Q1 $P = 6$ m

$6 = 2y + \frac{1}{2}\pi x + \frac{1}{2}\pi x \Rightarrow 6 = 2y + \pi x \Rightarrow y = \frac{6-\pi x}{2} = 3 - \frac{\pi}{2}x$

$A = xy + \pi(\frac{1}{2}x)^2 = xy + \frac{\pi}{4}x^2$

$A(x) = x(3 - \frac{\pi}{2}x) + \frac{\pi}{4}x^2 = 3x - \frac{\pi}{2}x^2 + \frac{\pi}{4}x^2 = 3x - \frac{\pi}{4}x^2$, max A=?

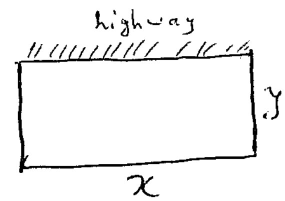
$A'(x) = 3 - 2(\frac{\pi}{4})x = 3 - \frac{\pi}{2}x \Rightarrow A'(x) = 0 \Rightarrow 3 - \frac{\pi}{2}x = 0 \Rightarrow x = \frac{3}{\frac{\pi}{2}} = \frac{6}{\pi}$ C.N.

$A''(x) = -\frac{\pi}{2} \Rightarrow A''(\frac{6}{\pi}) = -\frac{\pi}{2} < 0 \Rightarrow A$ has a max. at $x = \frac{6}{\pi}$

$y = 3 - \frac{\pi}{2}(\frac{6}{\pi}) = 3 - 3 = 0$

so for $x = \frac{6}{\pi}$ m and $y = 0$ m, the greatest amount of light is admitted

Q 2 let x and y be the dimensions of the field and let C be the total cost.



$C = 40x + 20(x + \cancel{y} + y)$

$C = 40x + 20x + 40y = 60x + 40y$

but $xy = 60{,}000$ so $y = \frac{60{,}000}{x}$

so $C(x) = 60x + \frac{2{,}400{,}000}{x}$, min C=?

$C'(x) = 60 - \frac{2{,}400{,}000}{x^2} = \frac{60x^2 - 2{,}400{,}000}{x^2}$

$C'(x) = 0 \Rightarrow 60x^2 - 2{,}400{,}000 = 0 \Rightarrow x^2 = \frac{2{,}400{,}000}{60} = 40000 \Rightarrow x = 200$ C.N.

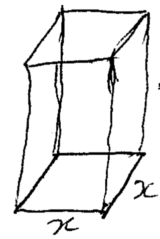
$C''(x) = 0 + \frac{2(2{,}400{,}000)}{x^3} \Rightarrow C''(200) > 0 \Rightarrow C$ has a min. at $x = 200$

but $y = \frac{60000}{200} = 300$. Thus if $x = 200$ m and $y = 300$ m, then the total cost will be minimum.

Q3

$P = 108$ cm

$108 = 4x + y \Rightarrow y = 108 - 4x$



$V = x^2 y$

$V(x) = x^2(108 - 4x) = 108x^2 - 4x^3$, $\max V = ?$

$V'(x) = 216x - 12x^2 = 12x(18 - x)$

$V'(x) = 0 \Rightarrow 12x = 0 \Rightarrow x = 0$ NA

$18 - x = 0 \Rightarrow x = 18$ cm

$V''(x) = 216 - 24x \Rightarrow V''(18) = 216 - 24(18) < 0$

so V has a max. at $x = 18$.

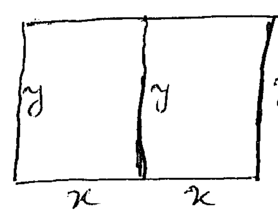
$y = 108 - 4(18) = 108 - 72 = 36$ cm

so if $x = 18$ cm and $y = 36$ cm, then the volume will be maximum.

Q 4

$P = 200$ total fence

$200 = 4x + 3y \Rightarrow y = \frac{200 - 4x}{3}$



$A = A_1 + A_2 = xy + xy = 2xy$

$A(x) = 2x\left(\frac{200 - 4x}{3}\right) = \frac{400}{3}x - \frac{8}{3}x^2$, $\max A = ?$

$A'(x) = \frac{400}{3} - \frac{16}{3}x \Rightarrow A'(x) = 0 \Rightarrow \frac{400}{3} - \frac{16}{3}x = 0 \Rightarrow x = \frac{\frac{400}{3}}{\frac{16}{3}} = 25$

$A''(x) = -\frac{16}{3} \Rightarrow A''(25) = -\frac{16}{3} < 0 \Rightarrow A$ has a max. at $x = 25$.

$y = \frac{200 - 4(25)}{3} = \frac{100}{3}$ m.

If $x = 25$ m and $y = \frac{100}{3}$ m, then the enclosed area will be maximum

Q5 Let x be the number dollars added to \$70 and let R be the total revenue. We know that $70+x$ is the new price of each book and $160-2x$ is the new number of books sold. Therefore

$R = (\text{price}) \times (\#\text{ of books})$

$R = (70+x)(160-2x) \Rightarrow R(x) = 1120 - 140x + 160x - 2x^2 \Rightarrow$

$R(x) = 1120 + 20x - 2x^2 \quad , \max R = ? \qquad x \geq 0$

$R'(x) = 0 + 20 - 4x \Rightarrow R'(x) = 0 \Rightarrow 20 - 4x = 0 \Rightarrow x = 5$ C.N.

$R''(x) = -4 \Rightarrow R''(5) = -4 < 0 \Rightarrow R$ has a max. at $x = 5$

If he put the price at $70+5=75$ dollars then his maximum revenue will be

$R = (75)(160-10) = (75)(150) = 1125$ dollars

Q6 (a) $\int f(x)\,dx = \int (4\sqrt{x} - \frac{1}{x^7} + \frac{3}{2x})\,dx$

$= 4(\frac{2}{3})x^{3/2} - (\frac{1}{-6})x^{-6} + \frac{3}{2}\ln|x| + C$

$= \frac{8}{3}x\sqrt{x} + \frac{1}{6x^6} + \frac{3}{2}\ln|x| + C$

(b) $\int g(x)\,dx = \int (\frac{3}{\sqrt{x}} + \frac{1}{e^{3x}} - 2)\,dx$

$= 3\int x^{-1/2}\,dx + \int e^{-3x}\,dx - 2\int 1\,dx$

$= 3(2)\sqrt{x} - \frac{1}{3}e^{-3x} - 2x + C$

Q6) $\rightarrow$ (c) $\int h(x)\,dx = \int \left(\frac{5}{2x+1} + \cos(\tfrac{x}{2}) - \sec^2 x\right) dx$

$$= 5\int \frac{1}{2x+1}\,dx + \int \cos(\tfrac{1}{2}x)\,dx - \int \sec^2 x\,dx$$

$$= \tfrac{5}{2}\ln|2x+1| + 2\sin(\tfrac{1}{2}x) - \tan x + C$$

(d) $\int k(x)\,dx = \int \left((x-1)^2 + 6e^{9x} - \frac{1}{\sin^2 x}\right) dx$

$$= \int (x-1)^2\,dx + 6\int e^{9x}\,dx - \int \frac{1}{\sin^2 x}\,dx$$

$$= \tfrac{1}{3}(x-1)^3 + \tfrac{6}{9}e^{9x} - \int \csc^2 x\,dx$$

$$= \tfrac{1}{3}(x-1)^3 + \tfrac{2}{3}e^{9x} - (-\cot x) + C$$

$$= \tfrac{1}{3}(x-1)^3 + \tfrac{2}{3}e^{9x} + \cot x + C$$

Q7 $x(t) = \int v(t)\,dt = \int (3t^2 + 4t - 5)\,dt = t^3 + 2t^2 - 5t + C$

since $x(1) = 2$ so $2 = 1^3 + 2(1)^2 - 5(1) + C \Rightarrow C = 4$

i.e $x(t) = t^3 + 2t^2 - 5t + C \Rightarrow x(5) = 5^3 + 2(5)^2 - 5(5) + 4 = 154$

Q8 $f'(x) = \int f''(x)\,dx = \int \left(12x + \frac{1}{x^2}\right) dx = 6x^2 - \frac{1}{x} + C_1$

$f(x) = \int f'(x)\,dx = \int \left(6x^2 - \frac{1}{x} + C_1\right) dx = 2x^3 - \ln|x| + C_1 x + C_2$

but $f'(1) = 2$ so $2 = 6(1)^2 - \frac{1}{1} + C_1 \Rightarrow C_1 = -3$

$f(1) = -2$ so $-2 = 2(1)^3 - \underbrace{\ln 1}_{0} - 3(1) + C_2 \Rightarrow C_2 = -1$

i.e $f(x) = 2x^3 - \ln|x| - 3x - 1$.