on frequency. Clearly if the inertial mass rises in the Earth's gravitational field, it will lose energy (but not speed) and shift to a longer (redder) wavelength.

4.4 Chapter 17-20

Dust clouds, rich in hydrogen and helium, but containing all the heavier elements accumulate throughout the universe in galaxies. In our Milky Way ("MW") spiral galaxy (Fig. 17-1 seems to show part of the Milky Way with a schematic of the Solar System ["SS"] superimposed) the dust also began to form rotating nebula clouds which formed our solar system (see Fig. 17-3 AND also the COBE image on p. 622), and similarly for the other star systems. See p. 469 for the Orion Nebula.

We think that the universe after the Big Bang should have had no net angular momentum, (remember that $L = mvr$, and for many bodies is a vector that must be summed over all of them) but that individual lumps could rotate with non-zero angular momentum as long as the sum zeroed. Similarly for the galaxies, one expects that they would rotate along all possible axes (recollect the right hand rule linking rotation and its axis). We should see spiral galaxies with all possible orientations with respect to us! (Some cartwheel, some are edge on, ... see Figure 22-2, 3, 6, 10).

4.5 Solar System

Fig. 17-1 (p. 462) indicates that the rotation of the SS is not in the same plane as the MW (which we see make an angle with the horizon) . Fig. 17-2, 3, 4 show our SS, outside which are the Kuiper Belt and the Oort Cloud of icy cometary objects. The Kuiper belt is a disk that stretches from the orbit of Neptune at $30AU$ ($1AU$ or Astronomical Unit is the mean distance of the Earth from the Sun, about $150 million km$) past Pluto at $40AU$ to some $100AU$. The data for KB are quite recent and at July 12, 1997 stands at 55 items, e.g. no. 1: 1992 QB1 at 43.9 AU with a diameter of $283km$; no. 4: 1993 RP $39.3AU$ $96km$; no. 18: 1994 VK8 $43.5AU$ $389km$; ... By contrast the Oort Cloud is about 0.5 to 1 light year (ly) distant ($1ly = 9.5x10^{15}m = 6.3x10^5AU$; $1Parsec = 3.26ly$) and is a spherical shell. The nearest stars are Proxima Centauri at $4.2ly$ and magnitude 11.3; Alpha Centauri A and B $4.3ly$ mags. .33 and 1.7 (a binary); Barnard's star at $5.96ly$. Sirius A at $8.7ly$ has mag -1.47 (the Sun has mag -26.9 and each increase in magnitude by 1 decreases the brightness by a factor of 2.5 - one needs at least binoculars to see stars fainter than mag 6.

4.6 The 3-body problem

Gravitational encounters between two or three of these objects can result in a radically changed orbit that enters the inner SS, and some may enter the Earth's atmosphere, and even impact if large enough. [Lagrange?]

Newton's explanation of Kepler's elliptical orbits for any two body system, e.g. the Earth moving around the Sun, or the Moon ... Earth, etc. assumes that there are only two bodies to describe, and thus ignores all others, e.g. the Moon when focussing on the Earth-Sun issue. In the 1770's Laplace (1749-1827) considered the restricted three body problem of a third very light mass among two massive ones and obtained an analytic solution (exact) for such very special cases. [PDL: 16.233, K.R.Symon's Mechanics, 3rd edition, p. 286-291). The results are interesting in that two so-called Laplacian stable points are found in the Moon's orbit around the Earth (at 60 degrees to the position of the Moon forming equilateral triangles with the Earth and the Moon) if one of the large masses contains more than about 96% of the mass (this is OK for the Earth-Moon system where the mass ratio is about 81). [L. G. Taff, Celestial Mechanics, p.192, gives the formula for 0.038521 which leads to a ration of 25 or more. Taff also discusses Hill's problem on p. 200.] However, in general the $3-$body problem is not soluble and one must resort to numerical calculations. With the advent of powerful computers by the late 1970's it became possible to study some interesting cases. Physics Today in Jan. 1979 (page 49-50) reported some of Leigh Palmer's (Simon Fraser) results for three-body systems (the diagram is for a star in a close encounter with a binary), in particular PDL in a visit to SFU around 1980 remembers being shown the $3-4-5$ problem of masses 3, 4, and 5 placed opposite those sides of a Pythagorean $3-4-5$ right angled triangle. Starting from rest they move under gravitational attraction towards the common centre of mass en route to a very close encounter.

4.7 Symmetric collisions

If two equal masses move from rest towards each other under gravity, they will bounce back and come to rest before repeating the motion in an oscillatory fashion. The same is true for an equilateral triangle of equal masses, and a square of four, or a regular tetrahedron of four, etc. These are not very interesting!

4.7.1 Binary Formation

Palmer was able to integrate the equations of motion through the close encounter by using high precision arithmetic and a fast computer. Single precision is $6-7$ digits, double about $12-13$ digits and is the minimum for serious work, and quadruple is nowadays available. Palmer required much more. The final scenario in his 345 problem is a close binary moving away from the triangle in the opposite direction to the third mass. A little thought shows that the close binary has a much greater potential energy of binding than the initial triplet, so that kinetic energy is available for the binary to escape the attraction of the third particle -

both are in a sense ejected from what might be a relatively localized situation like Laplace's. So this binary argument then applies to any 3-body system with the right range of masses - binary star formation then becomes common as single stars - as is indeed observed.

[see also Physics Today, Dec. 1971, p. 17 for the orbit of Toro.]

4.8 The Planets -

Gustav Holst!

With 9 planets and many moons our SS is clearly an N-body problem. However the $2-$body description is useful when the motion is dominated by a pair due to the others being so far away that their gravitational effect can be treated as a "perturbation" on the main motion. Thus all 9 planets undergo almost perfect Keplerian elliptical motion, as do the various planet-moon systems. [The rings of Saturn are thought to exhibit chaotic effects due to the proximity of a myriad of small objects.]

Once the dust has formed planetesimals and then planets (Fig. 17-3) the accretion in the planets increases in density and heats up so that the cores are very hot. Fig. 17-5 for Earth, 17-7 for Mercury; 17-8 for Venus; 17-9 for Mars; 17-10 for Jupiter, Saturn and Uranus.

With the help of the **Magnificent Cosmos** Spring 1998 (vol. 9, Number1) special issue of *Scientific American* we can piece together some of the more interesting aspects of our planets, and begin to connect this story with the emerging evidence for massive Jupiter scale planets around nearby stars in the MW (Globe and Mail, Sat. March 7, 1998 and listed Web sites).

[The 2nd edition of the sister textbook - *The Sciences - An Integrated Approach* by Trefil and Hazen, 1998, has a different table of planetary data than our present Appendix D, some updated information on the impact of comet Shoemaker-Levy on Jupiter, Pluto, and a longer list of meteor showers. They also place the "Planets" after "Stars" and "Cosmology"; as well as Concept Maps for each chapter.]

[Close approaches of Algol and Gliese 710 to Oort Cloud - L. A. Molnar, R. L. Mutel, Iowa - when?]

March 12, 1998 in Nature: three colatitudinal large impact craters $210 Myr$ ago on the supercontinent of Pangea (Fig 18-9) are now identified with St. Martin, Manitoba (near Gypsumville in the Interlake - seen on geological maps via concentric rings of magnetic anomalies - no surface features visible), the New Quebec Crater in Manicougan, and another in France.

4.9 Terrestrial and Jovian planets -

this is done quite well in the text and also in the Magnificent Cosmos. We note the discussion of the formation of the Moon on p. 474/5; the curious relationship between the day and the year on Mercury - a ratio of 2 : 3 (Magnificent Cosmos p. 29); the retrograde rotation of Venus and its very long day, Uranus (almost in the plane of the ecliptic), and Pluto; the Asteroid belt between Mars and Jupiter - Ceres has a diameter of $770km$ and Eros $32km$;

Tuesday, March 24, 1998

Note some other relevant information on the layered Earth in Fig. 18-21, the Milankovitch cycles in Fig. 19-17 for the $23,000year$ top-like precession of the equinoxes (vernal on March 21, autumnal on September 21; solstices on Dec. 21 and June 21); the tilt of the axis from 21.5 to 24.5 degrees every $41,000years$; and the variation of the eccentricity over $100,000years$ - all due to the N-body effects of the planets (since the nearest stars are too far away). [see also Physics Today which arrived Mar. 16, 1998]

Chemical Evolution - the Miller-Urey Experiment - Fig. 20-9 and p. 585-6.

[for later: compare p. 634 picture of SN 1987A (Shelton) with "Science -2nd. Ed." which is more recent!]

BEGIN ch. 21 - THE STARS

5 Chapter 21 The STARS

[c:\My Documents\np21.99.tex]

These supplementary notes use Scientific Workplace's synthesis of LaTeX and Maple which enable proper mathematical typesetting with calculation and graph plotting harmoniously integrated.

6 The Sun

Hazen and Trefil, p.612:

All the output of the Sun ($4.24 \times 10^{23}\mathrm{k\,W/m^2}$) passes through concentric spheres of radius r, area $4\pi r^2$.

p. 612: $1.5kW$ falls on the Earth, 50 % gets to the surface.

At the Earth's orbit this area is $2.83 \times 10^{23}\mathrm{m}^2$.

(check: $4.24 \times 10^{23} \times 10^3/(2.83 \times 10^{23}) = 1498.2.$)

Sunlight takes about $8minutes$ to travel to the Earth:

$150 \times 10^9 m/(3 \times 10^8 m/s) = 500.0s$

p.614: Venus: multiply by $(150/108)^2 = 1.929$

to get $1500 \times 1.929 = 2893.5$ kW

$R_s = 698000km$,

$R_e = 6400km$,

$\frac{R_s}{R_e} : 109.06$

Stellar core (p. 615 with p. 617) is about 10% of the volume of the Sun:half the radius would give:

$(1/2)^3 \times 100 = 12.5$ %; $(1/3)^3 \times 100 = 3.7037$ %

so the picture underemphasizes the volume of the core.

HT p. 615: In the core the fusion process is effectively (combining 3 steps):

$6p \rightarrow^4_2 He^{2+} + 2e^+ + 2p + 2\overline{v} + 3\gamma$, recycling 1/3 of the protons.

Outside the core is a large radiative zone surrounded by a convection zone, and then the photosphere which we see at a temperature of about $6000K$. N.B. Photons produced in the core take $10,000 years$ to reach the surface of the Sun. The **corona** extends further out but is usually only seen by us during solar eclipses.

6.0.1 Compare the sizes (angular) of the Sun and the Moon to an observer on Earth:

The angle of the disk of the Sun at the Earth is

$2 \times 698000km/(150 \times 10^6 km) = 9.3067 \times 10^{-3}$ rad

The angle of the disk of the Moon at the Earth is

$2 \times 1738km/(384000km) = 9.0521 \times 10^{-3}$ rad

Compare these with the angle subtended by a $1cm$ disk at a distance of $1m$: $1/100 = .01$ rad, which is quite similar.

HT p. 615: enough H for $75By$ (Billion years)! In fact it will only burn H for a lifetime of $11By$ ($4.5By$ already used up!), then burn He to end up as a carbon cinder (see later).

Solar neutrino problem: only 1/2 to 1/3 of the expected number of electron neutrinos are observed - thought to oscillate to v and τ neutrinos - very current problem related to possible mass of the neutrinos, and the missing dark matter thought to be in the universe.

The Sun emits charged particles, protons and oxygen ions mainly, through coronal ejections, and these take about $3-4$ days to reach the Earth.

6.1 Speed of gas particles

The "average" speed of gas particles at temperature T degrees Kelvin and uniform pressure is known from the **nineteenth century kinetic theory of gases**:

$\frac{1}{2}mv^2 = \frac{3}{2}kT$, for $v = \sqrt{\frac{3kT}{m}}$

where k is Boltzmann's constant: 1.38×10^{-23}J/K.

diatomic hydrogen molecules at 300K: $\sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{2 \times 1.67 \times 10^{-27}}}$: 1928.4 m/s

protons at 100 million degrees: $\sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 1 \times 10^8}{1.67 \times 10^{-27}}}$: 1.5745×10^6 m/s

6.2 Pressure in gases

Take the **ideal gas law**: $pV = nRT$, solve for p: Solution is : $\left\{p = nR\frac{T}{V}\right\}$, where R is the universal gas constant. Initially the temperature rise in the gas accumulating under gravitational attraction to form the Sun is due to release of gravitational energy. The internal pressure rises for two reasons: the increase in temperature and the reduction in volume. When there is the product of temperature and density reaches the Lawson criterion thermonuclear fusion begins to ignite the protostar.

6.3 Protostars

The accumulating gas will be denser at the centre, but if we assume that for the moment that it is uniform, then we can talk about the variation of gravity as we approach a gas ball from outside and then move through its surface to the centre.

Since Newton we know that outside any spherically symmetric mass that the gravity is as for a point mass, i.e. inverse square law. Inside what counts is only the mass closer to centre, for a gravity decreasing linearly to the centre.

The pressure as force per unit area still increases:
$p \propto g/(4\pi r^2) \propto \frac{r}{r^2} = \frac{1}{r}$

These ideas can be applied to a set of spherical onion shells in order to describe a stratified or vary density star (and to planets like Jupiter), but the general outcome is the similar.

6.4 Stellar distances

Triangulation is used up to distances of a few hundred light years. Since the Milky Way has a size of about $100,000 ly$, triangulation restricts astronomers to stars closer than the width of our spiral arm. The discovery and study of Cepheid variable stars in this same region (Henrietta Leavitt) lead to a correlation between the distance and luminosity of these variables, which can be applied to very distant Cepheids well beyond our galaxy.

6.5 Why Cepheid Variables Pulsate

("Magnificent Cosmos", Scientific American Presents, Spring 1998, Volume 9, Number 1, page 94.)

These are young stars several times more massive than the Sun which pulsate because the gravitational force on the atmosphere is not in balance with the pressure of the hot gases. Singly ionized helium in the outer layers is doubly ionized by photons from the core making the layer opaque to more photons so that they push this layer outward to give an increase in size and luminosity. It then cools and captures an electron to become transparent again and shrinks to a smaller size before repeating the process many times. See the graphs in HT Fig. 21-10.

6.6 Absolute magnitude (Luminosity)

This is the actual power output of a star, often expressed in terms of the output of our Sun (1 unit) - see Fig. 21-11. The range is from about 1000 times less to $100,000$ times greater than the Sun. **Apparent magnitude** depends on the luminosity divided by the distance squared.

In Example 21-2 the luminosity of a **Cepheid variable** is given incorrectly as 150 - it should be 1.5×10^5. The distance is correct at $2600 ly$.

6.7 Hertzsprung-Russell Diagram

HT give an overall colored version in fig. 21-11 and focus on the future evolution of our Sun in Fig. 21-15.

6.8 Degeneracy Pressure

Here we expand on the remarks in HT (p. 631).

This is due to **Pauli's Exclusion Principle** wherein no two (fermion) particles may have the same set of quantum numbers. this means that two fermions exert a kinematical repulsion when very close, in addition to any Coulomb repulsion if they are charged. Two equations describe this pressure where E_F is the Fermi energy, m is the fermion mass and N/V is the particle density (per unit volume):

$E_F = \frac{\hbar^2}{2m}\left(3\pi^2 \frac{N}{V}\right)^{\frac{2}{3}}$

$p = \frac{2}{5}\left(\frac{N}{V}\right) E_F = (\text{constants})\,\frac{1}{m}\left(\frac{N}{V}\right)^{\frac{5}{3}}$

Since electrons are the lightest of the elementary particles, in an electron-proton plasma in the Sun, the electron degeneracy pressure (DP) is about 2000 times the proton's DP. When our Sun has finished its hydrogen burning and then its helium burning, it will be unable to burn its carbon. then it will cool and shrink in volume under gravitation until the electron pressure prevents further collapse. The Sun will then be a **white dwarf** star about the size of the Earth. This happens for stars up to the **Chandrasekhar** crossover mass of about 1.4 times the mass of the Sun. For more massive stars which burn carbon to produce still heavier elements up to iron before exhausting the fusion processes, a similar collapse occurs to a much denser object where the electrons and protons have fused to become neutrons (**neutron stars**), and the neutron DP comes into play, aided by an even smaller volume with a radius of about $15 kilometers.$

6.9 Novas

See ("Magnificent Cosmos", Scientific American Presents, Spring 1998, Volume 9, Number 1, page 56.) for V1974 Cygni 1992.

These are quite different from Novas, which are binary stars where the larger becomes a red giant and envelopes the smaller in a gas cloud which slows it down and pulls it closer. When the red giant has exhausted itself and become a white dwarf the smaller orbiting companion will lose material which falls onto the white dwarf, getting compressed by the high gravity until it produces a fusion explosion which blows off a lot of material. this can happen many times.

6.10 Supernovas

Manitoba's Ian Shelton was the codiscoverer of SN 1987A while working on a Canadian project at an observatory in Chile which sees the southern hemisphere.

It exploded in the Greater Magellanic Cloud, a galaxy closer than the nearest spiral (M31) galaxy.

6.11 Black holes

A simple calculation can be done for the radius in which light orbits a black hole.

Equate centripetal acceleration with gravitational force:
$m_i c^2/r = G m_i M/r^2$
cancel the photon's inertial mass for:
$c = \sqrt{\frac{GM}{r}}$, or $r = GM/c^2$

The Schwartzchild radius can be found by arguing that the kinetic energy for the escape velocity has to be doubled (see chapter 4) yielding:
$r_S = 2GM/c^2$
For the Sun this gives:
$2 \times 6.67259 \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2} \times 1.99 \times 10^{30}\,\mathrm{kg}/(2.99792458 \times 10^8\,\mathrm{m\,s}^{-1})^2$: $2954.\,9\,\mathrm{m}$, or $3k\,\mathrm{m}$.

For a neutron star of 3 solar masses and radius $1.5 \times 10^4\,\mathrm{m}$ $(15k\,\mathrm{m})$ the corresponding value of $9k\,\mathrm{m}$ is too small because it is still within the surface of the neutron star and so does not contain enough mass.

For a heavier star (30 solar masses) which can collapse beyond a neutron star to a higher density, the Schwartzchild radius is larger but the mass is now entirely inside.

6.12 Densities of Solar and Stellar Objects

	radius(m)	mass(kg)	average density
Moon	1.74×10^6 m	7.36×10^{22} kg	$7.36 \times 10^{22}/(4\pi/3 \times (1.74 \times 10^6)^3) = 3340$
Earth	6.37×10^6 m	5.98×10^{24} kg	$5.98 \times 10^{24}/(4\pi/3 \times (6.37 \times 10^6)^3) = 5520$
Sun	6.96×10^8 m	1.99×10^{30} kg	$1.99 \times 10^{30}/(4\pi/3 \times (6.96 \times 10^8)^3) = 1410$
white dwarf	earth	0.65 Sun	$0.65 \times 1409.\,1 \times \left(\frac{6.96 \times 10^8}{6.37 \times 10^6}\right)^3 = 1.\,2 \times 10^9$
neutron star	1.5×10^4 m	3 Suns	$3 \times 1.\,2 \times 10^9 \times \left(\frac{6.37 \times 10^6}{1.5 \times 10^4}\right)^3 = 2.\,7 \times 10^{17}$
black hole	Schwarzchild	30 Suns	

For white dwarf this agree well with a figure of 1000 kg per cubic centimeter, and for neutron stars with a f figure of 10^{11}kg per cubic centimeter.

7 Chapter 22 Cosmology

[c:\My Documents\np22.99.tex]

7.1 Redshift

The relativistic Doppler shift is given by

$f_{observed} = f_{source}\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$

V. Slipher 1912 found a redshift (lower frequency) for most galaxies . This means that they are moving away from us. At the time it was not known how far away were these galaxies.

A few nearby have a blueshift, meaning that they move closer to us.

7.2 Hubble, 1924

Used Cepheid variable to measure the distance of the Andromeda galaxy in Orion (the only galaxy visible to the naked eye at $900,000ly$. Later a better calibration of the Cepheids revised this to $2Mly$. See Fig. 22-9 for the Milky Way and Andromeda in the upper left corner.

7.3 Hubble's Law 1929

Using more data and a plot such as that in Fig. 22-7, and described the trend by a linear relation:

$v = H \times d$

If we assume same velocity over time, then a special $time = \frac{d}{v} = \frac{1}{H}$ is found for ALL galaxies. This is the time since the Big Bang.

Hubble found a slope 50 to $100km/s/Mpc$ for time of 8 to $16Byr$.

7.4 Big Bang

Georges Lemaître in1927 proposed a primeval "atom" whose explosion resulted in these velocities. Later George Gamow in 1948 termed this the Big Bang. In the same year Hermann Bondi, Thomas Gold and (Sir) Fred Hoyle proposed the Steady State theory of the universe.

7.5 Large Scale Structure of the Universe

See HT 653. Fig. 22-9 and a diagram in Gurzadyan & Kocharyan's "Paradigms of the Large Scale Universe" by S. Simpson from "Sky and Telescope" in 1987, show the Milky Way and its close-by minor galaxies, including the Large (home of SN 1987A) and Small Magellanic Clouds, and the recently discovered Sagittarius dwarf galaxy. Our Local Group of major spiral galaxies includes Andromeda ($2Mly$) and is moving in the Virgo Supercluster (Figure at bottom of p. 654) which has a diameter of $50Mly$. In turn these are part of the Virgo Local Supercluster, and a group of these (including Perseus and Hydra Centaurus) are moving towards an unseen mass called the Shapley concentration (Great Attractor) some $180Mly$ away.

7.6 The Great Wall

Margaret Geller and her colleagues use 3D graphics to understand the distribution of galaxies from a large scale survey. Fig. 22-20 shows a 120° wide by 12°

thick (flattened) wedge of sky which has a scale of $15,000 km/s$ in recessional velocities. It looks like a slice through a foam of soap bubbles where one sees mainly the thin soap film on which the galaxies are concentrated, and relatively empty voids. This was totally unexpected.

7.7 Dark Matter

Vera Rubin studied the rotation velocities of spiral galaxies and found that they do not slow down away from the centre in the way that we understand from the planets in our solar system. The role of a massive centre like our Sun must be replace by a large amount of unseen mass. At present we do not know what this is, but it is thought to be 90% of the total mass in the universe!

7.8 COBE

George Smoot lead the team that studied the microwave measurements from the Cosmic Background Explorer. This shows ripples at the level of $10^{-4}K$ in the black body radiation at $3K$ left over from the time about $300,000 years$ after the Big Bang when the universe became transparent to electromagnetic radiation.

Symbols mol m^2 Btu cal erg J kcal dyn N lb Hz Å cm ft in km m mi g kg ° rad hp kW W atm Pa sr °C °F K h mn s y m^3 m/s $\mathrm{m/s}^2$ 1.6×10^{-3} eV