MATH 1010 Assignment 3 Winter 2008

1. Let $A = \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 4 \\ -2 & 0 & -1 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 1 \\ 3 & 4 \\ -1 & 5 \end{pmatrix}$, $D = \begin{pmatrix} 1 \\ -6 \\ -1 \end{pmatrix}$ and

   $E = \begin{pmatrix} -1 & 0 & 0 \\ 7 & 0 & -1 \\ 2 & -2 & 3 \end{pmatrix}$.

   Evaluate each of the following expressions or explain why it is not defined.

   $(a)\quad 3B^T - 2C$, $\qquad (b)\quad A^2 + BC$, $\qquad (c)\quad CB + 6ED$, $\qquad (d)\quad (B^T + C)A$

2. Let $A = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 4 \\ 2 & 1 \\ -5 & 0 \end{pmatrix}$, find the matrix X such that

   $AB - X = \begin{pmatrix} 5 & 2 \\ 1 & 0 \end{pmatrix}$.

3. The row reduced echelon form of the augmented matrix associated with a system of linear equations is

   given to be $\left(\begin{array}{ccccc|c} 1 & -4 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -2 & 6 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$.

   (a) What is the number of equations of this system?

   (b) What is the number of variables of this system?

   (c) What is the number of solutions of this system? Write all solutions.

4. Use augmented matrices and Gauss-Jordan elimination to find all solutions of the following systems of equations. Clearly indicate what elementary operations you are using.

   $$(a)\quad \begin{aligned} x + 3y \quad -z + u &= 1 \\ x + 4y \quad +2z - 2u &= 3 \\ 2x + 7y \quad +z - u &= 4 \end{aligned} \qquad (b)\quad \begin{aligned} -x_2 \quad -3x_3 \qquad &= -5 \\ 2x_1 \qquad +4x_3 + 2x_4 &= 20 \\ x_1 - x_2 \quad -x_3 + x_4 &= 5 \\ -x_1 - x_2 \quad -5x_3 - 2x_4 &= -21 \end{aligned}$$

   $$(c)\quad \begin{aligned} 5x_1 + 2x_2 \quad +x_3 - x_4 &= 3 \\ -2x_1 - x_2 \quad -2x_3 + x_4 &= -1 \\ -7x_1 - x_2 \quad +13x_3 - 4x_4 &= -2 \end{aligned}$$

5. Find the inverse of each of the following matrices or explain why it does not exist.

   $(a)\quad A = \begin{pmatrix} 3 & -5 \\ 11 & -3 \end{pmatrix}$ $\qquad (b)\quad B = \begin{pmatrix} 2 & 1 & 4 \\ 0 & -3 & 1 \\ 1 & 1 & 2 \end{pmatrix}$

   $(c)\quad C = \begin{pmatrix} 1 & -2 & 0 \\ -1 & 3 & -2 \\ -2 & -1 & -4 \end{pmatrix}$ $\qquad (d)\quad D = \begin{pmatrix} 2 & 3 & 6 \\ -1 & 2 & 4 \\ -5 & 3 & 6 \end{pmatrix}$