

THE UNIVERSITY OF MANITOBA

April 15, 2000

FINAL EXAMINATION

PAPER NO: 387

TITLE PAGE

DEPARTMENT & COURSE NO: 136.150

TIME: 2 HOURS

EXAMINATION: Introductory Calculus

EXAMINER: (Identified Below)

NAME: (PRINT) _____

STUDENT NUMBER: _____

SIGNATURE: _____

(I understand that cheating is a serious offense)

Please indicate your instructor and section by placing a check mark in the appropriate box below.

<u>SECTION</u>		<u>TIME</u>	<u>INSTRUCTOR</u>
<input type="checkbox"/> L08	M,W,F Tues.	10:30 - 11:20 10:00 - 10:50	P. Penner
<input type="checkbox"/> L09	Tues, Thurs	8:30 - 9:45	W. Korytowski
<input type="checkbox"/> L10	M, W, F	9:30 - 10:20	S. Kalajdziewski
<input type="checkbox"/> L11	Tues, Thurs.	11:30 - 12:45	A. Gumel
<input type="checkbox"/> L92	Challenge for credit		

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. Please show your work clearly.

No calculators, texts, notes or other aids are permitted.

This exam has a title page, 11 pages of questions and 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staples.

The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 120.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

DO NOT WRITE IN THIS COLUMN

1. _____ /15

2. _____ /10

3. _____ /14

4. _____ /10

5. _____ /10

6. _____ /11

7. _____ /18

8. _____ /11

9. _____ /11

10. _____ /10

TOTAL

_____ /120

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Values

[15] 1. Compute each limit, if it exists. If it does not exist determine if the expression tends to $+\infty$, $-\infty$ or neither.

(a) $\lim_{x \rightarrow 2^-} \frac{x^2 + 8}{x - 2}$

(b) $\lim_{x \rightarrow \infty} \sqrt{x^2 - 2x} - x$

(c) $\lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 2x}$

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Values

[10] 2. (a) State the definition of the derivative $f'(x)$ of a function $f(x)$.

(b) Use the definition in (a) to evaluate $f'(x)$ if $f(x) = x^2 - 2x$.

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EXAMINER: Various

Values

[14] 3. Find $\frac{dy}{dx}$ if

(a) $y = \frac{x}{\cos x}$

(b) $y = \sqrt{xe^x + \frac{1}{x}}$

(c) $y = (1 + \sin x)^x$.

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EXAMINER: Various

Values

[10] 4. (a) State the Mean-Value Theorem.

(b) Use the Mean-Value Theorem and show that if $f'(x) > 0$ for all x in the interval $[a, b]$ then $f(x)$ is increasing on $[a, b]$.

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[10] 5. Find the equation of the tangent line to the curve

$2x^3y^3 - y = x$ at the point $(1, 1)$.

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EXAMINER: Various

Values

- [11] 6. Find all local and absolute extrema of the function $f(x) = 2x^3 + 3x^2 - 12x + 1$ on the interval $[-1, 2]$. Identify the point(s) of absolute maximum and the point(s) of absolute minimum.

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EXAMINER: Various

Values

[18] 7. Let $f(x) = \frac{2x^2}{x^2 - 1}$. The first two derivatives are

$$f'(x) = \frac{-4x}{(x^2 - 1)^2} \text{ and}$$

$$f''(x) = \frac{12x^2 + 4}{(x^2 - 1)^3}$$

(a) Compile the following information about f and its graph (Give answers only; answer "NONE" if the function doesn't display a feature listed).

domain? _____

critical/singular points? _____

symmetry? _____

asymptotes? _____

x,y intercepts? _____

intervals of increase/decrease? _____

local extrema? _____

intervals of concavity? _____

points of inflection? _____

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7. (b) Sketch the graph of $y = f(x)$ on the given axes, reflecting all relevant information from a) and labelling any important features on the graph.

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- [11] 8. A boat is travelling along a straight line at 60 km/hr. A student on the boat accidentally drops a heavy calculus textbook in the water, which then sinks vertically at 2 km/hr. How fast are the student and the textbook separating 0.1 hours after the textbook is released in the water? (Assume the water is very deep.)

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- [11] 9. By cutting away identical squares from each corner of a rectangular piece of cardboard and folding up the resulting flaps, the cardboard may be turned into an open box. If the cardboard is 16 cm long and 10 cm wide, find the dimensions of the box that will yield the maximum volume.

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Values

[10] 10. (a) Find an antiderivative of the function $f(x) = -x^2 + 4x$.

(b) Compute the area of the region bounded by $f(x)$ and the x-axis.