Preserver Problems and Graph Theory

Marko Orel

- **University of Primorska**, Koper, Slovenia
  - Faculty of Mathematics, Natural Sciences and Information Technologies (**FAMNIT**)
  - IAM
- IMFM, Ljubljana, Slovenia

CMS Summer Meeting, Winnipeg, June 7, 2014

Outline

1. Preservers of a binary relation/Endomorphisms of a graph
2. Adjacency preservers
3. Hamiltonicity, Lovász problem

Outline

1. Preservers of a binary relation/Endomorphisms of a graph
2. Adjacency preservers
3. Hamiltonicity, Lovász problem

Outline

1. Preservers of a binary relation/Endomorphisms of a graph
2. Adjacency preservers
3. Hamiltonicity, Lovász problem

Outline

1. Preservers of a binary relation/Endomorphisms of a graph
2. Adjacency preservers
3. Hamiltonicity, Lovász problem

Preservers of a binary relation

$R \subseteq A \times A$ a binary relation on A $\quad (a_1 R a_2 \Leftrightarrow a_2 R a_1, \quad a \bar{R} a)$

$\varphi : A \to A$ preserves R in both directions, if

$$a_1 R a_2 \qquad \varphi(a_1) R \varphi(b_1)$$

Preservers of a binary relation

$R \subseteq A \times A$ a binary relation on A $\quad (a_1 R a_2 \Leftrightarrow a_2 R a_1, \quad a \bar{R} a)$

$\varphi : A \to A$ preserves R in both directions, if

$$a_1 R a_2 \qquad \varphi(a_1) R \varphi(b_1)$$

Preservers of a binary relation

$R \subseteq A \times A$ a binary relation on A $\quad (a_1 R a_2 \Leftrightarrow a_2 R a_1, \quad a\bar{R}a)$

$\varphi : A \to A$ preserves R in both directions, if

$$a_1 R a_2 \quad \Longrightarrow \quad \varphi(a_1) R \varphi(b_1)$$

$R \subseteq A \times A$ a binary relation on A $\quad (a_1 R a_2 \Leftrightarrow a_2 R a_1, \quad a\bar{R}a)$

$\varphi : A \to A$ preserves R in both directions, if

$$a_1 R a_2 \quad \Longleftrightarrow \quad \varphi(a_1) R \varphi(b_1)$$

Preservers of a binary relation

$\Gamma = (V, E)$, $V = A$, $E = \{\{a_1, a_2\} \in A \times A \ : \ a_1 R a_2\}$

(i) φ preserves $R \iff \varphi$ is an endomorphisms of Γ

(ii) φ bijective and preserves R in both directions

$$\iff$$

φ is an automorphisms of Γ

(iii) φ bijective and preserves R

$$\iff$$

φ is bijective endomorphism of Γ

(ii) and (iii) are equivalent if $|A| < \infty$

$$\Gamma = (V, E), \quad V = A, \quad E = \big\{\{a_1, a_2\} \in A \times A \ : \ a_1 R a_2\big\}$$

(i) φ preserves R $\iff$ φ is an endomorphisms of Γ

(ii) φ bijective and preserves R in both directions

$$\iff$$

φ is an automorphisms of Γ

(iii) φ bijective and preserves R

$$\iff$$

φ is bijective endomorphism of Γ

(ii) and (iii) are equivalent if $|A| < \infty$

Preservers of a binary relation

$$\Gamma = (V, E), \quad V = A, \quad E = \big\{\{a_1, a_2\} \in A \times A \ : \ a_1 R a_2\big\}$$

(i) φ preserves $R \iff \varphi$ is an endomorphisms of Γ

(ii) φ bijective and preserves R in both directions

$$\iff$$

φ is an automorphisms of Γ

(iii) φ bijective and preserves R

$$\iff$$

φ is bijective endomorphism of Γ

(ii) and (iii) are equivalent if $|A| < \infty$

Preservers of a binary relation

$$\Gamma = (V, E), \ \ V = A, \ \ E = \big\{\{a_1, a_2\} \in A \times A \ : \ a_1 R a_2\big\}$$

(i) φ preserves $R \iff \varphi$ is an endomorphisms of Γ

(ii) φ bijective and preserves R in both directions

$$\iff$$

φ is an automorphisms of Γ

(iii) φ bijective and preserves R

$$\iff$$

φ is bijective endomorphism of Γ

(ii) and (iii) are equivalent if $|A| < \infty$

Preservers of a binary relation

$$\Gamma = (V,E), \;\; V = A, \;\; E = \left\{\{a_1, a_2\} \in A \times A \;:\; a_1 R a_2\right\}$$

(i) φ preserves $R \iff \varphi$ is an endomorphisms of Γ

(ii) φ bijective and preserves R in both directions

$$\iff$$

φ is an automorphisms of Γ

(iii) φ bijective and preserves R

$$\iff$$

φ is bijective endomorphism of Γ

(ii) and (iii) are equivalent if $|A| < \infty$

Preservers of a binary relation

$$\Gamma = (V, E), \;\; V = A, \;\; E = \Big\{ \{a_1, a_2\} \in A \times A \;:\; a_1 R a_2 \Big\}$$

(i) φ preserves $R \iff \varphi$ is an endomorphisms of Γ

(ii) φ bijective and preserves R in both directions

$$\iff$$

φ is an automorphisms of Γ

(iii) φ bijective and preserves R

$$\iff$$

φ is bijective endomorphism of Γ

(ii) and (iii) are equivalent if $|A| < \infty$

Preservers of a binary relation

$$\Gamma = (V, E),\ \ V = A,\ \ E = \big\{\{a_1, a_2\} \in A \times A\ :\ a_1 R a_2\big\}$$

(i) φ preserves $R \iff \varphi$ is an endomorphisms of Γ
(ii) φ bijective and preserves R in both directions

$$\iff$$

φ is an automorphisms of Γ
(iii) φ bijective and preserves R

$$\iff$$

φ is bijective endomorphism of Γ

(ii) and (iii) are equivalent if $|A| < \infty$

Cores

Γ = a finite undirected graph with no loops/multiple edges

A graph is a *core* if any its endomorphism is an automorphism.

Examples: complete graphs K_n, odd cycles C_{2n+1}

A subgraph Γ' in Γ is a *core of* Γ if:

- Γ' is a core
- There exists a homomorphism $\varphi : \Gamma \to \Gamma'$

Example: $\mathrm{core}(C_4) = K_2$

Proposition

Every graph Γ has a core, which is an induced subgraph and is unique up to isomorphism.

Cores

Γ = a finite undirected graph with no loops/multiple edges

A graph is a *core* if any its endomorphism is an automorphism.

Examples: complete graphs K_n, odd cycles C_{2n+1}

A subgraph Γ' in Γ is a *core of* Γ if:

- Γ' is a core
- There exists a homomorphism $\varphi : \Gamma \to \Gamma'$

Example: $\mathrm{core}(C_4) = K_2$

Proposition

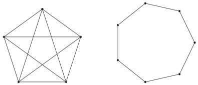
Every graph Γ has a core, which is an induced subgraph and is unique up to isomorphism.

Cores

Γ = a finite undirected graph with no loops/multiple edges

A graph is a *core* if any its endomorphism is an automorphism.

Examples: complete graphs K_n, odd cycles C_{2n+1}



A subgraph Γ' in Γ is a *core of* Γ if:

- Γ' is a core
- There exists a homomorphism $\varphi : \Gamma \to \Gamma'$

Example: $\mathrm{core}(C_4) = K_2$

Proposition

Every graph Γ has a core, which is an induced subgraph and is unique up to isomorphism.

Cores

Γ = a finite undirected graph with no loops/multiple edges

A graph is a *core* if any its endomorphism is an automorphism.

Examples: complete graphs K_n, odd cycles C_{2n+1}

A subgraph Γ' in Γ is a *core of* Γ if:

- Γ' is a core
- There exists a homomorphism $\varphi : \Gamma \to \Gamma'$

Example: $\mathrm{core}(C_4) = K_2$

Proposition

Every graph Γ has a core, which is an induced subgraph and is unique up to isomorphism.

Cores

Γ = a finite undirected graph with no loops/multiple edges

A graph is a *core* if any its endomorphism is an automorphism.

Examples: complete graphs K_n, odd cycles C_{2n+1}

A subgraph Γ' in Γ is a *core of* Γ if:

- Γ' is a core
- There exists a homomorphism $\varphi : \Gamma \to \Gamma'$

Example: $\mathrm{core}(C_4) = K_2$

Proposition

Every graph Γ has a core, which is an induced subgraph and is unique up to isomorphism.

Cores

Γ = a finite undirected graph with no loops/multiple edges

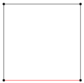
A graph is a *core* if any its endomorphism is an automorphism.

Examples: complete graphs K_n, odd cycles C_{2n+1}

A subgraph Γ' in Γ is a *core of* Γ if:

- Γ' is a core
- There exists a homomorphism $\varphi : \Gamma \to \Gamma'$

Example: $\mathrm{core}(C_4) = K_2$



Proposition

Every graph Γ has a core, which is an induced subgraph and is unique up to isomorphism.

Cores

Γ = a finite undirected graph with no loops/multiple edges

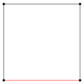
A graph is a *core* if any its endomorphism is an automorphism.

Examples: complete graphs K_n, odd cycles C_{2n+1}

A subgraph Γ' in Γ is a *core of* Γ if:

- Γ' is a core
- There exists a homomorphism $\varphi : \Gamma \to \Gamma'$

Example: $\mathrm{core}(C_4) = K_2$



Proposition

Every graph Γ has a core, which is an induced subgraph and is unique up to isomorphism.

Adjacency preservers

$\mathcal{A}$ = set of matrices

$X, Y \in \mathcal{A}$ are *adjacent* if $\mathrm{rk}(X - Y)$ is minimal and nonzero

$\mathcal{A} \in \{M_{m\times n}(\mathbb{F}), S_n(\mathbb{F}), H_n(\mathbb{F})\} \Longrightarrow \mathrm{rk}(A - B) = 1$

$\mathcal{A} = A_n(\mathbb{F}) \Longrightarrow \mathrm{rk}(A - B) = 2$

$XRY \Longleftrightarrow X \text{ and } Y \text{ are adjacent}$

Bijective maps that preserves adjacency in both directions on

$$\mathcal{A} \in \{M_{m\times n}(\mathbb{F}), S_n(\mathbb{F}), H_n(\mathbb{F}), A_n(\mathbb{F})\}$$

are characterized by fundamental theorem of geometry of matrices of appropriate type. (cf. Wan 1996)

$\mathcal{A} = H_n(\mathbb{F})$: $\Phi(A) = \lambda P A^\sigma P^* + B$

Adjacency preservers

A = set of matrices

$X, Y \in A$ are *adjacent* if $\mathrm{rk}(X - Y)$ is minimal and nonzero

$A \in \{M_{m\times n}(\mathbb{F}), S_n(\mathbb{F}), H_n(\mathbb{F})\} \Longrightarrow \mathrm{rk}(A - B) = 1$

$A = A_n(\mathbb{F}) \Longrightarrow \mathrm{rk}(A - B) = 2$

$XRY \iff X \text{ and } Y \text{ are adjacent}$

Bijective maps that preserves adjacency in both directions on

$$A \in \{M_{m\times n}(\mathbb{F}), S_n(\mathbb{F}), H_n(\mathbb{F}), A_n(\mathbb{F})\}$$

are characterized by fundamental theorem of geometry of matrices of appropriate type. (cf. Wan 1996)

$A = H_n(\mathbb{F})$: $\Phi(A) = \lambda P A^{\sigma} P^* + B$

Adjacency preservers

$\mathcal{A}$ = set of matrices

$X, Y \in \mathcal{A}$ are *adjacent* if $\mathrm{rk}(X - Y)$ is minimal and nonzero

$\mathcal{A} \in \{M_{m\times n}(\mathbb{F}), S_n(\mathbb{F}), H_n(\mathbb{F})\} \Longrightarrow \mathrm{rk}(A - B) = 1$

$\mathcal{A} = A_n(\mathbb{F}) \Longrightarrow \mathrm{rk}(A - B) = 2$

$X\mathcal{R}Y \Longleftrightarrow X$ and Y are adjacent

Bijective maps that preserves adjacency in both directions on

$$\mathcal{A} \in \{M_{m\times n}(\mathbb{F}), S_n(\mathbb{F}), H_n(\mathbb{F}), A_n(\mathbb{F})\}$$

are characterized by fundamental theorem of geometry of matrices of appropriate type. (cf. Wan 1996)

$\mathcal{A} = H_n(\mathbb{F})$: $\quad \Phi(A) = \lambda P A^\sigma P^* + B$

Adjacency preservers

$\mathcal{A}$ = set of matrices

$X, Y \in \mathcal{A}$ are *adjacent* if $\mathrm{rk}(X - Y)$ is minimal and nonzero

$\mathcal{A} \in \{M_{m\times n}(\mathbb{F}), S_n(\mathbb{F}), H_n(\mathbb{F})\} \Longrightarrow \mathrm{rk}(A - B) = 1$

$\mathcal{A} = A_n(\mathbb{F}) \Longrightarrow \mathrm{rk}(A - B) = 2$

$XRY \Longleftrightarrow X$ and Y are adjacent

Bijective maps that preserves adjacency in both directions on

$$\mathcal{A} \in \{M_{m\times n}(\mathbb{F}), S_n(\mathbb{F}), H_n(\mathbb{F}), A_n(\mathbb{F})\}$$

are characterized by fundamental theorem of geometry of matrices of appropriate type. (cf. Wan 1996)

$\mathcal{A} = H_n(\mathbb{F})$: $\quad \Phi(A) = \lambda P A^\sigma P^* + B$

Adjacency preservers

A = set of matrices

$X, Y \in A$ are *adjacent* if $\mathrm{rk}(X - Y)$ is minimal and nonzero

$A \in \{M_{m\times n}(\mathbb{F}), S_n(\mathbb{F}), H_n(\mathbb{F})\} \Longrightarrow \mathrm{rk}(A - B) = 1$

$A = A_n(\mathbb{F}) \Longrightarrow \mathrm{rk}(A - B) = 2$

$XRY \iff X$ and Y are adjacent

Bijective maps that preserves adjacency in both directions on

$$A \in \{M_{m\times n}(\mathbb{F}), S_n(\mathbb{F}), H_n(\mathbb{F}), A_n(\mathbb{F})\}$$

are characterized by fundamental theorem of geometry of matrices of appropriate type. (cf. Wan 1996)

$A = H_n(\mathbb{F})$: $\quad \Phi(A) = \lambda P A^\sigma P^* + B$

$\mathcal{A}$ = set of matrices

$X, Y \in \mathcal{A}$ are *adjacent* if $\mathrm{rk}(X - Y)$ is minimal and nonzero

$\mathcal{A} \in \{M_{m\times n}(\mathbb{F}), S_n(\mathbb{F}), H_n(\mathbb{F})\} \Longrightarrow \mathrm{rk}(A - B) = 1$

$\mathcal{A} = A_n(\mathbb{F}) \Longrightarrow \mathrm{rk}(A - B) = 2$

$XRY \iff X$ and Y are adjacent

Bijective maps that preserves adjacency in both directions on

$$\mathcal{A} \in \{M_{m\times n}(\mathbb{F}), S_n(\mathbb{F}), H_n(\mathbb{F}), A_n(\mathbb{F})\}$$

are characterized by fundamental theorem of geometry of matrices of appropriate type. (cf. Wan 1996)

$\mathcal{A} = H_n(\mathbb{F})$: $\Phi(A) = \lambda P A^\sigma P^* + B$

Adjacency preservers

$\mathcal{A}$ = set of matrices

$X, Y \in \mathcal{A}$ are *adjacent* if $\mathrm{rk}(X - Y)$ is minimal and nonzero

$\mathcal{A} \in \{M_{m\times n}(\mathbb{F}), S_n(\mathbb{F}), H_n(\mathbb{F})\} \Longrightarrow \mathrm{rk}(A - B) = 1$

$\mathcal{A} = A_n(\mathbb{F}) \Longrightarrow \mathrm{rk}(A - B) = 2$

$X\mathcal{R}Y \iff X \text{ and } Y \text{ are adjacent}$

Bijective maps that preserves adjacency in both directions on

$$\mathcal{A} \in \{M_{m\times n}(\mathbb{F}), S_n(\mathbb{F}), H_n(\mathbb{F}), A_n(\mathbb{F})\}$$

are characterized by fundamental theorem of geometry of matrices of appropriate type. (cf. Wan 1996)

$\mathcal{A} = H_n(\mathbb{F})$: $\quad \Phi(A) = \lambda P A^{\sigma} P^* + B$

Adjacency preservers (no bijectivity, one direction)

Characterizations of adjacency preservers:

- $H_n(\mathbb{C})$ (Šemrl, Huang 2008, Canad. J. Math.)
- $H_2(\mathbb{D})$ (Huang 2008, Aequationes Math.)
- $S_n(\mathbb{R})$ (Legiša 2011, Math. Commun.)
- $H_n(\mathbb{F}_{q^2})$ (Orel 2009, Finite Fields Appl.)
- $S_n(\mathbb{F}_q)$, $n \geq 3$ (Orel 2012, J. Algebraic Combin.)
- $M_{m\times n}(\mathbb{D})$
  (Šemrl 2014, Mem. Amer. Math. Soc.)
  (Šemrl & de Seguins Pazzis, preprint)
- $M_{m\times n}(\mathbb{F}_q)$
  (Li, Sze, Huang, Huang, 2014, Linear Algebra Appl.)
- $HGL_n(\mathbb{F}_{q^2})$, $q \geq 4$ (Orel, submitted; preprint on arXiv)

$$\Phi(A) = PA^\sigma P^* \qquad \Phi(A) = P(A^{-1})^\sigma P^*$$

Adjacency preservers (no bijectivity, one direction)

Characterizations of adjacency preservers:

- $H_n(\mathbb{C})$ (Šemrl, Huang 2008, Canad. J. Math.)
- $H_2(\mathbb{D})$ (Huang 2008, Aequationes Math.)
- $S_n(\mathbb{R})$ (Legiša 2011, Math. Commun.)
- $H_n(\mathbb{F}_{q^2})$ (Orel 2009, Finite Fields Appl.)
- $S_n(\mathbb{F}_q)$, $n \geq 3$ (Orel 2012, J. Algebraic Combin.)
- $M_{m\times n}(\mathbb{D})$
  (Šemrl 2014, Mem. Amer. Math. Soc.)
  (Šemrl & de Seguins Pazzis, preprint)
- $M_{m\times n}(\mathbb{F}_q)$
  (Li, Sze, Huang, Huang, 2014, Linear Algebra Appl.)
- $HGL_n(\mathbb{F}_{q^2})$, $q \geq 4$ (Orel, submitted; preprint on arXiv)

$$\Phi(A) = PA^\sigma P^* \qquad \Phi(A) = P(A^{-1})^\sigma P^*$$

Adjacency preservers (no bijectivity, one direction)

Characterizations of adjacency preservers:

- $H_n(\mathbb{C})$ (Šemrl, Huang 2008, Canad. J. Math.)
- $H_2(\mathbb{D})$ (Huang 2008, Aequationes Math.)
- $S_n(\mathbb{R})$ (Legiša 2011, Math. Commun.)
- $H_n(\mathbb{F}_{q^2})$ (Orel 2009, Finite Fields Appl.)
- $S_n(\mathbb{F}_q)$, $n \geq 3$ (Orel 2012, J. Algebraic Combin.)
- $M_{m\times n}(\mathbb{D})$
  (Šemrl 2014, Mem. Amer. Math. Soc.)
  (Šemrl & de Seguins Pazzis, preprint)
- $M_{m\times n}(\mathbb{F}_q)$
  (Li, Sze, Huang, Huang, 2014, Linear Algebra Appl.)
- $HGL_n(\mathbb{F}_{q^2})$, $q \geq 4$ (Orel, submitted; preprint on arXiv)

$$\Phi(A) = PA^\sigma P^* \qquad \Phi(A) = P(A^{-1})^\sigma P^*$$

Adjacency preservers (no bijectivity, one direction)

Characterizations of adjacency preservers:

- $H_n(\mathbb{C})$ (Šemrl, Huang 2008, Canad. J. Math.)
- $H_2(\mathbb{D})$ (Huang 2008, Aequationes Math.)
- $S_n(\mathbb{R})$ (Legiša 2011, Math. Commun.)
- $H_n(\mathbb{F}_{q^2})$ (Orel 2009, Finite Fields Appl.)
- $S_n(\mathbb{F}_q)$, $n \geq 3$ (Orel 2012, J. Algebraic Combin.)
- $M_{m\times n}(\mathbb{D})$
  (Šemrl 2014, Mem. Amer. Math. Soc.)
  (Šemrl & de Seguins Pazzis, preprint)
- $M_{m\times n}(\mathbb{F}_q)$
  (Li, Sze, Huang, Huang, 2014, Linear Algebra Appl.)
- $HGL_n(\mathbb{F}_{q^2})$, $q \geq 4$ (Orel, submitted; preprint on arXiv)

$$\Phi(A) = PA^\sigma P^* \qquad \Phi(A) = P(A^{-1})^\sigma P^*$$

Adjacency preservers (no bijectivity, one direction)

Characterizations of adjacency preservers:

- $H_n(\mathbb{C})$ (Šemrl, Huang 2008, Canad. J. Math.)
- $H_2(\mathbb{D})$ (Huang 2008, Aequationes Math.)
- $S_n(\mathbb{R})$ (Legiša 2011, Math. Commun.)
- $H_n(\mathbb{F}_{q^2})$ (Orel 2009, Finite Fields Appl.)
- $S_n(\mathbb{F}_q)$, $n \geq 3$ (Orel 2012, J. Algebraic Combin.)
- $M_{m\times n}(\mathbb{D})$
  (Šemrl 2014, Mem. Amer. Math. Soc.)
  (Šemrl & de Seguins Pazzis, preprint)
- $M_{m\times n}(\mathbb{F}_q)$
  (Li, Sze, Huang, Huang, 2014, Linear Algebra Appl.)
- $HGL_n(\mathbb{F}_{q^2})$, $q \geq 4$ (Orel, submitted; preprint on arXiv)

$$\Phi(A) = PA^{\sigma}P^* \qquad\qquad \Phi(A) = P(A^{-1})^{\sigma}P^*$$

Adjacency preservers (no bijectivity, one direction)

Characterizations of adjacency preservers:

- $H_n(\mathbb{C})$ (Šemrl, Huang 2008, Canad. J. Math.)
- $H_2(\mathbb{D})$ (Huang 2008, Aequationes Math.)
- $S_n(\mathbb{R})$ (Legiša 2011, Math. Commun.)
- $H_n(\mathbb{F}_{q^2})$ (Orel 2009, Finite Fields Appl.)
- $S_n(\mathbb{F}_q)$, $n \geq 3$ (Orel 2012, J. Algebraic Combin.)
- $M_{m\times n}(\mathbb{D})$
  (Šemrl 2014, Mem. Amer. Math. Soc.)
  (Šemrl & de Seguins Pazzis, preprint)
- $M_{m\times n}(\mathbb{F}_q)$
  (Li, Sze, Huang, Huang, 2014, Linear Algebra Appl.)
- $HGL_n(\mathbb{F}_{q^2})$, $q \geq 4$ (Orel, submitted; preprint on arXiv)

$$\Phi(A) = PA^\sigma P^* \qquad \Phi(A) = P(A^{-1})^\sigma P^*$$

Adjacency preservers (no bijectivity, one direction)

Characterizations of adjacency preservers:

- $H_n(\mathbb{C})$ (Šemrl, Huang 2008, Canad. J. Math.)
- $H_2(\mathbb{D})$ (Huang 2008, Aequationes Math.)
- $S_n(\mathbb{R})$ (Legiša 2011, Math. Commun.)
- $H_n(\mathbb{F}_{q^2})$ (Orel 2009, Finite Fields Appl.)
- $S_n(\mathbb{F}_q)$, $n \geq 3$ (Orel 2012, J. Algebraic Combin.)
- $M_{m\times n}(\mathbb{D})$
  (Šemrl 2014, Mem. Amer. Math. Soc.)
  (Šemrl & de Seguins Pazzis, preprint)
- $M_{m\times n}(\mathbb{F}_q)$
  (Li, Sze, Huang, Huang, 2014, Linear Algebra Appl.)
- $HGL_n(\mathbb{F}_{q^2})$, $q \geq 4$ (Orel, submitted; preprint on arXiv)

$$\Phi(A) = PA^\sigma P^* \qquad \Phi(A) = P(A^{-1})^\sigma P^*$$

Characterizations of adjacency preservers:

- $H_n(\mathbb{C})$ (Šemrl, Huang 2008, Canad. J. Math.)
- $H_2(\mathbb{D})$ (Huang 2008, Aequationes Math.)
- $S_n(\mathbb{R})$ (Legiša 2011, Math. Commun.)
- $H_n(\mathbb{F}_{q^2})$ (Orel 2009, Finite Fields Appl.)
- $S_n(\mathbb{F}_q)$, $n \geq 3$ (Orel 2012, J. Algebraic Combin.)
- $M_{m\times n}(\mathbb{D})$
  (Šemrl 2014, Mem. Amer. Math. Soc.)
  (Šemrl & de Seguins Pazzis, preprint)
- $M_{m\times n}(\mathbb{F}_q)$
  (Li, Sze, Huang, Huang, 2014, Linear Algebra Appl.)
- $HGL_n(\mathbb{F}_{q^2})$, $q \geq 4$ (Orel, submitted; preprint on arXiv)

$$\Phi(A) = PA^\sigma P^* \qquad \Phi(A) = P(A^{-1})^\sigma P^*$$

Adjacency preservers (no bijectivity, one direction)

Characterizations of adjacency preservers:

- $H_n(\mathbb{C})$ (Šemrl, Huang 2008, Canad. J. Math.)
- $H_2(\mathbb{D})$ (Huang 2008, Aequationes Math.)
- $S_n(\mathbb{R})$ (Legiša 2011, Math. Commun.)
- $H_n(\mathbb{F}_{q^2})$ (Orel 2009, Finite Fields Appl.)
- $S_n(\mathbb{F}_q)$, $n \geq 3$ (Orel 2012, J. Algebraic Combin.)
- $M_{m\times n}(\mathbb{D})$
  (Šemrl 2014, Mem. Amer. Math. Soc.)
  (Šemrl & de Seguins Pazzis, preprint)
- $M_{m\times n}(\mathbb{F}_q)$
  (Li, Sze, Huang, Huang, 2014, Linear Algebra Appl.)
- $HGL_n(\mathbb{F}_{q^2})$, $q \geq 4$ (Orel, submitted; preprint on arXiv)

$$\Phi(A) = PA^\sigma P^* \qquad\qquad \Phi(A) = P(A^{-1})^\sigma P^*$$

Some tools

Cameron, Kazanidis 2008, J. Aust. Math. Soc.

If $\mathrm{Aut}(\Gamma)$ acts transitively on pairs of non-adjacent vertices, then Γ is a core or its core is a complete graph.

Godsil, Royle 2011, Ann. Comb.

If Γ connected regular, $\mathrm{Aut}(\Gamma)$ acts transitively on pairs of vertices at distance 2, then Γ is a core or its core is a complete graph.

Cores

$H_n(\mathbb{F}_{q^2})$

$HGL_n(\mathbb{F}_{q^2})$, $q \geq 4$

$S_n(\mathbb{F}_q)$, $n \geq 3$

$HGL_n(\mathbb{F}_{2^2})$, $SGL_m(\mathbb{F}_2)$, $m \geq 3$

Complete cores

$M_{m\times n}(\mathbb{F}_q)$ (Li, Sze, Huang, Huang)

$S_2(\mathbb{F}_q)$

Some tools

Cameron, Kazanidis 2008, J. Aust. Math. Soc.

If $\mathrm{Aut}(\Gamma)$ acts transitively on pairs of non-adjacent vertices, then Γ is a core or its core is a complete graph.

Godsil, Royle 2011, Ann. Comb.

If Γ connected regular, $\mathrm{Aut}(\Gamma)$ acts transitively on pairs of vertices at distance 2, then Γ is a core or its core is a complete graph.

Cores

$H_n(\mathbb{F}_{q^2})$ $\quad$ $S_n(\mathbb{F}_q)$, $n \geq 3$

$HGL_n(\mathbb{F}_{q^2})$, $q \geq 4$ $\quad$ $HGL_n(\mathbb{F}_{2^2})$, $SGL_m(\mathbb{F}_2)$, $m \geq 3$

Complete cores

$M_{m\times n}(\mathbb{F}_q)$ (Li, Sze, Huang, Huang)

$S_2(\mathbb{F}_q)$

Some tools

Cameron, Kazanidis 2008, J. Aust. Math. Soc.

If $\mathrm{Aut}(\Gamma)$ acts transitively on pairs of non-adjacent vertices, then Γ is a core or its core is a complete graph.

Godsil, Royle 2011, Ann. Comb.

If Γ connected regular, $\mathrm{Aut}(\Gamma)$ acts transitively on pairs of vertices at distance 2, then Γ is a core or its core is a complete graph.

Cores

$H_n(\mathbb{F}_{q^2})$ $\quad$ $S_n(\mathbb{F}_q)$, $n \geq 3$

$HGL_n(\mathbb{F}_{q^2})$, $q \geq 4$ $\quad$ $HGL_n(\mathbb{F}_{2^2})$, $SGL_m(\mathbb{F}_2)$, $m \geq 3$

Complete cores

$M_{m\times n}(\mathbb{F}_q)$ (Li, Sze, Huang, Huang)

$S_2(\mathbb{F}_q)$

Some tools

Cameron, Kazanidis 2008, J. Aust. Math. Soc.

If $\mathrm{Aut}(\Gamma)$ acts transitively on pairs of non-adjacent vertices, then Γ is a core or its core is a complete graph.

Godsil, Royle 2011, Ann. Comb.

If Γ connected regular, $\mathrm{Aut}(\Gamma)$ acts transitively on pairs of vertices at distance 2, then Γ is a core or its core is a complete graph.

Cores

$H_n(\mathbb{F}_{q^2})$ $\quad$ $S_n(\mathbb{F}_q)$, $n \geq 3$

$HGL_n(\mathbb{F}_{q^2})$, $q \geq 4$ $\quad$ $HGL_n(\mathbb{F}_{2^2})$, $SGL_m(\mathbb{F}_2)$, $m \geq 3$

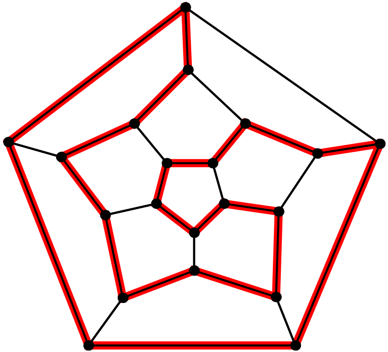
Complete cores

$M_{m\times n}(\mathbb{F}_q)$ (Li, Sze, Huang, Huang)

$S_2(\mathbb{F}_q)$

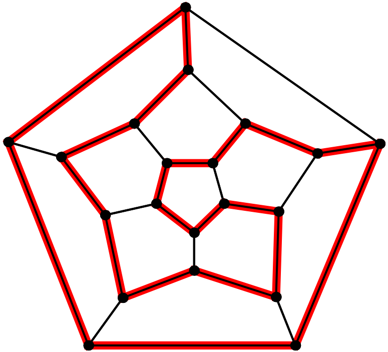
Problem on hamiltonicity related to Lovász problem

A cycle in a graph is Hamiltonian if it goes true every vertex.



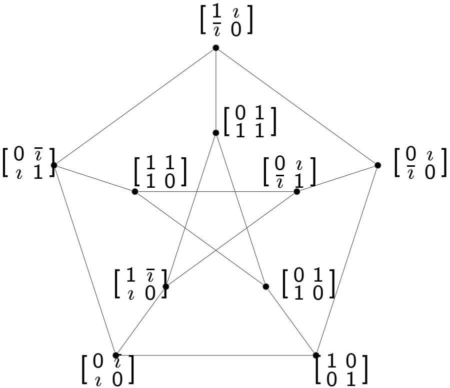
There are only 5 known connected vertex-transitive graphs **without** a Hamiltonian cycle: K_2, Petersen graph, Coxeter graph, two graphs derived from Petersen/Coxeter graph

A cycle in a graph is Hamiltonian if it goes true every vertex.

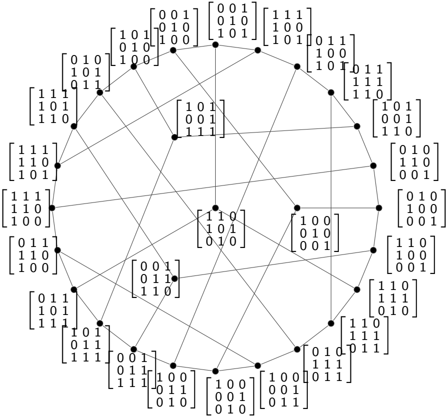


There are only 5 known connected vertex-transitive graphs **without** a Hamiltonian cycle: K_2, Petersen graph, Coxeter graph, two graphs derived from Petersen/Coxeter graph

Graph $HGL_2(\mathbb{F}_4)$ is the Petersen graph.



Graph $SGL_3(\mathbb{F}_2)$ is the Coxeter graph.



$HGL_n(\mathbb{F}_4)$ vertex transitive
$SGL_n(\mathbb{F}_2)$ vertex transitive for odd n

Problem

Do graphs $HGL_n(\mathbb{F}_4)$ and $SGL_m(\mathbb{F}_2)$ contain a Hamiltonian cycle for $n \geq 3$ and $m \geq 4$?
How to construct a hamiltonian cycle if it exists?

Concorde TSP Solver: yes, if $n = 3$ and $m \in \{4, 5\}$

$HGL_n(\mathbb{F}_4)$ vertex transitive
$SGL_n(\mathbb{F}_2)$ vertex transitive for odd n

Problem

Do graphs $HGL_n(\mathbb{F}_4)$ and $SGL_m(\mathbb{F}_2)$ contain a Hamiltonian cycle for $n \geq 3$ and $m \geq 4$?
How to construct a hamiltonian cycle if it exists?

Concorde TSP Solver: yes, if $n = 3$ and $m \in \{4, 5\}$

$HGL_n(\mathbb{F}_4)$ vertex transitive
$SGL_n(\mathbb{F}_2)$ vertex transitive for odd n

Problem

Do graphs $HGL_n(\mathbb{F}_4)$ and $SGL_m(\mathbb{F}_2)$ contain a Hamiltonian cycle for $n \geq 3$ and $m \geq 4$?
How to construct a hamiltonian cycle if it exists?

Concorde TSP Solver: yes, if $n = 3$ and $m \in \{4, 5\}$

Thank you for your attention!