(Q1) $(g\circ f)(x) = g(f(x)) = g\left(\frac{1}{x-2}\right) = \sqrt{\frac{2}{x-2}+1}$, $D(f) = (-\infty, 2)\cup(2,\infty)$

$D(g) = [-\frac{1}{2}, \infty)$

$D(g\circ f) = \{x \in (-\infty,2)\cup(2,\infty) \mid \frac{1}{x-2} \in [-\frac{1}{2}, \infty)\}$

$= \{ \quad " \quad " \quad \mid x \le 0 \text{ or } x > 2\}$

$= (-\infty, 0] \cup (2, \infty)$

- $(f\circ g)(x) = f(g(x)) = f(\sqrt{2x+1}) = \dfrac{1}{\sqrt{2x+1}-2}$

$D(f\circ g) = \{x \in [-\frac{1}{2}, \infty) \mid \sqrt{2x+1} \in (-\infty, 2)\cup(2,\infty)\}$

$= \{ \quad " \quad \mid x \ne \frac{3}{2}\}$

$= [-\frac{1}{2}, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$

- $(f\circ f)(x) = f(f(x)) = f\left(\frac{1}{x-2}\right) = \dfrac{1}{\frac{1}{x-2}-2} = \dfrac{x-2}{5-2x}$

$D(f\circ f) = \{x \in (-\infty, 2)\cup(2,\infty) \mid \frac{1}{x-2} \in (-\infty, 2)\cup(2,\infty)\}$

$= \{ \quad " \quad \mid x \ne \frac{5}{2}\}$

$= (-\infty, 2) \cup (2, \frac{5}{2}) \cup (\frac{5}{2}, \infty)$

- $(g\circ g)(x) = g(g(x)) = g(\sqrt{2x+1}) = \sqrt{2\sqrt{2x+1}+1}$

$D(g\circ g) = \{x \in [-\frac{1}{2}, \infty) \mid \sqrt{2x+1} \in [-\frac{1}{2}, \infty)\}$

$= [-\frac{1}{2}, \infty)$ (because $\sqrt{2x+1} \ge 0$)

Q(2) (i) $\lim_{x\to 4} f(x) = 1$, (ii) $\lim_{x\to 2^-} f(x) = -\infty$, (iii) $\lim_{x\to 2^+} f(x) = 3$

(iv) $\lim_{x\to 0} f(x) = +\infty$, (v) $\lim_{x\to (-3)^+} f(x) = 0$, (vi) $\lim_{x\to(-3)^-} f(x) = -2$

Q(3) (1) $\lim_{x\to 2} \frac{x^2+3x-10}{-3x-6} = \frac{4+6-10}{-6-6} = \frac{0}{-12} = 0$

(2) $\lim_{x\to -3} \frac{x^3-9x}{x^2+2x-3} = \lim_{x\to -3} \frac{x(x+3)(x-3)}{(x+3)(x-1)} = \lim_{x\to -3} \frac{x(x-3)}{(x-1)} = \frac{-3(-6)}{-4} = -\frac{9}{2}$

(3) $\lim_{x\to 0} \frac{x}{\sqrt{x+1}-1} = \lim_{x\to 0} \frac{x(\sqrt{x+1}+1)}{x+1-1} = \lim_{x\to 0} (\sqrt{x+1}+1) = \sqrt{0+1}+1 = 2$

(4) $\lim_{x\to 3} \left(\frac{1}{x-3} - \frac{6}{x^2-9}\right) = \lim_{x\to 3} \frac{x+3-6}{x^2-9} = \lim_{x\to 3} \frac{x-3}{(x-3)(x+3)} = \lim_{x\to 3} \frac{1}{x+3} = \frac{1}{6}$

(5) $\lim_{x\to 1} \frac{\sqrt{3x}-\sqrt{3}}{x-1} = \lim_{x\to 1} \frac{\sqrt{3}(\sqrt{x}-1)}{(\sqrt{x}-1)(\sqrt{x}+1)} = \lim_{x\to 1} \frac{\sqrt{3}}{\sqrt{x}+1} = \frac{\sqrt{3}}{\sqrt{1}+1} = \frac{\sqrt{3}}{2}$

(6) $\lim_{t\to 5^-} \frac{t-5}{|t^2-25|} = \lim_{t\to 5^-} \frac{t-5}{-(t^2-25)} = \lim_{t\to 5^-} \frac{t-5}{-(t-5)(t+5)} = \lim_{t\to 5^-} \frac{1}{-(t+5)} = -\frac{1}{\ldots}$

(7) $\lim_{h\to 0} \frac{\frac{1}{\sqrt{1+h}-1}}{h} = \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{\sqrt{1+h}-1}\right] = \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{\sqrt{1+h}-1} \cdot \frac{\sqrt{1+h}+1}{\sqrt{1+h}+1}\right] = \lim_{h\to 0} \frac{1}{h}\left[\frac{\sqrt{1+h}+1}{1+h-1}\right.$

$= \lim_{h\to 0} \frac{(\sqrt{1+h}+1)}{h^2} = +\infty$

(8) $\lim_{x\to 2^-} \frac{5}{\sqrt{2-x}}$ it does not enist and $\lim_{x\to 2^-} \frac{5}{\sqrt{2-x}} = +\infty$

(9) $\lim_{x\to 1} \frac{x^3\sqrt{x}-\sqrt{x}}{(x^2-1)(x^2+x+1)} = \lim_{x\to 1} \frac{\sqrt{x}(x^3-1)}{(x-1)(x+1)(x^2+x+1)} = \lim_{x\to 1} \frac{\sqrt{x}(x-1)(x^2+x+1)}{(x-1)(x+1)(x^2+x+1)}$

$= \lim_{x\to 1} \frac{\sqrt{x}}{x+1} = \frac{\sqrt{1}}{1+1} = \frac{1}{2}$

Q(4) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(3x+a) = 3+a$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} bx = b$ $\Rightarrow$ $3+a=b \Rightarrow \boxed{a-b=-3}$ ①

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} bx = 2b$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(ax+2a) = 2a+2a = 4a$ $\Rightarrow$ $4a=2b \Rightarrow \boxed{2a-b=0}$ ②

From ① and ② we get $\begin{cases} a-b=-3 \\ 2a-b=0 \end{cases} \Rightarrow a=3\ ,\ b=6$

Q(5) we know that $-1 \le \cos\frac{3}{x} \le 1$ so $-\sqrt[3]{x^4} \le \sqrt[3]{x^4}\cos\frac{3}{x} \le \sqrt[3]{x^4}$

but $\lim_{x\to 0}\sqrt[3]{x^4} = 0$ and $\lim_{x\to 0} -\sqrt[3]{x^4} = 0$ by the squeeze theorem must $\lim_{x\to 0}\sqrt[3]{x^4}\cos\frac{3}{x} = 0$.

Q(6) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(x^2+1) = 0^2+1 = 1$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\sqrt{x^2+1} = \sqrt{0^2+1} = 1$ $\Rightarrow \lim_{x\to 0^-} f(x) = 1 = \lim_{x\to 0^+} f(x)$

so yes $\lim_{x\to 0} f(x) = 1$

If $a=1$ then $\lim_{x\to 0} f(x) = 1 = f(0)$ i.e. f is continuous at $x=0$.

Q(7) must $\lim_{x\to a} f(x) = f(a) = 2$, but

$$\lim_{x\to a} f(x) = \lim_{x\to a} \frac{x^2-a^4}{x-a^2} = \lim_{x\to a} \frac{(x-a^2)(x+a^2)}{x-a^2}$$

$$= \lim_{x\to a} (x+a^2)$$

$$= a+a^2$$

So must $a+a^2=2 \Rightarrow a^2+a-2=0 \Rightarrow (a-1)(a+2)=0$

$\Rightarrow a=1, \quad a=-2.$

Q(8) since it is continuous everywhere, so it is continuous at $x=-1$ and $x=3$ as well. But, $f(-1)=2$ and

$$\lim_{x\to(-1)^-} f(x) = \lim_{x\to(-1)^-} (2) = 2$$

$$\lim_{x\to(-1)^+} f(x) = \lim_{x\to(-1)^+} (ax+b) = -a+b$$

so must $\boxed{-a+b=2}$ ①

Also $f(3)=-2$ and

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (ax+b) = 3a+b \Rightarrow \boxed{3a+b=-2} \text{ ②}$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (-2) = -2$$

from ① and ②: $\begin{cases} -a+b=2 \\ 3a+b=-2 \end{cases} \Rightarrow a=-1, \ b=1$

Q(9) The Intermediate theorem says:
(a)

If f is continuous on $[a,b]$ and N is a number between $f(a)$ and $f(b)$, then there exist a number "c" between "a" and "b" such that $f(c)=N$.

(b) $f(-1) = e^{-1} + (-1) - 2 = \frac{1}{e} - 3 < 0$

$f(1) = e^{1} + 1 - 2 = e - 1 > 0$

since $f(x) = e^{x} + x - 2$ is continuous on $[-1, 1]$ and 0 is between $f(-1)$ and $f(1)$ so there exist c between -1 and 1 such that $f(c)=0$. i.e c is a real root of f.

Q(10) It is false. (in fact if $\lim_{x\to 0} f(x)$ exists and $f(x) > 1$, then $\lim_{x\to 0} f(x) \geq 1$)

A counter example: let $f(x) = \begin{cases} x^2+1 & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$

then $f(x) > 1$ for all x. but

$\lim_{x\to 0} f(x) = \lim_{x\to 0} (x^2+1) = 0^2+1 = 1$