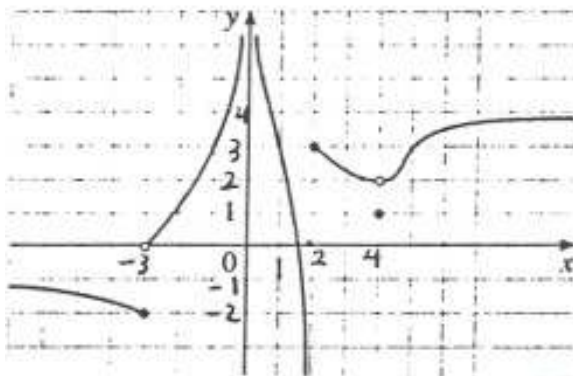


MATH 1500 A01 Assignment 1 Winter 2010 (Due date is Friday January 29)

- [12] 1. Let  $f(x) = \frac{1}{x-2}$  and  $g(x) = \sqrt{2x+1}$ ; find  $g \circ f$ ,  $f \circ g$ ,  $f \circ f$ , and  $g \circ g$ . Also find their domains.
- [6] 2. The graph of a function  $f$  is given. Find each of the following if they exist. If a limit does not exist indicate why not and whether the function tends to  $\infty$  or  $-\infty$ .



- (i)  $\lim_{x \rightarrow 4} f(x)$                       (ii)  $\lim_{x \rightarrow 2^-} f(x)$                       (iii)  $\lim_{x \rightarrow 2^+} f(x)$
- (iv)  $\lim_{x \rightarrow 0} f(x)$                       (v)  $\lim_{x \rightarrow (-3)^+} f(x)$                       (vi)  $\lim_{x \rightarrow (-3)^-} f(x)$

- [36] 3. Evaluate each of the following limits or explain why it does not exist.

- (1)  $\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{-3x - 6}$       (2)  $\lim_{x \rightarrow -3} \frac{x^3 - 9x}{x^2 + 2x - 3}$       (3)  $\lim_{x \rightarrow 0} \frac{x}{\sqrt{x+1} - 1}$
- (4)  $\lim_{x \rightarrow 3} \left( \frac{1}{x-3} - \frac{6}{x^2-9} \right)$       (5)  $\lim_{x \rightarrow 1} \frac{\sqrt{3x} - \sqrt{3}}{x-1}$       (6)  $\lim_{t \rightarrow 5^-} \frac{t-5}{|t^2-25|}$
- (7)  $\lim_{h \rightarrow 0} \frac{1}{\sqrt{1+h}-1}$       (8)  $\lim_{x \rightarrow 2^-} \frac{5}{\sqrt{2-x}}$       (9)  $\lim_{x \rightarrow 1} \frac{x^3 \sqrt{x} - \sqrt{x}}{(x^2-1)(x^2+x+1)}$

- [8] 4. Let  $f(x) = \begin{cases} 3x + a & \text{if } x < 1 \\ bx & \text{if } 1 \leq x \leq 2 \\ ax + 2a & \text{if } x > 2 \end{cases}$ . Find values of  $a$  and  $b$  such that the function has limit at both 1 and 2.

- [6] 5. Show that  $\lim_{x \rightarrow 0} \sqrt[3]{x^4} \cos \frac{3}{x} = 0$ .

[6] 6. Let  $f(x) = \begin{cases} \sqrt{x^2 + 1} & \text{if } x > 0 \\ x^2 + 1 & \text{if } x < 0 \\ a & \text{if } x = 0 \end{cases}$ . Does the limit of  $f(x)$  exist at  $x = 0$ ? If yes, is

there a value of  $a$  that makes this function continuous at  $x = 0$ ? (Give reasons for your answer.)

[6] 7. Find **all** values of  $a$  for which  $f(x) = \begin{cases} \frac{x^2 - a^4}{x - a^2} & \text{if } x \neq a \\ 2 & \text{if } x = a \end{cases}$  is continuous at  $x = a$ .

[8] 8. Find the constants  $a$  and  $b$  such that  $f(x) = \begin{cases} 2 & \text{if } x \leq -1 \\ ax + b & \text{if } -1 < x < 3 \\ -2 & \text{if } x \geq 3 \end{cases}$  is continuous everywhere. Use limits to justify your answers.

[6] 9. (a) State the Intermediate Value Theorem.

(b) Show that the function  $f(x) = e^x + x - 2$  has at least one real root.

[6] 10. Is it true that if  $f(x) > 1$  for all  $x$  and  $\lim_{x \rightarrow 0} f(x)$  exists, then  $\lim_{x \rightarrow 0} f(x) > 1$ ? If yes prove it and if no give a counter example.