Preservers of Unextendible Product Bases
and
Local Distinguishability of Quantum States

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Overview

- Nonlocality without entanglement
- Unextendible product bases
- Preservers

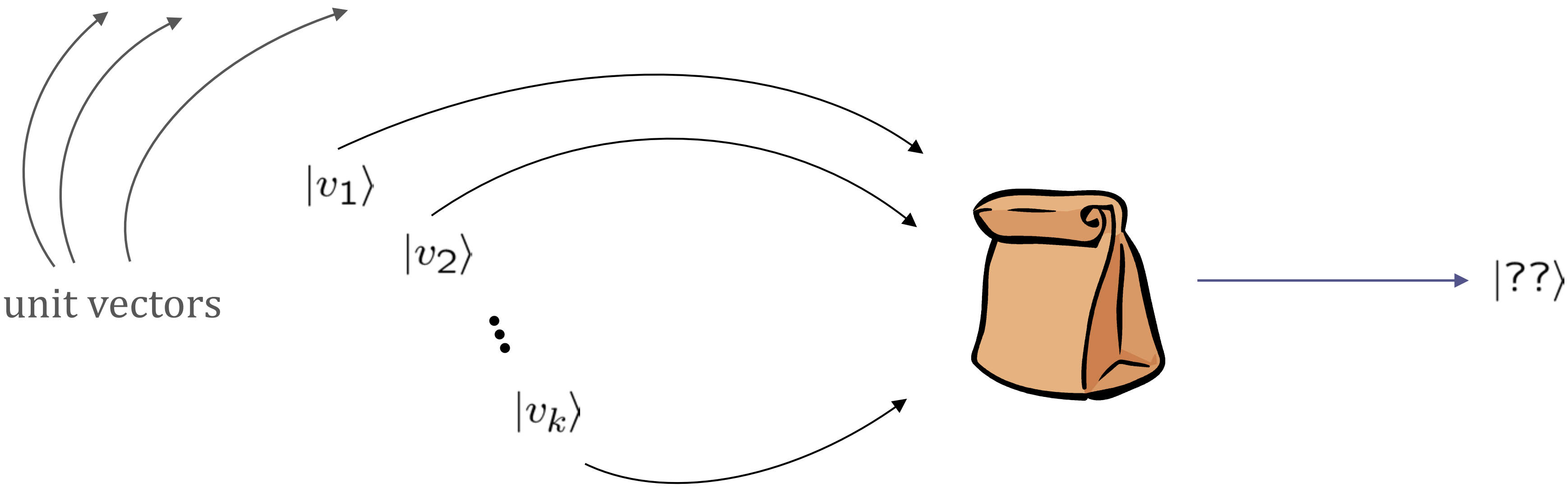
Overview

- **Nonlocality without entanglement**
  - Can prepare states locally that we then cannot distinguish locally
  - That's weird!
- Unextendible product bases
- Preservers

Nonlocality without entanglement

It is well-known that a set of pure quantum states

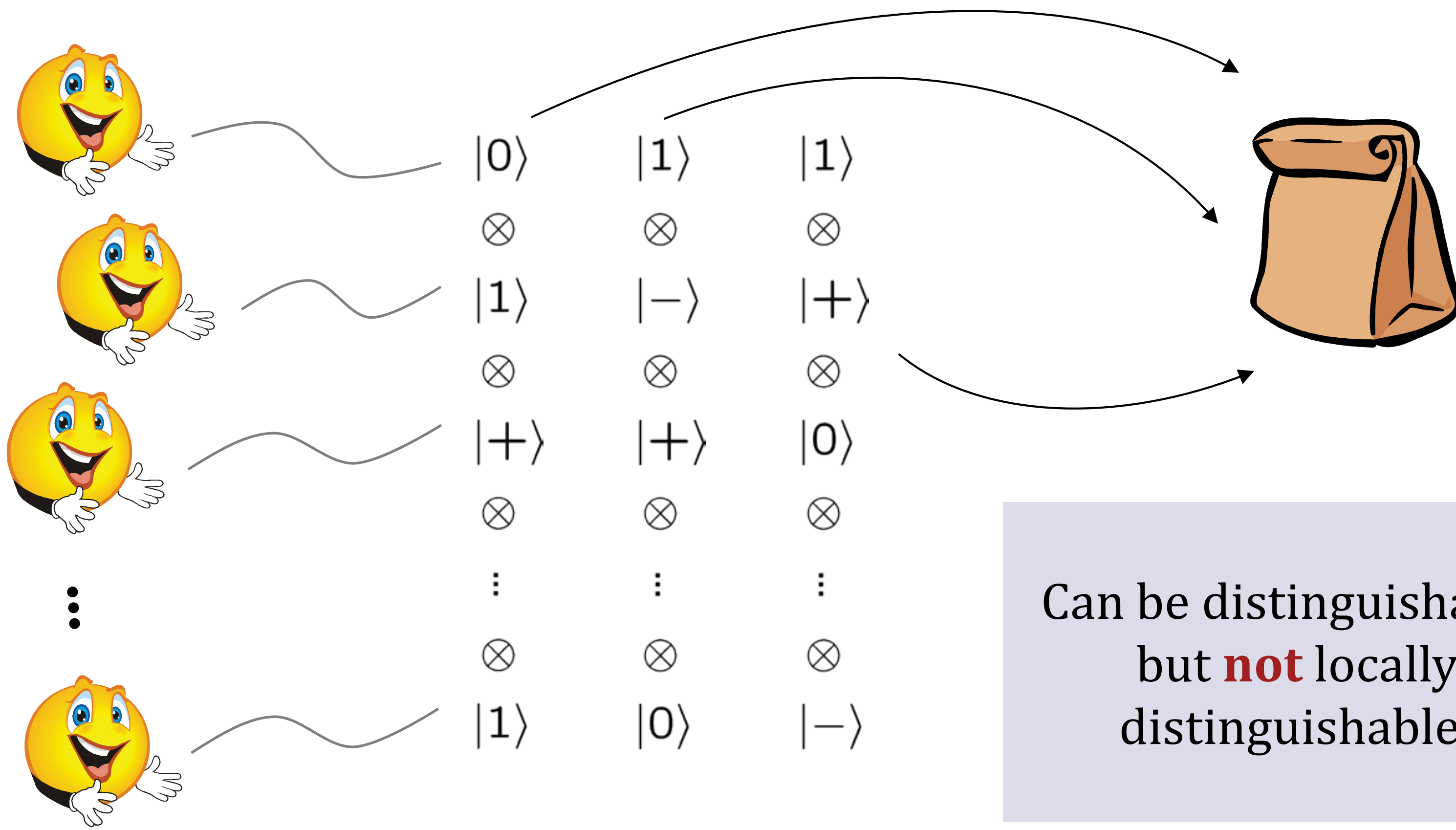
$$|v_1\rangle, |v_2\rangle, \ldots, |v_k\rangle \in \mathbb{C}^d$$



if and only if they are mutually orthogonal.

Nonlocality without entanglement

Many parties locally prepare some pure (product) quantum states:



Can be distinguishable, but **not** locally distinguishable!

Nonlocality without entanglement

One way to create such states:

unextendible product bases

Overview

- Nonlocality without entanglement
- **Unextendible product bases**
  - Exhibit nonlocality without entanglement
  - Also useful for other things
- Preservers

Unextendible product bases

An unextendible product basis (UPB) is a set of vectors

$$S \subset \mathbb{C}^{d_1} \otimes \mathbb{C}^{d_2} \otimes \cdots \otimes \mathbb{C}^{d_p}$$

satisfying three properties:

unextendibility

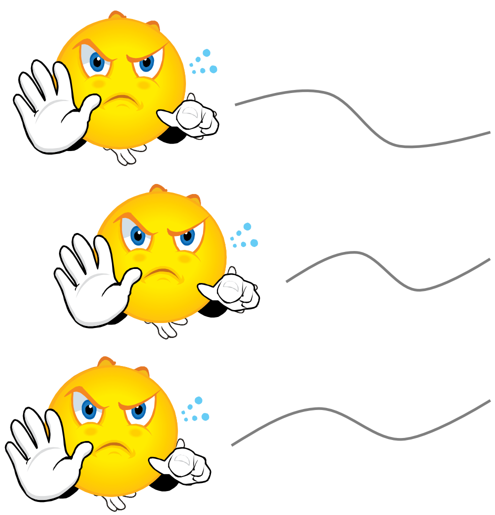
1. Each $|v\rangle \in S$ is a product state (i.e., $|v\rangle = |v_1\rangle \otimes |v_2\rangle \otimes \cdots \otimes |v_p\rangle$).

2. Mutual orthogonality: $\langle w|v\rangle = 0 \quad \forall\, |v\rangle, |w\rangle \in S.$

3. There is no product state $|z\rangle$ satisfying $\langle z|v\rangle = 0 \quad \forall\, |v\rangle \in S.$

Unextendible product bases

A set is a UPB iff we get "stuck" when locally preparing them:



$$\begin{array}{cccc} |0\rangle & |1\rangle & |-\rangle & |+\rangle \\ \otimes & \otimes & \otimes & \otimes \\ |0\rangle & |+\rangle & |1\rangle & |-\rangle \\ \otimes & \otimes & \otimes & \otimes \\ |0\rangle & |-\rangle & |+\rangle & |1\rangle \end{array}$$

...stuck!

- No product state is orthogonal to them all, so this is a UPB.
- This is called the "shifts" UPB.

Unextendible product bases

Theorem (Bennett et. al., 1998)

The states of a UPB are not perfectly locally distinguishable in a finite amount of time.

What if we have infinite time or allow for arbitrarily small error?

- Seems to be a harder question.
- This is where preservers come in!

Overview

- Nonlocality without entanglement
- Unextendible product bases
- **Preservers**
  - Useful for answering the infinite time distinguishability question
  - But we still don’t know what they look like…

Preservers

We have two main questions about what the preservers of UPBs look like. ("Yes" to both $\Longrightarrow$ infinite time indistinguishability)

Question 1

Given a UPB $S \subset \mathbb{C}^{d_1} \otimes \mathbb{C}^{d_2} \otimes \cdots \otimes \mathbb{C}^{d_p}$, suppose there exist $A_i \in M_{d_i}$ for $1 \leq i \leq p$ such that:

- $\langle v|(A_1 \otimes \cdots \otimes A_p)|v\rangle \neq 0 \quad \forall\, |v\rangle \in S$, and
- $(A_1 \otimes \cdots \otimes A_p)S$ is a set of mutually orthogonal vectors.

Does this imply that each A_i has full rank?

need unextendibility of S

maybe not unit vectors

Preservers

Question 2

Given a UPB $S \subset \mathbb{C}^{d_1} \otimes \mathbb{C}^{d_2} \otimes \cdots \otimes \mathbb{C}^{d_p}$, suppose there exist full rank $A_i \in M_{d_i}$ for $1 \leq i \leq p$ such that $(A_1 \otimes \cdots \otimes A_p)S$ is a UPB.

Does this imply that each A_i is a multiple of a unitary matrix?

members of this UPB don't need to have unit length

- Multiples of unitary matrices preserve UPBs
- Question 2 asks whether or not these are *all* preservers

Preservers

What is known?

- In $3 \otimes 3$ systems, the answer to both questions is "yes" (Fu-Leung-Mančinska, arXiv:1312.5350).
- In $2 \otimes 2 \otimes \cdots \otimes 2$ systems, the answer to both questions is "yes" (unpublished).

- Proof techniques don't generalize to other cases
- Don't know the answer any other cases