Solution of MATH 1010 Assignment 1 Winter 2008

1. (a)

$$\begin{aligned} -10 + 5x + 3 - 6x &= 12x + 1 \\ -7 - x &= 12x + 1 \\ -13x &= 8 \\ x &= -\frac{8}{13} \end{aligned}$$

(b)

$$\begin{aligned} -3x + 8x - 40 &= 6 - 16x \\ 5x - 40 &= 12x + 1 \\ 21x &= 46 \\ x &= \frac{46}{21} \end{aligned}$$

(c)

$$\begin{aligned} 6 - 21 + 7x + 4x &= 2x - 14 + 9x - 1 \\ -15 + 11x &= 11x - 15 \\ 0 &= 0 \end{aligned}$$

All real numbers satisfy this equation.

(d)

$$\begin{aligned} 4\Big(5 - \frac{2-3x}{2}\Big) &= 4\Big(1 - \frac{6x+1}{4}\Big) \\ 20 - 4 + 6x &= 4 - 6x - 1 \\ 12x &= -13 \\ x &= -\frac{13}{12} \end{aligned}$$

2. Find all solutions of each of the following inequalities:

(a)

$$\begin{aligned} 4x - 3 - 2 - 2x &\geq x + 12 \\ 2x - 5 &\geq x + 12 \\ x &\geq 17 \end{aligned}$$

(b)

$$\begin{aligned} 6\Big(\frac{1}{2}(2-x) - \frac{1}{3}(5+x)\Big) &< 6(4) \\ 3(2-x) - 2(5+x) &< 24 \\ 6 - 3x - 10 - 2x &< 24 \\ -5x &< 28 \\ x &> -\frac{28}{5} \end{aligned}$$

(c)

$$\begin{aligned}
6\left(\frac{3x}{2}+2x\right) &\le 6\left(\frac{8x}{-3}-1\right)\\
3(3x)+6(2x) &\le -2(8x)-6(1)\\
9x+12x &\le -16x-6\\
37x &\le -6\\
x &\le -\frac{6}{37}
\end{aligned}$$

3. For the line $5x-2y=7$, find each of the following and then draw the line.

   (a) Set $y=0$ then, $5x-2(0)=7$ so $x=\frac{7}{5}$.

   (b) Set $x=0$ then, $5(0)-2y=7$ so $y=-\frac{7}{2}$.

   (c) We write the equation in the form

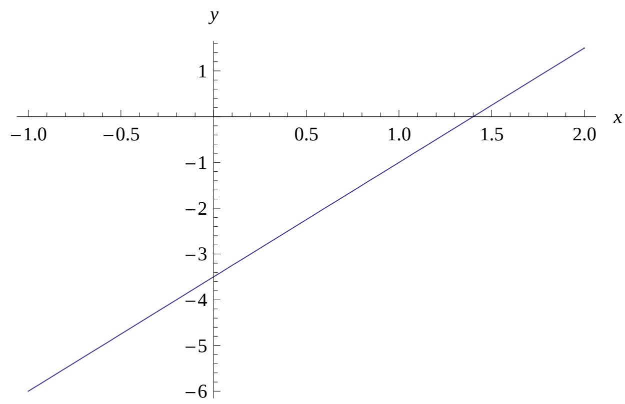
$$\begin{aligned}
-2y &= -5x+7\\
y &= \frac{5}{2}x-\frac{7}{2}
\end{aligned}$$

Therefore the slope of the line is $\frac{5}{2}$.

   (d) When we set $y=3$,

$$\begin{aligned}
5x-2(3) &= 7\\
5x &= 13\\
x &= \frac{13}{5}
\end{aligned}$$

Therefore the point is $(\frac{13}{5}, 3)$.



4. We write the equation $6y-5x=-1$in the form

$$\begin{aligned}
6y &= 5x-1\\
y &= \frac{5}{6}x-\frac{1}{6}
\end{aligned}$$

Therefore the slope of this line is $m_1 = \frac{5}{6}$. Let m_2 be the slope of the new line then must $m_1 m_2 = -1$ i.e. $\frac{5}{6} m_2 = -1$ which means $m_2 = -\frac{6}{5}$; so the equation of the new line is

$$\begin{aligned} y - 8 &= -\frac{6}{5}(x+1) \\ y &= -\frac{6}{5}x + \frac{34}{5} \\ [\text{or} \quad 6x + 5y &= 34\,] \end{aligned}$$

5. We write the equation $\dfrac{11-8x}{8} = -1 + y$ in the form

$$\begin{aligned} \frac{11}{8} - x &= -1 + y \\ y &= -x + \frac{19}{8} \end{aligned}$$

Therefore the slope of this line is $m_1 = -1$. Let m_2 be the slope of the new line then must $m_1 = m_2$ which means $m_2 = -1$; so the equation of the new line is

$$\begin{aligned} y - 0 &= -1(x - 0) \\ y &= -x \end{aligned}$$

6. When we write $-6x + 3y + 1 = 0$ in the form $y = 2x - \frac{1}{3}$, we see that its slope is 2. This is also the slope of the required line. i.e. $m = 2$. On the other hand in order to find the point of intersection of the two given lines, we multiply the first equation by -3, so the equations become:

$$\begin{aligned} 9x - 6y &= 24 \\ 5x + 6y &= 4 \end{aligned}$$

When we add these equations,

$$\begin{aligned} 14x &= 28 \\ x &= 2 \end{aligned}$$

Substitution of $x = 2$ into $-3x + 2y = -8$ gives

$$\begin{aligned} -3(2) + 2y &= -8 \\ y &= -1 \end{aligned}$$

Therefore the point of intersection is $(2, -1)$. Now the equation of the line with slope $m = 2$ through the point $(2, -1)$ is

$$\begin{aligned} y - (-1) &= 2(x - 2) \\ y &= 2x - 5 \\ [\text{or} \quad 2x - y &= 5\,] \end{aligned}$$

7. In order to find possible intersection point, we multiply the first equation by 4 and the second equation by3, so the equations become:

$$\begin{aligned} 28x - 12y &= -4 \\ 6x + 12y &= 21 \end{aligned}$$

When we add these equations,

$$\begin{aligned} 34x &= 17 \\ x &= \frac{17}{34} = \frac{1}{2} \end{aligned}$$

Substitution of $x = \frac{1}{2}$ into $2x + 4y = 7$ gives

$$2(\frac{1}{2}) + 4y = 7$$
$$y = \frac{3}{2}$$

Therefore the point of intersection is $(\frac{1}{2}, \frac{3}{2})$.

8. We need to find two points on each border lines:

$$\begin{aligned} x + 2y = 4 \quad &\Rightarrow \quad (4, 0),\ (0, 2) \\ x - y = 1 \quad &\Rightarrow \quad (1, 0),\ (0, -1) \\ 3x + 2y = 6 \quad &\Rightarrow \quad (2, 0),\ (0, 3) \end{aligned}$$

In order to find the corner point B where the two lines $x - y = 1$ and $3x + 2y = 6$ meet, we multiply $x - y = 1$ by 2 and add to $3x + 2y = 6$ to get $x = \frac{8}{5}$ and then substitution gives $y = \frac{3}{5}$. Hence the coordinates of B are $(\frac{8}{5}, \frac{3}{5})$.

In order to find the corner point C where the two lines $x + 2y = 4$ and $3x + 2y = 6$ meet, we multiply $x + 2y = 4$ by -1 and add to $3x + 2y = 6$ to get $x = 1$ and then substitution gives $y = \frac{3}{2}$. Hence the coordinates of C are $(1, \frac{3}{2})$.

Therefore $\{ (0, 0), (1, 0), (\frac{8}{5}, \frac{3}{5}), (1, \frac{3}{2}), (0, 2) \}$ are all corner points.