University of Manitoba

Department of Electrical and Computer Engineering

ECE 7670

Optimization Methods For Computer-aided Design

Assignment 1

Due Date: February 10, 2009 **Instructor: J. LoVetri**

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1) Let V be the vector space of polynomial functions $p : \mathbb{R} \to \mathbb{R}$ which have degree less than or equal to 2. Show that $B_1 = \{1, x, x^2\}$ and $B_2 = \{1, x+t, (x+t)^2\}$ for $t \in \mathbb{R}$ are both bases for this vector space and find the transformation matrices which transform the coordinates of any $p \in V$ between the two bases. Let D be the differentiation operator and show that it is a linear operator on the vector space V. Determine the matrix representation of D in B_1 and B_2. Call these M_1 and M_2, respectively, and determine the transformation equation between the two matrices.

2) Consider the basis $B_X = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ for $X = \mathbb{R}^3$ and $B_Y = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \right\}$ for $Y \subset \mathbb{R}^4$ and the linear operator $\mathcal{L} : X \to Y$ having the matrix representation:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

under these two bases. Find $y = \mathcal{L}x$ and its coordinates under the basis B_Y for $x = \begin{bmatrix} 5 \\ 10 \\ 20 \end{bmatrix}$.

3) Find the derivative $Df(x)$ of $f : \mathbb{R}^4 \to \mathbb{R}$ given by $f(x) = \|Ax - b\| + \lambda\|Cx\| + \gamma\|Db\|$, where $x, b \in \mathbb{R}^4$, $A, C, D \in \mathbb{R}^{4 \times 4}$, and $\lambda, \gamma \in \mathbb{R}$.

4) Write a progam in MATLAB that takes as input a function $f : \mathbb{R}^n \to \mathbb{R}$, it's gradient $\nabla f : \mathbb{R}^n \to \mathbb{R}^n$, a direction $d \in \mathbb{R}^n$, a starting point $x_{(0)} \in \mathbb{R}^n$, and the size of the desired final bounding interval size Δ. It then bounds the minimum in the d-direction using Swann's method with a starting $\delta = 0.1$ and magnification parameter 2. Finally, it performs the Fibonacci search for the required number of function evaluations to reach the specified final bounding interval size. The midpoint of the final bounding interval is output as the minimum as well as the norm of the gradient.

5) Use the program of question (4) to find the parameter vector $x \in \mathbb{R}^4$ in the model

$$f(t\,;x) = x_1 + x_2 e^{x_4 t} + x_3 e^{x_5 t}$$

that fits, in a least-squares sense, the data $y(t)$ given in the table below. Starting at $x = 0$, minimize along the steepest decent direction with your Fibonacci line-search program using 5 function evaluations in the Fibonacci search (*i.e.*, after bounding). Restart the line search with a new gradient until the norm of the gradient reaches below 10^{-4}. Output the total number of function evaluations, bounding and interval reduction, the number of steepest decent iterations, and the final parameter vector.

Table 1: Data to be Modeled

$y(t)$	t
1.75	0
1.65	1
1.56	2
1.49	3
1.43	4
1.37	5
1.33	6
1.29	7
1.26	8
1.23	9