UNIVERSITY OF MANITOBA

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

24.8200 Engineering Electromagnetics

ASSIGNMENT 7

Due Date: Tuesday December 5, 2006 **Instructor: J. LoVetri**

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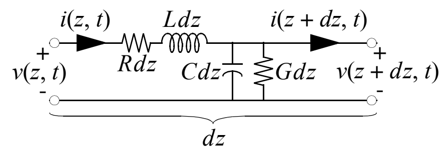
1) (a) Show that the time-harmonic steady-state voltage and current on a transmission-line satisfy the one-dimensional Helmholtz equations

$$\begin{cases} \dfrac{d^2}{dz}\boldsymbol{V}(z) - ZY\,\boldsymbol{V}(z) = 0 \\ \dfrac{d^2}{dz^2}\boldsymbol{I}(z) - ZY\,\boldsymbol{I}(z) = 0 \end{cases}$$



where $Z = R + j\omega L$, $Y = G + j\omega C$, R, L, G, C, are the per-unit-length resistance [Ω/m], inductance [H/m], conductance [S/m], and capacitance [F/m] respectively, and ω is the angular frequency [rad/sec]. (Use the lumped-element model shown.)

(b) Show that the solutions for waves, propagating in the positive z-direction, are given by

$$\boldsymbol{V}^+ = V_0 e^{-\gamma z}, \qquad \boldsymbol{I}^+ = I_0 e^{-\gamma z}$$

where $\gamma = \sqrt{ZY}$ is the propagation constant.

(c) Show that if $R \ll \omega L$ and $G \ll \omega C$

$$\alpha \approx \frac{R}{2\sqrt{L/C}} + \frac{G\sqrt{L/C}}{2} \qquad \beta \approx \omega\sqrt{LC}$$

where $\gamma = \alpha + j\beta$, and determine the relationship between V_0 and I_0.

2) Given the following inhomogeneous Sturm-Liouville system:

(O.D.E.) $$\frac{d}{dx}\left[p(x)\frac{d\psi}{dx}\right] + [q(x) + \lambda s(x)]\psi = -f(x) \quad , \qquad 0 \le x \le a$$

(B.C.s) $$k_1\psi(0) + k_2\left.\frac{d\psi}{dx}\right|_{x=0} = k_3 \quad , \quad k_4\psi(a) + k_5\left.\frac{d\psi}{dx}\right|_{x=a} = k_6$$

where k_1, k_2, k_3, k_4, k_5, and k_6, are constants.

a)Determine an oppropriate ordinary differential equation and boundary conditions for the related Green's function problem.

b)Express the solution to the above system, symbolically, in terms of the Green's function, *G(x; x')*, which is the solution of part (a).

3) Consider, first, the lossless transmission-line, ($R = G = 0$), with distributed time-harmonic current source $\boldsymbol{I_s}(z)$ [A/m] (shunt). Derive the appropriate inhomogeneous Helmholtz equation for the voltage on this line and determine expressions for the voltage, $\boldsymbol{V}(z)$, using the Green's function technique for the following four cases:

(a) an infinite homogeneous transmission line;

(b) a semi-infinite ($z > 0$) transmission line short-circuited at $z = 0$;

(c) a semi-infinite ($z > 0$) transmission line open-circuited at $z = 0$; and

(d) a finite line ($0 < z < b$) terminated by impedances Z_a and Z_b.