Question. *Suppose an art object purchased for \$60,000 is expected to appreciate in value at a constant rate of \$4000 per year. Assuming that the object's value is a linear function of the time (in years):*

1. *Find a linear equation for the problem*
2. *What would be the value of the object after 3 years?*
3. *After how many years will the value of the object be \$104,000?*

Solution.

(a) $V = 60000 + 4000t$ (in dollars) or $V = 60 + 4t$ (in thousands of dollars).

(b)

$$\begin{aligned} V &= 60 + 4(3) \\ &= 60 + 12 \\ &= 72 \end{aligned}$$

Therefore the art object is worth \$72,000 after 3 years.

(c)

$$\begin{aligned} 104 &= 60 + 4t \\ 104 - 60 &= 4t \\ 44 &= 4t \\ t &= 11 \end{aligned}$$

Therefore the art object is worth \$104,000 after 11 years.

Question. *Find the feasible set and all corner points of the region described by the following constraints and then find the maximum and minimum values of the objective function $z = x + 10y$:*

$$x + 4y \geq 12$$

$$x - 2y \leq 0$$

$$2y - x \leq 6$$

$$x \leq 6$$

Solution.

First, I will graph the lines. To do so, I will make tables of values:

$$x + 4y = 12$$

x	y
0	3
12	0

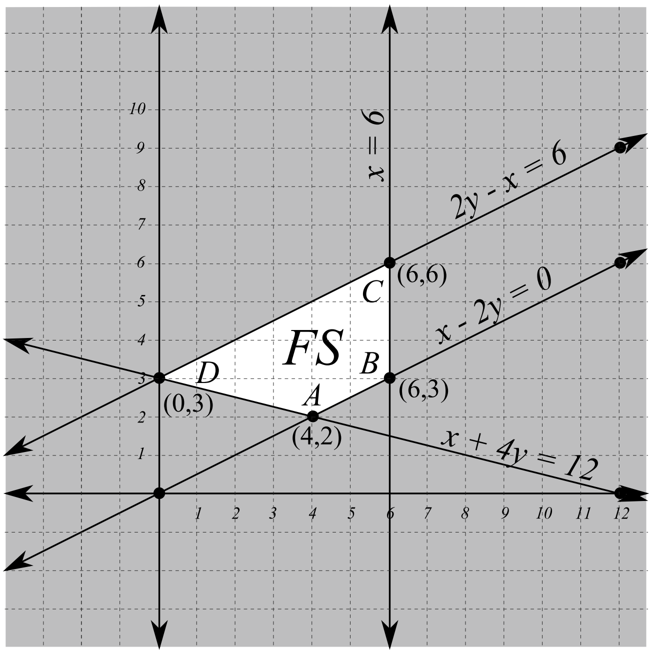
$$x - 2y = 0$$

x	y
0	0
12	6

$$2y - x = 6$$

x	y
0	3
12	9

Next, I graph and shade:



The points A, B, C, and D can all be found as the intersection of lines.

The objective function is then checked at each of the corner points to find the maximum and minimum values.

x	y	$z = x + 10y$	
0	3	30	
4	2	24	$\leftarrow$ Minimum
6	3	36	
6	6	66	$\leftarrow$ Maximum

Therefore the objective function is minimized at $x = 4$, $y = 2$, with a value of 24, and the objective function is maximized at $x = 6$, $y = 6$, with a value of 66.

Question. *Maximize the function* $P = 5x + 5y - 10$ *subject to the following constraints:*

$$x + y \leq 16$$

$$x + 3y \leq 36$$

$$0 \leq x \leq 10$$

$$y \geq 0$$

Solution.

First, turn each inequality into a line:

$$x + y = 16$$

$$x + 3y = 36$$

$$x = 0$$

$$x = 10$$

$$y = 0$$

Then graph each of the lines, using tables of values for the non-trivial ones:

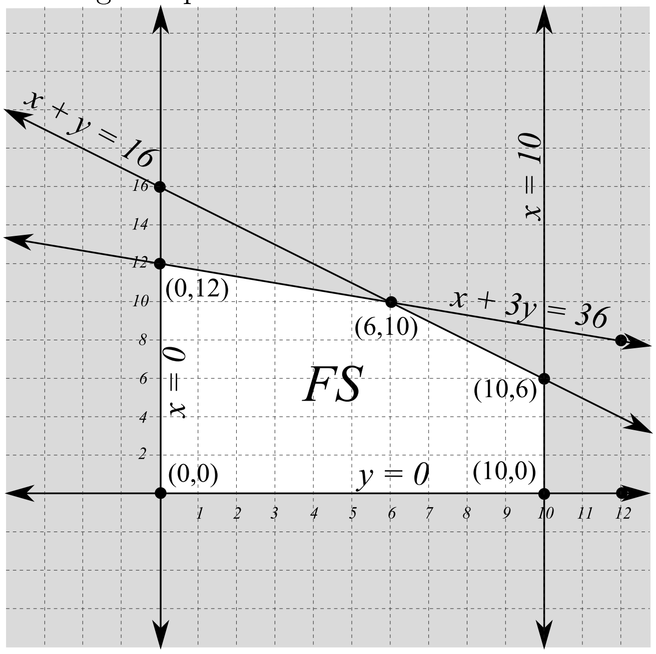
$$x + y = 16$$

x	y
0	16
10	6

$$x + 3y = 36$$

x	y
0	12
12	8

Now graph the lines, and shade correctly using test points:



The intersection points can all be found fairly quickly.

The objective function is then checked at each of the corner points to find the maximum value.

x	y	$P = 5x + 5y - 10$	
0	0	-10	
0	12	50	
6	10	70	$\leftarrow$ Maximum
10	6	70	$\leftarrow$ Maximum
10	0	40	

Therefore the objective function is maximized at every point on the line segment connecting $(6, 10)$ and $(10, 6)$. On every one of these points, the objective function has a value of 70.

Question. *Formulate the following as a linear programming problem. Label your variables clearly, write down all the constraints and the objective function.* ***Set up by do not solve.***

A 4-H club member raises geese and goats. She wants to raise no more than 16 animals, including no more than 10 geese. She spends \$15 to raise a good and \$12 to raise a goat, and she has \$540 available for the project. Find a maximum profit she can make if each goose produces a profit of \$10 and each goat a profit of \$15.

Solution.

Let x be the number of geese to raise, and let y be the number of goats to raise.

We seek to maximize the objective function:

$$P = 10x + 15y.$$

The constraints are:

$$x + y \leq 16$$

$$x \leq 10$$

$$15x + 12y \leq 540$$

$$x \geq 0$$

$$y \geq 0$$

Question. *Formulate the following as a linear programming problem. Label your variables clearly, write down all the constraints and the objective function. Then* ***solve the problem.***

A physical fitness enthusiast decides to devote her exercise time to a combination of jogging and cycling. She wants to earn "aerobic points" (a measure of the benefit of the exercise to strengthening the heart and lungs). She jogs at 6 kilometers per hour and cycles at 18 kilometers per hour. An hour of jogging earns 12 aerobic points, and an hour of cycling earns 9 aerobic points. Each week she would like to earn at least 36 aerobic points, cover at least 54 kilometers, and cycle at least as much as she jogs. How many hours of each activity should she do each week to meet the above requirements, but minimize the total time spent working out?

Solution.

Let x be the number of hours of jogging to do per week, and let y be the number of hours of cycling to do per week.

The objective function is the total number of hours working out, that is

$$H = x + y.$$

We seek to minimize this function.

The constraints are:

$$6x + 18y \geq 54,$$

$$12x + 9y \geq 36,$$

$$y \geq x,$$

$$x \geq 0,$$

$$y \geq 0.$$

Turning each inequality into a line, we get the set of lines:

$$6x + 18y = 54,$$

$$12x + 9y = 36,$$

$$y = x,$$

$$x = 0,$$

$$y = 0.$$

Next we graph these lines. For the non-trivial ones, use a table of values:

$$6x + 18y = 54$$

x	y
0	3
9	0

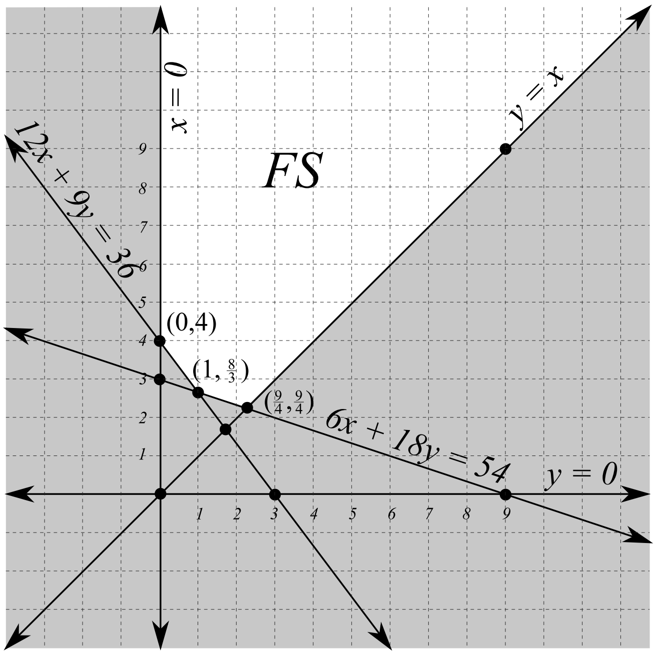
$$12x + 9y = 36$$

x	y
0	4
3	0

$$y = x$$

x	y
0	0
9	9

Now graph, and shade correctly by checking test points:



The intersection points can all be found fairly quickly.

The objective function is then checked at each of the corner points to find the minimum value.

x	y	$H = x + y$	
0	4	4	
1	$\frac{8}{3}$	$\frac{11}{3} = 3.666...$	$\leftarrow$ Minimum
$\frac{9}{4}$	$\frac{9}{4}$	$\frac{18}{4} = 4.5$	

Therefore the objective function is minimized at $x = 1$, $y = \frac{8}{3}$, at which point the value of the objective function is $\frac{11}{3}$. What this means to the word problem is that the person should go jogging for 1 hour per week, and cycle for $\frac{8}{3}$ hours per week (that is, $\frac{8}{3}60 = 160$ minutes $= 2$ hours and 40 minutes). With these times, she will achieve her goals, and work out the least amount necessary–a total of $\frac{11}{3}$ hours per week (that is, $\frac{11}{3}60 = 220$ minutes $= 3$ hours and 40 minutes).