UNIVERSITY OF MANITOBA

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

ECE 7670

OPTIMIZATION METHODS FOR COMPUTER-AIDED DESIGN

ASSIGNMENT 4

Due Date: April 9, 2009 **Instructor: J. LoVetri**

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1) Consider the designof a digital filter in the form

$$y(n) = \sum_{k=0}^{N-1} b_k x(n-k) - \sum_{k=1}^{M} a_k y(n-k) \qquad \text{(ARMA)}$$

where the N and M parameters are specified. Assume that the magnitude of the frequency response of this filter is of interest. Discuss how you would formulate the design of this filter, *i.e.*, the choice of the coefficients a_k and b_k, as an optimization problem. Recall that if we take the z-transform of the above ARMA model we get the transfer function

$$H(z) \triangleq \frac{Y(z)}{X(z)} = \left[\sum_{k=0}^{N-1} b_k z^{-k}\right] / \left[1 + \sum_{k=1}^{M} a_k z^{-k}\right],$$

which can be written as a rational function where the poles and zeros are factored out. For example, for $N = 3$ and $M = 2$ we would get

$$H(z) = G\frac{(z-z_0)(z-z_1)(z-z_2)}{(z-p_1)(z-p_2)}.$$

The frequency response is then obtained as $\tilde{H}(\omega) = H(e^{j\omega})$.

As a test for your formulation download the file names "mystery.mat" from the course website. This file contains two vectors named *t* (time) and *corrupt* (voice data). There is a 60 Hz additive noise superimposed on the voice data. Formulate the design of a 60 Hz notch filter as an optimization problem and use any optimization technique that we've learned so far in the course to reveal the secret message. Plot the mystery signal before and after you apply the filter. Compare for different values of M and N.

2) Table 1 lists the nutritional value of some common foods. Determine a diet which meets the minimum daily requirements listed at the bottom of the table while minimizing the total energy content. Give your results as proportions of the units of each food item listed (for example 1.35 slices of ham, 375 ml. of milk, *etc.*). Formulate the problem as a linear program and use the "lp" routine in Matlab to find the solution. What happens if the calories in bran muffins are increased by a factor of 2 (say by adding icing sugar)?

Table 1: Nutritional Value of Typical Foods

variable	Food	Energy (kcal)	Protein (g)	Carbohydrates (g)	fat (g)	Vitamin A (RE)	Vitamin B1 Thiamin (mg)	Vitamin B2 Riboflavin (mg)	Vitamin C (mg)	fibre (g)
x1	Provolone (15ml)	158	12	0	12	119	0	.14	0	0
x2	Mozzarella (15ml)	132	9	1	10	113	0	.11	0	0
x3	2% milk (250ml)	128	9	12	5	147	.10	.43	2	0
x4	salami (1 slice)	25	1	0	2	0	.04	.02	0	0
x5	ham (1 slice)	49	5	0	3	0	.23	.07	0	0
x6	brussel sprouts (250ml)	64	4	14	0	119	.18	.13	102	5
x7	lettuce (250ml)	11	0	2	0	112	.03	.05	11	.9
x8	french fries (10 strips)	158	2	20	8	0	.09	.01	5	0
x9	orange (1 medium)	62	1	15	0	28	.11	.05	70	2.6
x10	whole wheat bread (1 slice)	61	3	12	0	0	.06	.03	0	1.4
x11	bran muffin (1 medium)	104	3	17	4	18	.05	.08	0	1.8
	daily requirement (128 lb female)	-	60	300	40	800	1.0	1.2	60	10

3) Do Problem 19.13 in the textbook.

4) Consider the following constrained minimization problem:

$$\begin{aligned}
\text{Minimize} \quad f(\boldsymbol{x}) &= \frac{1}{3}(x_1+1)^3 + x_2^2 \\
\text{Subject to:} \quad g_1(\boldsymbol{x}) &= -x_1 + 1 \le 0 \\
g_2(\boldsymbol{x}) &= -x_2 \le 0
\end{aligned}$$

(a) Formulate the problem using an interior penalty method.

(b) Solve the problem, by computer, using any method you wish. Explain your method and discuss the results.

5) A thermal station contains three units which consume fuel at the following rates in [gigajoules/hr] (with electrical power P_i in [MW]):

(A) 250 [MW] Unit: $\quad F_a = 350 + 8.3P_a + 0.0024P_a^2$

(B) 350 [MW] Unit: $\quad F_b = 400 + 8.2P_b + 0.0020P_b^2$

(C) 300 [MW] Unit: $\quad F_c = 430 + 8.2P_c + 0.0017P_c^2$

The total losses are given by $L(P) = 0.1P_a + 0.1P_b + 0.2P_c$ and the *efficiency ratio*, η, is defined as the ratio of power supplied to all the loads, P_L, to the total fuel per hour consumption, $F_T = F_a + F_b + F_c$.

(i) If the total power to be supplied to all loads, P_L, is 500 [MW], what is the power supplied by each unit which minimizes the **total** fuel consumption? Use the Lagrange multiplier method and explain the steps in the solution.
What is the optimal total fuel consumption in [gigajoules/hr]?

(ii) If the total load, P_L, is slightly increased will the efficiency ratio increase or decrease? Justify your answer?

(iii) At what total load is the least energy per hour consumed? Does this correspond to the best efficiency ratio (*i.e.,* greatest efficiency ratio)?

(iv) If unit C is shut down ***totally***, *i.e.*, $F_c = 0$, what is the new loading on units A and B for minimum fuel consumption? What is the change in the **total fuel consumption** because of the shutdown of unit C? From an economic standpoint, should you keep unit C running?