UNIVERSITY OF MANITOBA
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

24.767 OPTIMIZATION METHODS FOR COMPUTER-AIDED DESIGN

ASSIGNMENT 4

Due Date: March 21, 2005 **Instructor: J. LoVetri**

General Instructions: This assignment is to be written up neatly since messy work will not be marked. All programs which are written for the assignment are to be fully commented and included in an Appendix at the end of the assignment. Relevant program output used to answer a question is to be transferred into tables which should be included in the main body of the report at the location where each table is being referred to.

1) Download the file names “mystery.mat” from the course web-site. This file contains two vectors named *t* (time) and *corrupt* (voice data). There is a 60 Hz additive noise superimposed on the voice data. Using any optimization technique that we’ve learned so far in the course, design a digital filter of the form:

$$y(n) = \sum_{k=0}^{N-1} b_k x(n-k) - \sum_{k=1}^{M} a_k y(n-k) \qquad \text{(ARMA)}$$

to remove the 60 Hz additive noise so that you can determine the mystery message. Choose whatever N and M you require to expose the message. Fully describe the optimization technique you are using and give the coefficients that your optimization algorithm comes up with. Plot the frequency response of the filter you design.

(***Digital Signal Processing Review***: If we take the Z-transform of the above ARMA model we get the transfer function

$$H(z) \triangleq \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{N-1} b_k z^{-k}}{1 + \sum_{k=1}^{M} a_k z^{-k}}.$$

This can be written as a rational function and the numerator and denominator can be factored. For example:

$$H(z) = G\frac{(z-z_0)(z-z_1)(z-z_2)}{(z-p_1)(z-p_2)}.$$

The frequency response is obtained by $\tilde{H}(\omega) = H(e^{j\omega})$.

2) A common problem in network theory is finding the shortest path between a source node and a terminal node in the network. The *node-arc* incidence matrix A is an nxm matrix corresponding to a network with n nodes and m arcs. The n nodes are labelled n_i, $i = 1 \ldots n$, and the m arcs are labelled e_j, $j = 1 \ldots m$. The elements of A are set as

$$a_{ij} = \begin{cases} 1 & \text{if arc } e_j \text{ leaves node } n_i \\ -1 & \text{if arc } e_j \text{ enters node } n_i . \\ 0 & \text{otherwize} \end{cases}$$

The shortest path problem between a source node, s, and terminal node, t, can be solved as a linear programming problem. Given a network in terms of the node-arc incidence matrix, A, and a *cost vector*, $\boldsymbol{c}$, such that c_j is the cost of traversing arc e_j, the linear programming problem equivalent to the shortest path problem can be written as

$$\text{minimize} \qquad \boldsymbol{c}^T \boldsymbol{f}$$

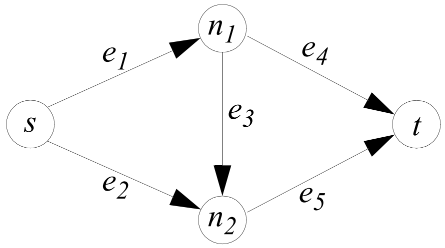
$$\text{subject to:} \qquad A\boldsymbol{f} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \begin{array}{l} \longleftarrow \textit{source node row} \\ \longleftarrow \textit{terminal node row} \\ \\ \\ \\ \\ \end{array}$$

$$f_j \geq 0 , j = 1(1)m$$

The n x m node-arc incidence matrix has rank of (m-1). The vector $\boldsymbol{f}$ represents the flow in each of the arcs, that is f_j represents the flow in arc e_j.

(a) What are the possible values of each f_j once the problem is solved and what do these values mean?

(b) Consider the problem of finding the shortest path from s to t in the network at the right. Formulate this problem as a linear programming problem as described above.



$$\boldsymbol{c} = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 3 \\ 1 \end{bmatrix}$$

(c) <u>Solve</u> the linear programming problem by enumerating all the possible *basic vectors* defined by the problem. State which ones are feasible and which ones are not.

3) Table 1 lists the nutritional value of some common foods. Determine a diet which meets the minimum daily requirements listed at the bottom of the table while minimizing the total energy content. Give your results as proportions of the units of each food item listed (for example 1.35 slices of ham, 375 ml. of milk, *etc*.). Formulate the problem as a linear program and use the “lp” routine in Matlab to find the solution. What happens if the calories in bran muffins are increased by a factor of 2 (say by adding icing sugar)?

Table 1: Nutritional Value of Typical Foods

variable	Food	Energy (kcal)	Protein (g)	Carbohydrates (g)	fat (g)	Vitamin A (RE)	Vitamin B1 Thiamin (mg)	Vitamin B2 Riboflavin (mg)	Vitamin C (mg)	fibre (g)
x1	Provolone (15ml)	158	12	0	12	119	0	.14	0	0
x2	Mozzarella (15ml)	132	9	1	10	113	0	.11	0	0
x3	2% milk (250ml)	128	9	12	5	147	.10	.43	2	0
x4	salami (1 slice)	25	1	0	2	0	.04	.02	0	0
x5	ham (1 slice)	49	5	0	3	0	.23	.07	0	0
x6	brussel sprouts (250ml)	64	4	14	0	119	.18	.13	102	5
x7	lettuce (250ml)	11	0	2	0	112	.03	.05	11	.9
x8	french fries (10 strips)	158	2	20	8	0	.09	.01	5	0
x9	orange (1 medium)	62	1	15	0	28	.11	.05	70	2.6
x10	whole wheat bread (1 slice)	61	3	12	0	0	.06	.03	0	1.4
x11	bran muffin (1 medium)	104	3	17	4	18	.05	.08	0	1.8
	daily requirement (128 lb female)	-	60	300	40	800	1.0	1.2	60	10