Q1 (a) f is continuous on $[-2, 0]$.

$f'(x) = 6x^5 - 6x = 6x(x^4-1)$

$f'(x) = 0 \Rightarrow 6x(x^4-1) = 0 \Rightarrow \underset{\text{C.N.}}{x=0}$, $x^4 - 1 = 0 \Rightarrow x^4 = 1 \Rightarrow \underset{\text{C.N.}}{x = \mp 1}$

but $x=1$ is not in $[-2, 0]$, so only $x=0$, $x=-1$ are acceptable.

$f(0) = 0$
$f(-1) = (-1)^6 - 3(-1)^2 = 1 - 3 = -2$
$f(-2) = (-2)^6 - 3(-2)^2 = 64 - 12 = 52$

so abs. max. of f is 52 at $x=-2$.
abs. min. of f is -2 at $x=-1$.

(b) $g(t) = t^5 - 5t^4 + 5t^3$ is continuous on $[-1, 2]$.

$g'(t) = 5t^4 - 20t^3 + 15t^2 = 5t^2(t^2 - 4t + 3)$

$g'(t) = 0 \Rightarrow 5t^2(t^2 - 4t + 3) = 0 \Rightarrow 5t^2 = 0 \Rightarrow t = 0$

C.N.'s

$t^2 - 4t + 3 = 0 \Rightarrow (t-1)(t+3) = 0 \Rightarrow t = 1, t = 3$

but $t=3$ is not in $[-1, 2]$, so only $t=0$, $t=1$ are acceptable.

$g(0) = 0$

$g(1) = 1 - 5 + 5 = 1$

$g(-1) = -1 - 5 - 5 = -11$

$g(2) = 2^5 - 5(2^4) + 5(2^3) = 32 - 80 + 40 = -8$

so abs. max. of f is 1 at $t=1$

abs. min. of f is -11 at $t=-1$

$\Rightarrow$ Q1

(c) $h(x) = 4\sqrt{x+1} - x$ is continuous on $[0, 8]$.

$$h'(x) = \frac{4}{2\sqrt{x+1}} - 1 = \frac{2-\sqrt{x+1}}{\sqrt{x+1}}$$

$h'(x) = 0 \Rightarrow 2 - \sqrt{x+1} = 0 \Rightarrow \sqrt{x+1} = 2 \Rightarrow x+1 = 4$

$\Rightarrow x = 3$ C.N.

$f(3) = 4\sqrt{3+1} - 3 = 4(2) - 3 = 5$

$f(0) = 4\sqrt{0+1} - 0 = 4$

$f(8) = 4\sqrt{8+1} - 8 = 4(3) - 8 = 4$

So abs. max is 5 at $x = 3$.

abs. min. is 4 at $x = 0$ and $x = 8$.

(d) $k(x) = x + \sqrt{1-x^2}$ is continuous on $[-1, 1]$.

$$k'(x) = 1 - \frac{2x}{2\sqrt{1-x^2}} = \frac{\sqrt{1-x^2} - x}{\sqrt{1-x^2}}$$

$k'(x) = 0 \Rightarrow \sqrt{1-x^2} - x = 0 \Rightarrow \sqrt{1-x^2} = x \Rightarrow 1 - x^2 = x^2$

$\Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$ C.N.

$$k\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} + \sqrt{1-\frac{1}{2}} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$k\left(-\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}} + \sqrt{1-\frac{1}{2}} = \frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = 0$$

$k(1) = 1 + \sqrt{1-1} = 1$

$k(-1) = -1 + \sqrt{1-1} = -1$

So abs. max. is $\sqrt{2}$ at $x = \frac{1}{\sqrt{2}}$.

abs. min. is -1 at $x = -1$.

Q2

(1) $y' = (\sec^2 x\, e^{\tan x}) \log_7 (x^3 + \cos x) + \left[\frac{1}{\ln 7}\left(\frac{3x^2 - \sin x}{x^3 + \cos x}\right)\right] e^{\tan x}$

(2) $y' = 2(\ln x)(\frac{1}{x}) + 2^{\ln x}(\frac{1}{x})(\ln 2) + 0 = \frac{2\ln x}{x} + \frac{(\ln 2) \cdot 2^{\ln x}}{x}$

(3) $y' = \dfrac{(e^{\sin(\ln x)} \cos(\ln x) \cdot \frac{1}{x})(1 + \sqrt{\ln x}) - [0 + \frac{1/x}{2\sqrt{\ln x}}](e^{\sin(\ln x)})}{[1 + \sqrt{\ln x}]^2}$

(4) $f'(x) = \dfrac{1}{2\sqrt{\dfrac{e^{x\ln(x^2+1)}}{1+x^2}}} \cdot \left[\dfrac{e^{x\ln(x^2+1)}}{1+x^2}\right]'$

$= \dfrac{1}{2\sqrt{\dfrac{e^{x\ln(x^2+1)}}{1+x^2}}} \cdot \left[\dfrac{\left[e^{x\ln(x^2+1)} \cdot (1 \ln(x^2+1) + (\frac{2x}{x^2+1})(x))\right](1+x^2) - (2x)\, e^{x\ln(x^2+1)}}{(1+x^2)^2}\right]$

(5) $g'(x) = 2\ln[\sin x \cos x - e^{x^2}] \cdot [(\cos x)(\cos x) - (\sin x)(\sin x) - 2x e^{x^2}]$

(6) $h(x) = x^2 \ln(2 + \sin x)$

$h'(x) = 2x \ln(2 + \sin x) + (\frac{0 + \cos x}{2 + \sin x})(x^2)$

(7) $\ln(k(x)) = \ln x \ln \frac{1}{x} \Rightarrow \frac{k'(x)}{k(x)} = \frac{1}{x} \ln \frac{1}{x} + \frac{-1/x^2}{1/x} \ln x \Rightarrow$

$k'(x) = [\frac{1}{x} \ln \frac{1}{x} - \frac{1}{x} \ln x](\frac{1}{x})^{\ln x}$

(8) $l'(x) = 10[\ln|4x| - \frac{1}{x} + x^{\ln \pi}]^9 [\frac{4}{4x} + \frac{1}{x^2} + (\ln \pi)\, x^{\ln \pi - 1}]$

Q3 (a) $\frac{1+3y^2y'}{x+y^3}+3x^2y+x^3y'=0-y'e^y$

at $(1,0)$: $\frac{1+0}{1+0}+3(0)+1y'=-y'(1) \Rightarrow 1+y'=-y' \Rightarrow y'=-\frac{1}{2}$

(b) $x^y=y^x \Rightarrow \ln x^y=\ln y^x \Rightarrow y\ln x = x\ln y \Rightarrow$

$y'\ln x+\frac{y}{x}=1\ln y+\frac{y'}{y}(x)$

at $(1,1)$: $y'\ln 1+\frac{1}{1}=\ln 1+\frac{y'}{1}(1) \Rightarrow 0+1=0+y' \Rightarrow y'=1$

(c) let $f(x)=x^{x^2}$ and $g(x)=x^{\sec(x-1)}$ so $y=f(x)+g(x)$

therefore $y'=f'(x)+g'(x)$. To find $f'(x)$ and $g'(x)$:

$f(x)=x^{x^2} \Rightarrow \ln(f(x))=x^2\ln x \Rightarrow \frac{f'(x)}{f(x)}=2x\ln x+\frac{x^2}{x}$

$\Rightarrow f'(x)=(2x\ln x+x)(x^{x^2})$

$g(x)=x^{\sec(x-1)} \Rightarrow \ln(g(x))=\sec(x-1)\ln x \Rightarrow$

$\frac{g'(x)}{g(x)}=\sec(x-1)\tan(x-1)\ln x+\sec(x-1)\cdot\frac{1}{x} \Rightarrow$

$g'(x)=\left[\sec(x-1)\tan(x-1)\ln x+\frac{\sec(x-1)}{x}\right](x^{\sec(x-1)})$

Hence $y'=(2x\ln x+x)x^{x^2}+\left[\sec(x-1)\tan(x-1)\ln x+\frac{\sec(x-1)}{x}\right]x^{\sec(x-1)}$

at $(1,2)$: $y'=(2(0)+1)1^1+\left[1(0)(0)+\frac{1}{1}\right]1^1=1+1=2$

Q 4) (1)

5

$f(x) = 5 - 8x^2 + x^4$

Domain $= (-\infty, \infty)$

$\lim\limits_{x \to \pm\infty} (5 - 8x^2 + x^4) = +\infty \Rightarrow$ no V. A.
no H. A.

x-intercept: If $y = 0$ then $x^4 - 8x^2 + 5 = 0 \Rightarrow x^2 = \dfrac{8 \pm \sqrt{64-20}}{2} = 4 \pm \sqrt{11}$

If $x^2 = 4 + \sqrt{11}$ then $x = \pm\sqrt{4+\sqrt{11}}$

If $x^2 = 4 - \sqrt{11}$ then $x = \pm\sqrt{4-\sqrt{11}}$

y-intercept: If $x = 0$ then $y = f(0) = 5$

$f'(x) = -16x + 4x^3 = 4x(x^2 - 4) \Rightarrow f'(x) = 0 \Rightarrow 4x(x^2-4) = 0 \Rightarrow x = 0,\ x = 2,\ x = -2$

$f(0) = 5$

$f(\pm 2) = 5 - 32 + 16 =$

x	$-\infty$	-2		0		2	$+\infty$
$f'(x)$	$-$		$+$		$-$		$+$
$f(x)$	$\searrow$	-11	$\nearrow$	$+5$	$\searrow$	-11	$\nearrow$
		loc. min		loc. max		loc. min	

$f''(x) = -16 + 12x^2 \Rightarrow f''(x) = 0 \Rightarrow x^2 = \dfrac{16}{12} = \dfrac{4}{3} \Rightarrow x = \pm\dfrac{2}{\sqrt{3}} \Rightarrow f(\pm 2/\sqrt{3}) = 5 - \frac{32}{3} + \frac{16}{9}$

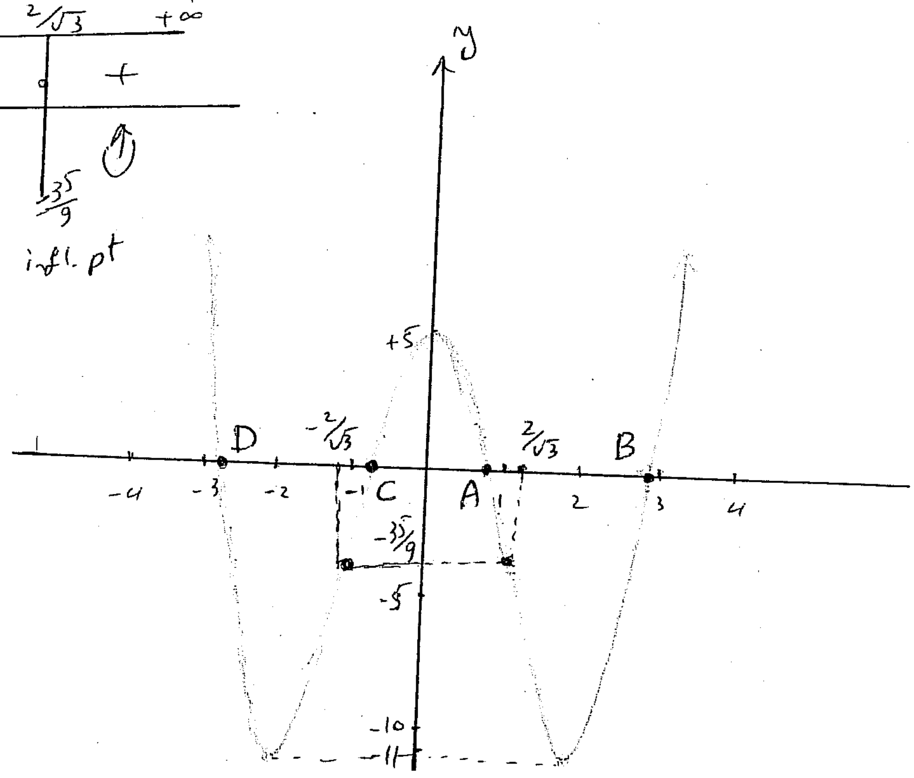
x	$-\infty$	$-2/\sqrt{3}$		$2/\sqrt{3}$	$+\infty$
$f''(x)$	$+$		$-$		$+$
$f(x)$	$\cup$	$-\frac{35}{9}$	$\cap$	$-\frac{35}{9}$	$\cup$
		infl. pt		infl. pt	

$A(\sqrt{4-\sqrt{11}}, 0)$

$B(\sqrt{4+\sqrt{11}}, 0)$

$C(-\sqrt{4-\sqrt{11}}, 0)$

$D(-\sqrt{4+\sqrt{11}}, 0)$



Q(4) (2) $g(x) = \dfrac{3x^2-6}{(x-1)^2}$

Domain $= (-\infty, 1) \cup (1, \infty)$

$\lim\limits_{x\to\pm\infty} \dfrac{3x^2-6}{(x-1)^2} = 3 \Rightarrow y = 3$ H.A , $\quad (x-1)^2 = 0 \Rightarrow x = 1$ V.A.

x-intercept: if $y=0 \Rightarrow \dfrac{3x^2-6}{(x-1)^2} = 0 \Rightarrow 3x^2-6=0 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

y-intercept: if $x=0 \Rightarrow y = \dfrac{0-6}{(0-1)^2} = \dfrac{-6}{1} = -6$

$$g'(x) = \frac{+6(x-1)^2 - 2(x-1)(3x^2-6)}{(x-1)^4} = \frac{6x(x-1) - 2(3x^2-6)}{(x-1)^3} = \frac{6x^2-6x-6x^2+12}{(x-1)^3} = \frac{6(2-x)}{(x-1)^3}$$

$g'(x)=0 \Rightarrow \dfrac{6(2-x)}{(x-1)^3} = 0 \Rightarrow 2-x=0 \Rightarrow x=2 \Rightarrow g(2) = \dfrac{3(4)-6}{(2-1)^2} = 6$

C.N.

x	$-\infty$	1		2		$+\infty$
$g'(x)$	$-$		$+$		$-$	
$g(x)$	↘		↗	6	↘	

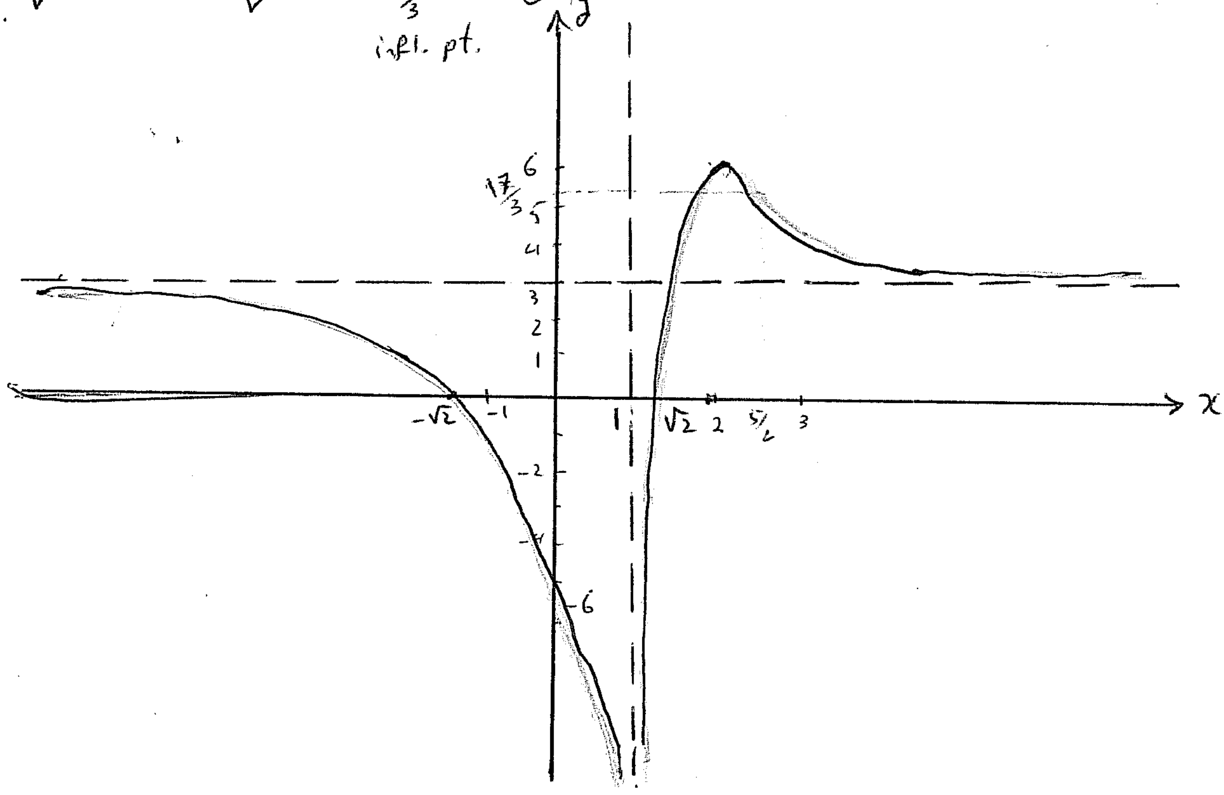
loc. max.

$$g''(x) = \frac{-6(x-1)^3 - 3(x-1)^2(12-6x)}{(x-1)^6} = \frac{-6(x-1)-3(12-6x)}{(x-1)^4} = \frac{-6x+6-36+18x}{(x-1)^4} = \frac{6(2x-5)}{(x-1)^4}$$

$g''(x) = 0 \Rightarrow \dfrac{6(2x-5)}{(x-1)^4} = 0 \Rightarrow 2x-5=0 \Rightarrow x = 5/2 \Rightarrow g(5/2) = \dfrac{3(\frac{25}{4})-6}{(5/2-1)^2} = \dfrac{51}{9} = \dfrac{17}{3}$

x	$-\infty$	1		$5/2$		$+\infty$
$g''(x)$	$-$		$-$		$+$	
$g(x)$	∩		∩	$17/3$	∪	

infl. pt.



Q4) (3) $h(x)=\frac{8(x-2)}{x^2}$

Domain $=(-\infty,0)\cup(0,\infty)$

$\lim\limits_{x\to\pm\infty}\frac{8(x-2)}{x^2}=0 \Rightarrow y=0$ H.A., $\quad x^2=0 \Rightarrow x=0$ V.A.

x-intercept: if $y=0 \Rightarrow \frac{8(x-2)}{x^2}=0 \Rightarrow x=2$

y-intercept: since $x=0$ is not in the domain, there is no y-intercept.

$h'(x)=\frac{8x^2-2x(8x-16)}{x^4}=\frac{8x-2(8x-16)}{x^3}=\frac{8(4-x)}{x^3}$

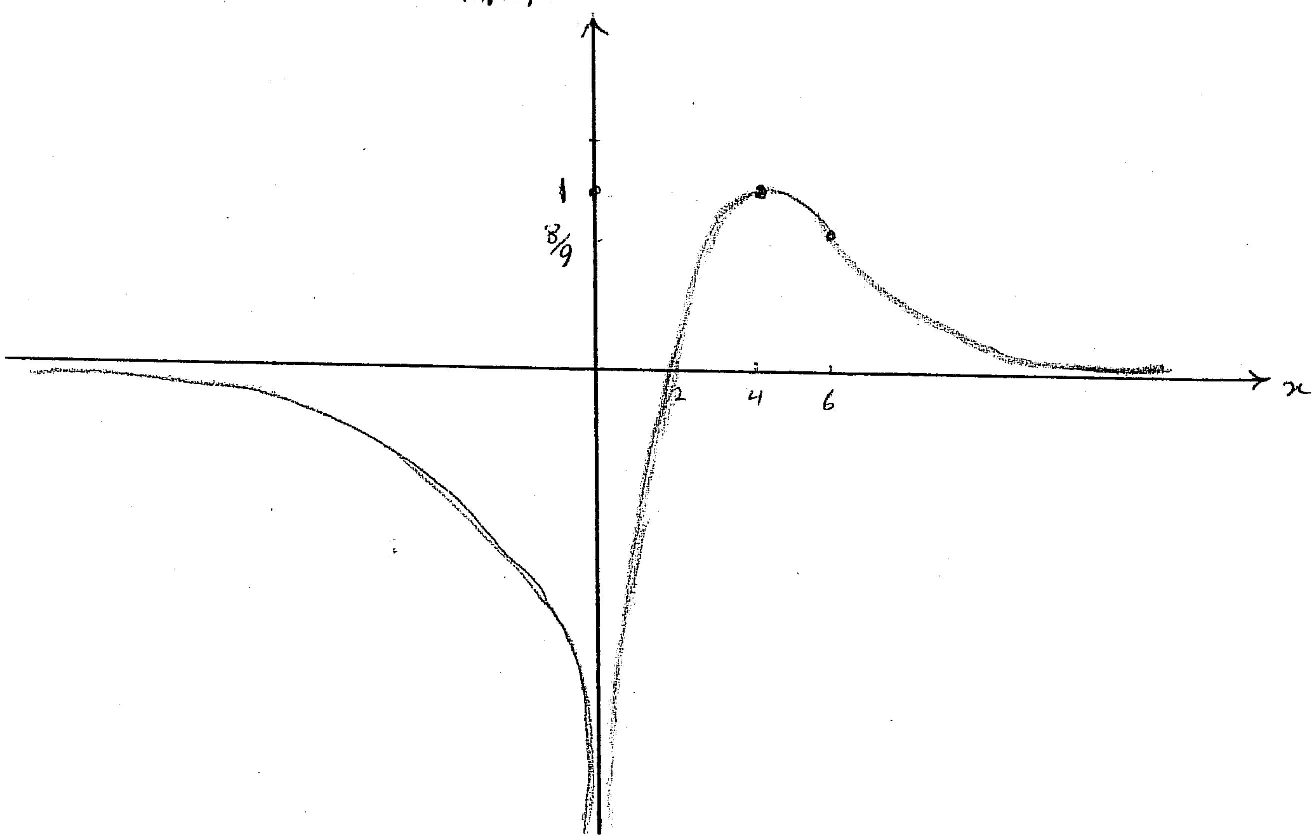
$h'(x)=0 \Rightarrow \frac{8(4-x)}{x^3}=0 \Rightarrow 4-x=0 \Rightarrow x=4$ C.N. $\Rightarrow h(4)=\frac{8(4-2)}{16}=1$

x	$-\infty$		0		4		$+\infty$
$h'(x)$		$-$		$+$		$-$	
$h(x)$		↘		↗	1 loc.max.	↘	

$h''(x)=\frac{-8x^3-3x^2(32-8x)}{x^6}=\frac{-8x-3(32-8x)}{x^4}=\frac{16(x-6)}{x^4}$

$h''(x)=0 \Rightarrow \frac{16(x-6)}{x^4}=0 \Rightarrow x-6=0 \Rightarrow x=6 \Rightarrow h(6)=\frac{8(6-2)}{36}=\frac{8}{9}$

x	$-\infty$		0		6		$+\infty$
$h''(x)$		$-$		$-$		$+$	
$h(x)$		↓		↓	$\frac{8}{9}$ infl. pt	↑	



Q4) (4) $k(x) = x e^{-x}$

Domain $= (-\infty, \infty)$

$$\lim_{x \to +\infty} x e^{-x} = \lim_{x \to +\infty} \frac{x}{e^x} = 0 \Rightarrow y = 0 \text{ H.A.}$$

There is no vertical asymptote.

x-intercept: if $y = 0 \Rightarrow x e^{-x} = 0$ but always $e^{-x} \neq 0$ so must $x = 0$

y-intercept: if $x = 0 \Rightarrow y = 0 e^{0} = 0(1) = 0$

$$k'(x) = 1 e^{-x} - x e^{-x} = e^{-x}(1 - x) \Rightarrow k'(x) = 0 \Rightarrow e^{-x}(1-x) = 0 \Rightarrow 1 - x = 0$$

$$\Rightarrow x = 1 \text{ C.N.} \Rightarrow k(1) = 1 e^{-1} = \frac{1}{e}$$

x	$-\infty$		1		$+\infty$
$k'(x)$		$+$	0	$-$	
$k(x)$		$\nearrow$	$1/e$	$\searrow$	

loc. max.

$$k''(x) = -e^{-x} - 1 e^{-x} + x e^{-x} = e^{-x}(x - 2) \Rightarrow k''(x) = 0 \Rightarrow e^{-x}(x-2) = 0 \Rightarrow x - 2 = 0$$

$$\Rightarrow x = 2 \Rightarrow k(2) = 2 e^{-2} = \frac{2}{e^2}$$

x	$-\infty$		2		$+\infty$
$k''(x)$		$-$	0	$+$	
$k(x)$		$\cap$	$2/e^2$	$\cup$	

infl. pt

Note 1: $\frac{1}{e} = \frac{e}{e^2} > \frac{2}{e^2}$

Note 2: $\lim_{x \to -\infty} x e^{-x} = -\infty$