A Logical Way of Ordering the Hexagrams of the Yijing and the Trigrams of the bagua.

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INTRODUCTION AND CONTEXT

In describing a new, logical ordering of the hexagrams and trigrams, I hope to increase understanding of the structure of the Yijing and the eight-fold bagua *[1].* The traditional, or "current" *[2],* sequence of the sixty four hexagrams in the "received text", going back at least as far back as 296 BCE*,* has always been a puzzle because, aside from a pairing of the four symmetric ones with their complements and the remaining ones with their inverts, no other logic seems to exist *[3].* Considerably more logic underpins the sequence in the recently discovered oldest complete text of the Mawangdui silk manuscript *[4]*, discovered in 1973 more than two millennia after its 168 BCE burial, and the object of considerable current research. The linear order of this manuscript is in groups of eight hexagrams, within each of which there is a deliberate shift of those with the same upper and lower trigrams to the right *[5].* The eight groups can then be arranged in a sequential stack to form an eight-by-eight square array in which the top trigrams have one sequence, while the bottom trigrams follow another sequence *[6].*

Shao Yung (1011-1077 CE) *[7]*, seeking a more logical ordering of the hexagrams, devised a systematic linear ordering of adjacent stacks of six black and white rectangles which is called a segregation table *[8]*. These stacks are clearly equivalent to the usual hexagrams. Shao Yung used this linear sequence as the basis for a square eight-by-eight array obtained by taking successive groups of eight hexagrams and stacking them in successive lines. He also devised an ingenious circular arrangement.

A dichotomous, or binary, nature of the yin and yang lines has long been appreciated and its transmission to the West may well have spurred Leibnitz towards his invention of binary arithmetic in 1679 *[9]* and published in 1701 *[10]*. Needham *[11]* recounts that the 20 year old Leibnitz began reading Chinese philosophy in 1666. Three yin-yang lines are stacked to create the eight trigrams, with the sixty-four hexagrams then viewed as a stacked pair of trigrams. In the binary system each line may be treated as a binary bit, with values 0 and 1, respectively for yin and yang *[12]*. Shao Yung clearly indexed a linear sequence of the sixty-four six-bit binary patterns in a manner that suggested to Leibnitz *[13]* that he had been scooped by some six and a half centuries in inventing the binary number system.

Z. D. Sung in 1934 at Shanghai *[14]* used a remarkable binary-geometric approach that combines three dimensional Cartesian geometry with the binary number system to produce novel three-dimensional diagrams, some of which have independently reappeared during work on modern binary computers since the 1950's.

An 8-by-8 index table of the hexagrams is often provided in books about the Yijing by taking both the upper and lower trigrams, running respectively along the top and down the array, from the same trigram sequence *[15]*. I suspect that it is in large measure these binary 8-by-8 binary arrays which the mathematically minded among us try to interpret as (square) matrices of linear algebra, although these were not invented until the 19th century!

There have been many modern efforts to try and find an underlying logic to the sequencing of the hexagrams. This situation was reviewed some time ago by Martin Gardner, mathematician and skeptic *[16]*. Some authors have come tantalizingly close, as we shall see during the course of our discussion below.

I have developed a comprehensive dichotomous scheme which accommodates particularly well various schema used in China over the past three millennia. The Chinese patterns of concern here include: (a) four-fold typologies based on a "compass" arrangement, which use a new "logical" sequence of the four two-line digrams (or bigrams, kua), and **not** the order of the Shao Yung-Leibnitz binary sequence; (b) a logical reordering of eight-fold circular (or octagonal) patterns of the eight bagua trigrams; (c) the sixty four hexagrams themselves in a new and harmonious arrangement; and (d) an alternative segregation table in both linear and circular forms.

TYPOLOGY

My starting point was outside the field of Chinese studies, namely the arrangement of personality types *[17]* in the Myers-Briggs Type Indicator ® (MBTI) *[18],* which is itself a development of Carl Jung's 1921 work on "Psychological Types" *[19]*. It is also well known that later in his career Jung was very much interested in the Yijing *[20*].

The sixteen types of the MBTI use a four-letter code where each letter is obtained from a pair of opposites (e.g. Introversion and Extroversion, I & E) *[21]*. Katherine (n. Cook)

Briggs and her daughter Isabel Briggs Myers arranged the sixteen types in a four-by-four tabular array in which only one of the four letters (bits) changes between adjacent horizontal or vertical types. I will refer to this characteristic as the "one-bit" (1-bit) property. Their elegant representation appears to be entirely original and dates probably no later than 1962. The essence of their arrangement is an ingenious coding of two binary dimensions in each ordinary dimension, leading to a square motif representing four binary variables, i.e. four dimensions.

Two things were clear to me several years ago, first that the Myers-Briggs 4-by-4 array of four letter codes could be extended to a six letter dichotomous scheme by simply adding the third physical dimension to the planar 4-by-4 square, and second, that the 64 six-bit types would exactly fit the hexagrams of the Yijing. Before I could follow those directions I developed a 3D version *[22]* of the MBTI type table by exploiting similarities with wrapping 4-by-4 magic squares *[23]* on the surface of a bagel or torus *[24],* which is an interesting artifact since it enables one to hold a 4D in the hand**.** This then led me to a line of inquiry through the issue of magic squares*,* which eventually brought me back to the hexagrams. I had put off studying the question of whether the MBTI array was a magic square, largely because I wanted to avoid assigning numbers to personality types. Tacitly, I had assumed that a numerical rendering of the MBTI would yield a magic square, quite possibly a special variety. However several readings of Marie-Louise von Franz's "Number and Time:…" *[25],* particularly her frequent references to the 3-by-3 Lo-shu magic square, eventually goaded me to study the question. I soon found that the

MBTI was NOT a 4-by-4 magic square, but a new variety of non-magic pandiagonal squares that is even more relevant to the Yijing *[26]*.

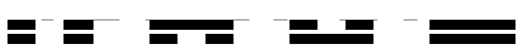
Since 8, the number of trigrams is not a square number, though it can be folded *[27]* into a double-sided square or cube *[28],* I began a detailed study of the simpler square 2-by-2, or 4-fold "precursor" typology. The obvious 2-by-2 binary labeled square is shown in Figure 1. The standard binary labeling is obtained by concatenating the left and right bits, each of which has the values of 0 and 1. Observe that changing both bits in any quadrant gives the opposite type along a diagonal.

L \ R	0	1
0	00→0	01→1
1	10→2	11→3

Figure 1 - The 2-by-2-logic square outlined by double lines showing the left and right binary indices indices and their corresponding integer values.

Unfolded about the right edge, this array gives the edge logic of the 4-by-4 MBTI, namely 00, 01; 11, 10, which has the 1-bit property. This is not the standard Leibnitz sequence of 00, 01, 10, 11. This was my second insight into the folding property *[29]*. In the context of (Jungian) type this is a 4-fold system which results from a 2D Cartesian axial system comprised of two function pairs, up and down, and left and right. These can be used in the compass motif for North and South, East and West. The 4 resulting quadrants represent 4 types, e.g. NW, NE, SE, SW. This double four is however not an 8-fold typology, rather it has two pairs of functions and 4 types. The classical example would be the four types of Earth, Air, Fire and Water, which are pairwise combinations of the function pairs, hot and cold, dry and wet *[30]*.

The standard numerical (integer) rendering of this array which follows by adding the result of multiplying the right bit by "1" and the left bit by "2" *[31]* is shown back in Figure 1: thus 00 becomes integer 0; 01 becomes 1; 10 becomes 2; and 11 becomes 3. Unfolded, the integer sequence is 0, 1, 3, 2, and not the standard Leibnitz sequence of integers 0, 1, 2, 3, i.e. the lexical order. Now let "0" be the yin broken line, and "1" be the yang solid line, and draw the four digrams for the 2-by-2 in Figure 2 as a rising stack of



yin and yang lines.

Figure 2a: The linear digram sequence of the standard (Leibnitz) binary scheme.

Figure 2b: The digrams in my logical linear sequence where one line changes between adjacent digrams (including the ends), and

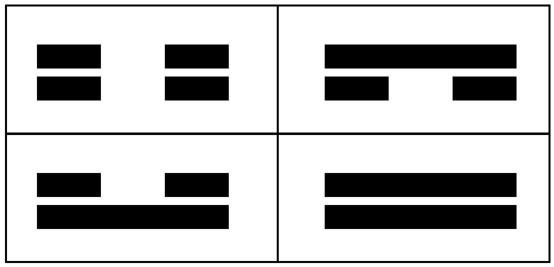
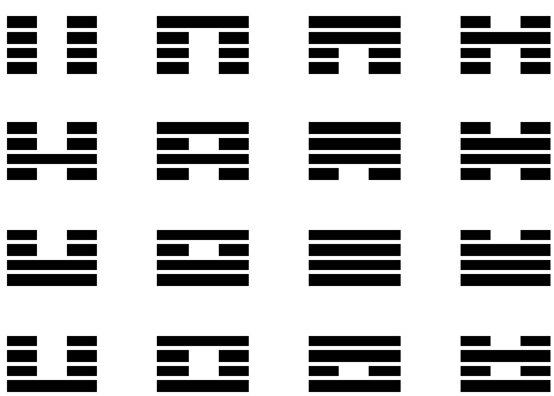


Figure 2c: The logical sequence of Figure 2b folded about the middle into a square array to give a "compass" motif of four orthogonal directions. N.B. the rising vertical orientation of the digrams in (b) has been carefully preserved in (c). Observe that the template for the 5-fold Chinese system of fire, water, earth, wood and metal is just a square with a central space for earth.

The essential characteristic of the MBTI which attracted my attention needs to be explicated in a way that makes contact with the trigrams and hexagrams. To this end I begin by reworking a standard four-by-four square array of the sixteen MBTI types in terms of binary 4-bit labels in the quadgram style of double digrams as shown in Figure



3.

Figure 3: The 4-by-4 array of the 16 quadgrams which are indexed horizontally for the upper digrams, and vertically for the lower digrams, by the digram sequence of Fig. 2b.

The feature that I find particularly harmonious is that for an inner cell, its horizontal and vertical neighbors differ by just one of the four lines in their respective quadgrams, a situation that is also true for edge and corner cells if the opposite edges are joined to each other *[32]*. In the original alphabetic version Isabel Briggs Myers *[33]* noted this elegant property. The essence of my work is to extend this property of changing just one line at a time between adjacent quadgrams, backward to digrams and trigrams, and forward to include hexagrams. My sixteen-fold pattern has potential relevance to other sixteen-fold schema, which are found from Africa (in the Ifa tradition) to Scandinavia (European geomancy), as well as the 256 (or 16-by-16) element geomantic tables *[34]*. The folding of the digram sequence in Fig. 2b also generalizes to the trigrams, quadgrams and hexagrams. As an example, the quadgrams of the 4-by-4 square of Figure 3 can be unfolded row-by-row into a chain of sixteen with the 1-bit property. Generally, my scheme can be seen as a string or chain of binary line stacks that have been assembled from a folded (or reflected) construction. Squares and cubes that result from simple foldings also have the 1-bit property between adjacent line stacks.

TRIGRAMS

Unlike the digrams, quadgrams and hexagrams, the 8 trigrams do not form a square, unless they are placed around a 9th central cell as in the style of the famous Lo-Shu magic square *[35]* which is much used in the context of Feng Shui *[36]*. However the most logical procedure is to use the third bit as the third dimension to create a 2-by-2-by-2 cube, just as Sung did in 1934 *[37]*. This cube unfolds to a 4-by-2 (or 2-by-4) rectangle, and then to the logical chain of 8 trigrams. In Figure 4a I show the standard binary order of Shao Yung-Leibnitz,

Figure 4a: The linear trigram sequence of the standard binary scheme.

and in Figure 4b my logical order of the trigrams.

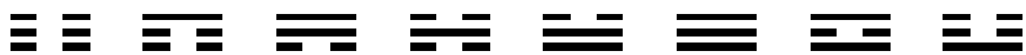


Figure 4b: The 8 trigrams in my logical order.

The corresponding cube is shown in Figure 4c *[38].*

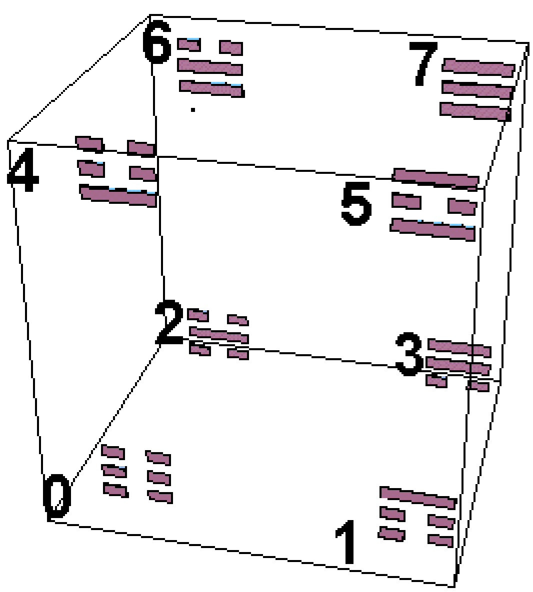


Figure 4c: The 3D cube for the 8 trigrams where I have added the integers from the standard Leibnitz algorithm.

Another general property of my 1-bit linear sequences is that they may be closed into a circle because their endpoints differ by just one line (see Fig. 2b and Fig. 4b). A fundamental consequence of this is that opposite trigrams are no longer diametrically opposite (antipodal) in a circular pattern, unlike the 4 digrams which do have the antipodal property. For the trigrams it is the cube which has the antipodal property, as can readily be seen from Figure 4c. My logical sequence is different from either of the two most common extant trigram “circles” (often shown as octagons) of Fu-Hsi (standard binary order of Figure 4a split across a diameter to form pairs of opposites *[39]* as per Shao Yung's circular diagram for the 64 hexagrams *[40]* or King Wen version, and also different from a third discussed recently by Steve Moore *[41]*.

Now use the Leibnitz algorithm to find the integer values of the trigrams: reading up the stack of the yang-yang-yin trigram produces the code "110" in which the right bit multiplies "1", the middle bit multiplies "2", and the left bit multiplies 4, from 2-times-2, for "110" -> 4+2=6. The pure yin trigram gives 0 and the pure yang 7. It then follows that the logical order that I have proposed for the trigrams is rendered for the linear sequence in Figure 4b as 0, 1, 3, 2, 6, 7, 5, 4. These values are also shown in the cube of Figure 4c. In contrast, the order of ShaoYung, which would also be that of Leibnitz, is the lexical: 0, 1, 2, 3, 4, 5, 6, 7. In Figure 2c I showed how the first four terms in this sequence, the four digrams, fold into a square. Observe that the two diagonals of the digram square each sum to 3, while the four tridiagonals of the trigram cube in Figure 4c sum to 7 *[42]*.

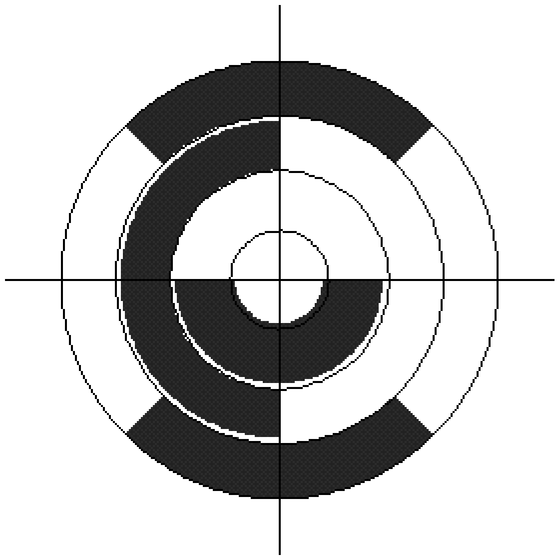


Figure 4d – Inspired by diagrams first drawn by Shao Yung, this diagram joins the 8 "trigrams" into a circle (take white arcs for the yang lines and black for the yin lines) – note how the opposite (yin and yang interchanged) "trigrams" no longer face each other across a diameter, but they are approximately 1/3 (or 2/3) of the way around the circle.

This aspect may have some implications for Feng Shui *[43],* although it is not my intention to change that basic art. Figure 4d differs from that of Sung *[44]* and similar ones by Diana ffarrington Hook *[45]* because it uses my logical sequence, which splits the yin and yang segments in the outer ring. The linear version of this circular form constitutes an alternative segregation table. Although not included here, I have drawn the linear and circular diagrams up the six levels corresponding to the Yijing.

HEXAGRAMS

A 4-by-4-by-4 cube, indexed along each edge by the digram sequence of Figure 2b, can then be unfolded to give the 8-by-8 array in Figure 5. That array is then identical with the result of indexing the edges of such an array by the 8 trigrams in the order of Figure 4b so that double trigrams (hexagrams) result. This square array may be unfolded into a linear sequence, which can then formed into a circle which is different from that of Shao Yung. Aside from a different sequence, he put the first half of his sequence around a semicircle, and then crossed the diameter before continuing to complete the circle so that all the opposite trigrams are antipodal. All my versions (linear, square, cubic and circular) maintain the property that only one line changes between adjacent hexagrams in horizontal, vertical, depth, or circular, moves**.**

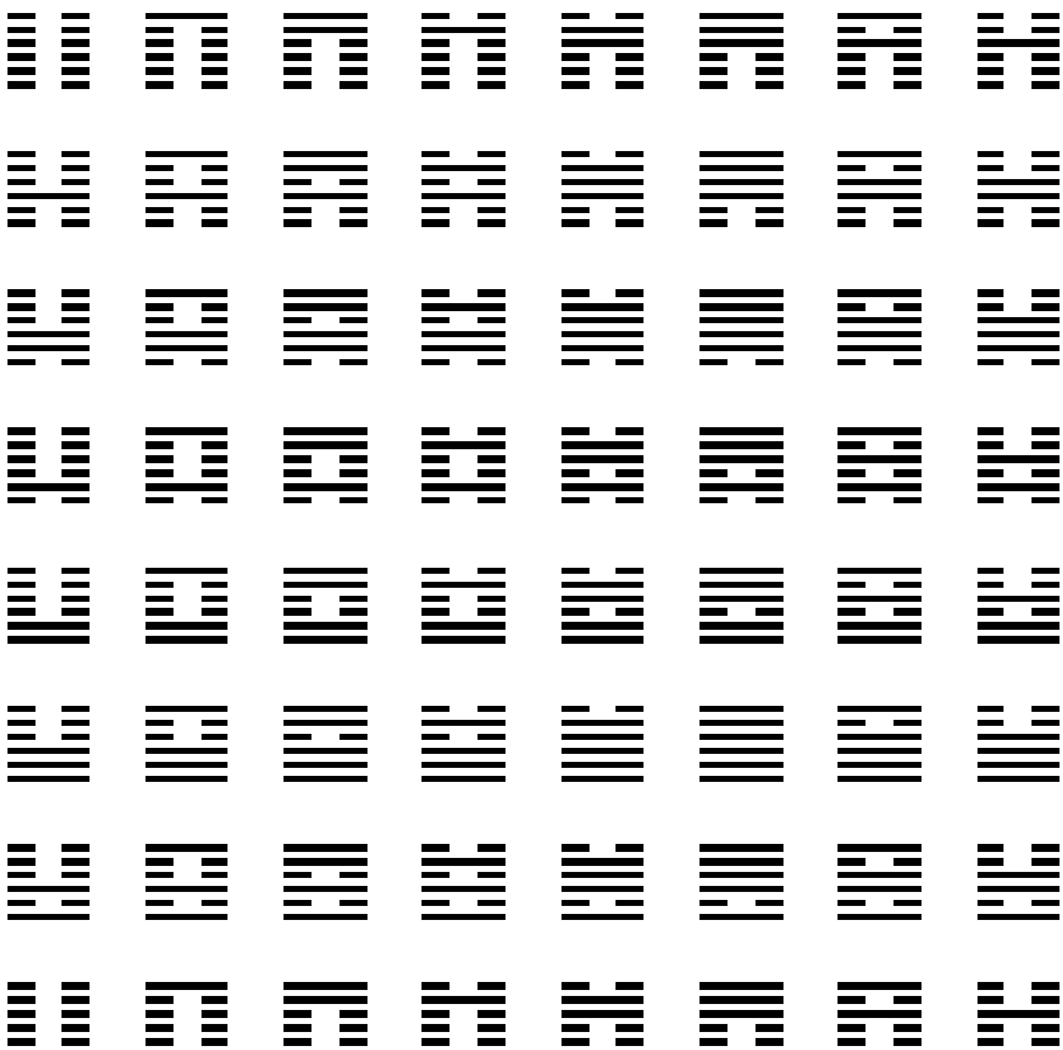


Figure 5: 8-by-8 array of hexagrams spanned by the "logical" order of trigrams**.** Note that while opposites are displaced 2 steps along lines parallel to a body diagonal in the cube, this is more complicated to see in the unfolded 8-by-8.)

GRAY'S CODE

A breakthrough in my understanding of these patterns came from some of my electrical and computer engineering students in 1997 following a brief introduction to the MBTI. They pointed out a connection between the 4-by-4 MBTI array and Karnaugh maps *[46]*, which were developed for digital logic design. The concurrent development of Frank Gray's *[47]* "reflected number code" *[48]* (both were published in 1953) quickly became paired in that arena *[49]*. My logical sequences in Figures 1b and 3b are examples of the Gray code, just convert yin to 0 and yang to 1, and they then form the edges of Karnaugh's arrays. Note that my "folding" is the converse of Gray's "reflected". McKenna and Mair *[50]* have used the Gray sequence to order 32 hexagrams, which are then paired with their complements (opposites) to form double columns of an 8-by-8 square array. Their complementary columns also follow a Gray code, but my work, which began without knowledge of the Gray code, is a much simpler application of the Gray code.

It is worth stressing here that the mathematical insight afforded by the Gray code is not essential to the development of my "logical" ordering of the trigrams and hexagrams such as I have discussed here, but that it does reveal a deeper underlying structure, with further possibilities.

I should note a subtle point here - it was Gray's intention for the integer representation of his code to be monotonic, e.g. for 3 bits it would have to be 0, 1, 2, 3, 4, 5, 6, 7, which means that he did not use the Leibnitz algorithm. Gray would have obtained a different numerical square than that in Figure 1 because the 2 and 3 in that diagram would be interchanged, but he would have the same symbolic square for the digrams in Figure 2c!

CONCLUSIONS

I hope that the framework presented here will facilitate a comparison of extant ancient manuscripts, as well as subsequent efforts at finding congenial patterns. The 2-by-2 pattern (Fig. 2c) clearly exists in all four-fold "compass" motifs. My linear order in Figure 2b agrees with the order of the digrams used for the four seasons and for the four phases of the Moon *[51]*. Moreover the MBTI and Karnaugh maps are in frequent use today, albeit in rather different contexts from that of Yijing studies. It would be satisfying if my "logical" order of the 8 trigrams were to be discovered in an ancient artifact. Given all the other remarkable arrangements devised in ancient China, my trigram sequence would have been no more difficult to devise than the Lo-shu magic square, which also has an origin lost in antiquity. Perhaps my logical sequence will be found on an ancient cave wall, or lying unrecognized in a museum somewhere! If I have convinced you that my logical order of the trigrams was attainable long ago, what then are persuasive reasons to adopt a different order?

The 1973 Mawangdui discovery is important, not only for the origins of the Yijing itself, but also for what one may learn about the origins of the trigrams. However it was not until I saw photographs of the silk manuscript *[52]* that I understood that the upper and lower trigrams are separated by a vertical space which makes it quite clear that the hexagrams are combinations of the trigrams. These are found at the deteriorated top edge of the silk text.

The binary basis for my ideas connects strongly with Sung's work in 1934, and is also interesting in connection with Leibnitz's development of modern binary arithmetic. While Martin Gardner *[53]* is certainly aware of Gray codes and Karnaugh maps, none of the works that he cites have developed a coherent formulation of schemes based on several dichotomous dimensions. Gardner shows Sung's 2-by-2-by-2 cube of the eight trigrams in one chapter, and in another, discusses Hamiltonian paths *[54]* along the edges of the cube where the bits change one-at-a-time. Such a path gives my (Gray code) order of the trigrams. In contrast the lexical order of the decimal (integer) rendering of the binary indices gives a three-dimensional "zigzag" path through the corners of the cube in Figure 4c *[55]*.

With my main account complete, I can now incorporate two interesting efforts that have recently come to my attention.

Firstly, a Web site by Stephen J. Cullinane *[56]* describes the use of Karnaugh maps for the Yijing. His conclusion is very close to mine, with the following differences: (a) he has a 4-by-4-by-4 Karnaugh cube, which I agree with, but, (b) he creates an 8-by-8 array which does not have the 1-bit property because he simply arranges the four levels of the cube side-by-side instead of unfolding as I have done, and (c) he gives the resulting matrix in terms of the traditional numbering of the hexagrams, which takes some effort to square with the integers obtained from the Leibnitz algorithm.

Secondly, Richard Grant *[57]* has published a slim booklet on extending the MBTI to the Yijing. Grant uses a combination of the sixteen-fold MBTI and a four-fold temperament scheme *[58]* to construct the 64 hexagrams. In essence Grant translates the MBTI array as quadgrams, as I have done, and then "multiplies" each quadgram by the four digrams by taking the lower digram line as the bottom line 1 of the hexagrams, and the upper digram line as line 6. Unfortunately Grant does not show the usual form of the 8-by-8 array, but instead leaves a diamond (˜) motif of quartets of hexagrams in place of each of the 16 quadgrams. Effectively Grant has made a compressed 4-by-4-by-4 Karnaugh cube. It is clear from these two efforts that sooner or later someone else would have tidied this up into my scheme and recognized the elegance of the 1-bit property.

In closing let me note an interesting Web site by Tony Smith *[59]* in which he uses Clifford algebras to include a wide range of schemes, including the Yijing. It does not appear to me that he has developed the 1-bit property.

Notes and References

1 – During the later stages of the development of this work, I enjoyed helpful email exchanges with Steve Moore, Andreas Schl`ter, Greg Whincup, and Edward Shaughnessy, as well as helpful conversations with Bill Kocay and Marcus Steeds.

2 - Moore, Steve: *The Trigrams of Han: Inner Structures of the I Ching*. The Aquarian Press, Thorsons Publishing Group, Wellingborough, Northamptonshire, NN8 2RQ, England, 1989.

3 - Shaughnessy, Edward L.: *I Ching: The Classic of Changes*. Ballantine Books, New York, 1997. See p. 18.

4 - Fu Juyou and Chen Songchang (editors): *Mawangdui Han mu wenwu* (Added English title: *The Cultural Relics Unearthed from the Han Tombs at Mawangdui*), Hunan Publishing House, 1992. This was the only source that I could obtain which has the photographs of the silk text from tomb number three. It is a large sumptuous monograph in a beautiful box with an English translation by Zhou Shiyi and Chen Kefeng.

5 - Shaughnessy, op. cit., p. 17.

6 - Walter, Katya: *The Tao of Chaos*. Element Books, Shaftesbury, Dorset, England, 1996. See p. 184.

7 - See e.g. Master Alfred Huang: *The Numerology of the I Ching: A Sourcebook of Symbols, Structures, and Traditional Wisdom*. Inner Traditions, Rochester, Vermont, 2000. See p. 51 for the square and the circle, and p. 43 for the segregation table.

8 - Needham, J.: *Science and Civilization in China*. Volume II, Cambridge, England, 1956. See p. 276, Figure 41.

9 - Leibnitz, G. W.: *De Progressione Dyadica, Pars I*," (manuscript, 15 March 1679).

10 - See also http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Leibniz.html

11 - Needham, op. cit. [6], p. 497.

12 - Needham, op. cit. p. 341 and Walter, op. cit. p. 116.

13 - Leibnitz, G. W.: *Explication de l'ArithmJtique Binaire, qui se sert des seuls caractPres 0 et 1, avec des Remarques sur son Utilit, et sur ce qu'elle donne les sens des anciennes Figures Chinoises de Fohy*, MJmoires de l'Academie Royale des Sciences, 1703, vol. 85.

14 - Sung, Z. D.: *The Symbols of Yi King, or The Symbols of the Chinese Logic of Changes.* Shanghai, 1934; Paragon Book Reprint Corp., NY, 1969. See the review by Andreas Schl`ter: http://www.weft.telinco.co.uk/changes/reviews/sung.html.

15 - Certainly this dates at least from Richard Wilhelm's 1923 translation of the Yijing into German, which appeared much later in English: *The I Ching or Book of Changes, The Richard Wilhelm Translation.* Routledge & Kegan Paul, London, 1950. Another trigram sequence is used in Whincup, Gregory: *Rediscovering the I Ching*. St. Martin's Griffin, New York, 1986. I am grateful to Whincup for pointing out that his 8-by-8 table of the hexagrams is intended strictly as an index to them, and by extension, not intended to suggest that such square arrangements predated that of Shao Yung. An 8-by-8 table arranged row by row according to the traditional sequence is shown by James L. Legge in his 1882 translation: *I Ching - Book of Changes*, Gramercy Books, New York, 1996.

16 - Gardner, M.: *The combinatorial basis of the I Ching*. Scientific American, January 1974, p. 108 -113; reprinted with an update in Gardner, M.: *Knotted Doughnuts and other Mathematical Entertainment's*. F. H. Freeman and Co., New York, 1986; see chapter 20: *The I Ching.*

17 - Here type is used for the polarity of a psychological function, as distinct from the value of a trait, e.g. IQ score.

18 - Myers, Isabel Briggs with Peter B. Myers: *Gifts Differing: Understanding Personality Type*. CPP Books, Palo Alto, California, 1980, 1993.

19 – Jung, C. G.: *Psychological Types: or The Psychology of Individuation*. (1921) Translated by H. Godwin Baynes, 1924, Kegan Paul, Trench, Trubner & Co., New York.

20 – Jung, C. G.: *Foreword* to Wilhelm, op. cit. 15. - Wilhelm's translation of the Yijing into German was published in 1923, but Jung's famous foreword was written in 1949 for Wilhelm's memorial; and Jung, C. G.: *Foreword and Commentary* to Richard Wilhelm's translation (1928) of *The Secret of the Golden Flower*. Harcourt Brace Jovanovich, 1931. The latter is concerned with the 8-fold "flower" pattern and thus the trigrams.

21 - The other three pairs are (1) Sensing and iNtuition, S & N, (2) Thinking and Feeling, T & F, and (3) Judging and Perceiving, J & P.

22 – Loly, P. D.: *Type Wheels and Square Bagels*. Bull. Psych. Type 22:4, Summer 1999, pp. 33-34.

23 - The basic property of a numerical magic square is that all rows, columns, and the two diagonals give the same arithmetic sum. For 3-by-3 of the integers 1 to 9 there is only the unique Lo-shu square, while for 4-by-4 there are 880 distinct squares. The number for 5-by-5 is some 275 million, and there are more 17 million million million (to avoid confusion between British and American "billions") for 6-by-6. The number for the Yijing size of 8-by-8 must be truly humungous.

24 - Jenkins, G. and Wild, A.: *Mathematical Curiosities I*, Tarquin Publications, Stradbrooke, Diss, Norfolk, 1980.

25 - Von Franz, Marie-Louise: *Number and Time: Reflections Leading toward a Unification of Depth Psychology and Physics*. Northwestern University Press, Evanston, Illinois, 1974. Von Franz's thoughts on the Lo-shu magic square are also found to a lesser extent in the richly illustrated: *Time: Rhythm and Repose*, London: Thames & Hudson (Art and Imagination Series), 1978; in *Psyche and Matter*, **Shambala**, 1992; and in *On Divination and Synchronicity. The Psychology of Meaningful Chance*. Originally presented as Lectures at the C.G. Jung Institute, Zurich, Inner City Books, Toronto, Canada, 1980.

26 - Loly, P. D.: *A purely pandiagonal 4*4 square and the Myers-Briggs Type Table.* J. Rec. Math. - The publication of this 1999 issue is now expected in 2001!

27 - The ideas in the present article about folding are my own.

28 - I have a deep feeling that the relationship of the 8 trigrams to the 8 corners of a cube, and therefore the corners of a standard room, are a fundamental expression of the three-dimensionality of the space that we inhabit. A ninth central point just adds to the connections with a plethora of ancient motifs.

29 - My first insight into folding involved an eight-fold Jungian typology in a 2-by-4 (or 4-by-2) rectangular arrangement folded into a double-sided 2-by-2 square, or 2-by-2-by-2 cube.

30 - Stevens, Anthony: *On Jung*, Penguin Books, 1990. See p. 69.

31 - Walter, op. cit., p. 116. Mathematically it is convenient to use zero for the starting number.

32 - Loly, op. cit. *[22]*.

33 - Myers, op. cit. p. 41.

34 - Pennick, Nigel: *The Oracle of Geomancy: The Divinatory Arts of Raml, Geomatria, Sikidy and I Ching*. Capall Bann, Freshfields, Chieveley, Berks., England, 1995.

35 - Von Franz, op. cit., pp. 23, 123, and 238-241.

36 – Master Lam Kam Chuen: *The Personal Feng Shui Manual*. Owl Books, New York, 1998.

37 - Sung, op. cit. p. 122, Figure 46.

38- Sung, op. cit. p. 12, Figure 1, and Gardner, op. cit. *[16],* chapter 2: *The binary Gray code.*

39 - Moore, Steve: *Some New Evidence for Dating the Trigrams of the I Ching*, The Oracle, No. 2, Winter 1995/1996, pp. 3-8.

40 - Huang, op. cit. p. 51.

41 - Moore, op. cit. *[2],* p. 110.

42 - Often a "1" is added to all cases so that they range from 1 to 8. This is especially so for examining potential connections with the 3-by-3 Lo-shu magic square of the numbers 1 to 9, for which the dates are probably in the first millennium BCE, but are even more uncertain than those for the Yijing. For the square the previous diagonal sum of "3" becomes "5", and for the cube the previous triagonal sum of "7" becomes "9".

43 - Chuen, op. cit.

44 - Sung, op. cit., p.122 for Figure 46.

45 - Hook, Diana ffarington: *The I Ching and Its Associations*. Arkana, Penguin Books, 1980. See p. 116-120.

46 – Karnaugh, M.: *The Map Method for Synthesis of Combinatorial Logic Circuits*. Trans. AIEE. Pt. I, vol. 72, no. 9, (1953) 593-599.

47 – Gray, F.: *Pulse Code Communication*. U.S. Patent 2632058, March 17, 1953.

48 - To understand the essence of Gray's reflection construction, take 0 and 1, reflect a copy to give: 0, 1; 1, 0, where I have placed a semicolon to emphasize the reflection (mid) point. Next add a leading bit of 0 for the first half and 1 for the second: 00, 01; 11, 10. Then repeat this process to find my trigram sequence, again for the quadgrams, and twice more for the hexagrams.

49 - Mano, Morris: *Digital Design*, Second Edition, Prentice Hall, 1984.

50 - McKenna, S. E. and Mair, V. H.*: A reordering of the hexagrams of the I Ching*. Philosophy East and West, vol. 29 (1979) pp. 37-49.

51 - Brennan, J. H. *The Magical I Ching*, Llewellyn Publications, St. Paul, Minnesota, 2000. See pp. 6-7.

52 - Juyou and Songchang, op. cit.

53 - Gardner, op. cit., see Chapter 2: *The binary Gray code*.

54 - Gardner, op. cit.

55 - Trace a sequential line through the corners 0, 1, 2, 3, 4, 5, 6, 7.

56 - Cullinane, Steven H.: http://m759.freeservers.com/PHiching.html

57 - Grant, Richard D.*: The I Ching: Images of Psychological typology and Development*, Type & Temperament, Inc. PO Box 200, Gladwyne, PA 19035-0200, 1990.

58 - Keirsey, D. and -Bates, Marilyn*: Please Understand Me - Character & Temperament Types*, Prometheus Nemesis Book Company, Del Mar, CA, 1978. I should note that this popular work is based on the 16 MBTI types, which are grouped into 4 temperaments in combinations that have proved to be a useful simplification. However I am not comfortable with Grant's re-use of the original four dimensions of the MBTI, and would prefer the other two dimensions to hold further dichotomous functions.

59 - Smith, Tony [http://www.innerx.net/personal/tsmith/

BIOGRAPHICAL

Peter Loly is a professor of physics at the University of Manitoba, Winnipeg, Canada. His long term research interests concern theoretical studies of simple models of crystals. Nearly ten years ago he encountered the Myers-Briggs Type Indicator® (MBTI) and soon became fascinated by its structure, its implementation of Carl Jung's psychological types, and to Jung's dialogue with Wolfgang Pauli, one of the most eminent physicists of the 20th century. He has trained to administer the MBTI and gives workshops related to type. His research and publications now include contributions to type and to new aspects of magic squares, as well as an interesting new family of non-magic squares.

Email: loly@cc.umanitoba.ca

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