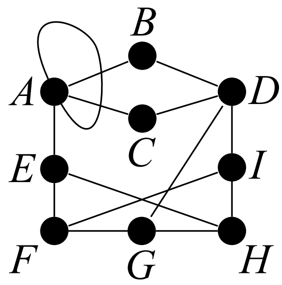
MATH 1010 Assignment 4 Winter 2008

Fill in the following table for the graph to the right:



1.

vertex	A	B	C	D	E	F	G	H	I
degree									

2. Which of the following are degree sets for simple graphs? For each set that is a degree set for a simple graph, draw such a graph, and for each set that is not a degree set for a simple graph, explain why not.

   (a) $\{2, 2, 2, 3, 4, 6\}$

   (b) $\{2, 3, 3, 4\}$

   (c) $\{2, 2, 3, 3\}$

   (d) $\{0, 1, 2, 2, 2, 5\}$

   (e) $\{2, 2, 2, 4, 4\}$

   (f) $\{1, 2, 3, 3, 4, 5\}$

3. Give an example of a simple graph

   (a) having no vertices of odd degree.

   (b) having all vertices of odd degree.

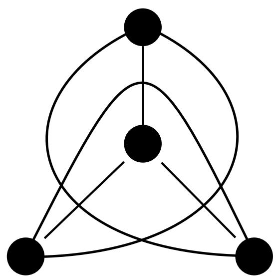
   (c) that is connected, has exactly 6 vertices, and exactly 5 edges.

   (d) that is connected, but has the property that removing any edge will disconnect the graph.

4. Is it possible for a graph to have 11 edges and each vertex have degree 3? If yes, how many vertices does it have? If not, explain why not.

5. A graph has 24 edges, 4 vertices of degree 5, and all other vertices of degree 2. How many nodes does it have in total?

6. Determine if the following graph is planar. If it is, redraw it with no edges crossing.



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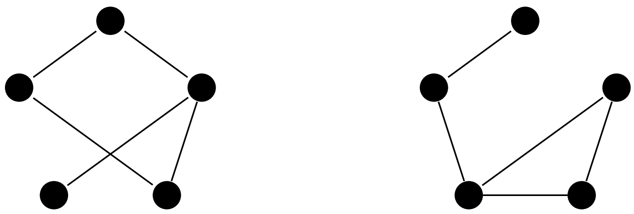
7. For each of the following pairs of graphs, explain why they are **not** equivalent.

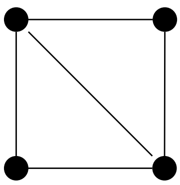
(a)

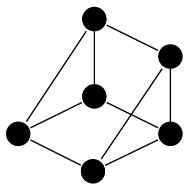
(b)

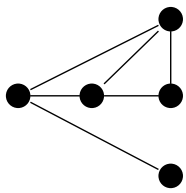
(c)

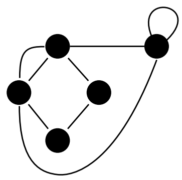


8. For each of the following graphs, fill in the below table (check mark if yes, leave blank if no)


(a)


(b)


(c)


(d)

graph	simple	planar	connected	has an Euler circuit	has an Euler path	has a Hamilton circuit
(a)						
(b)						
(c)						
(d)						

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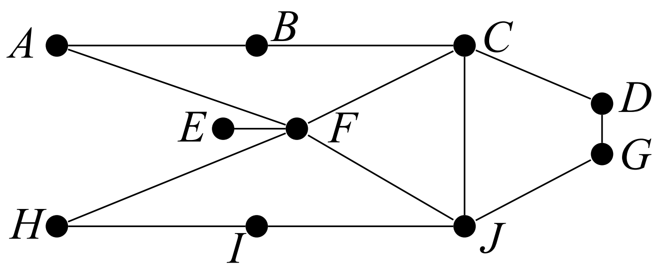
9. Consider the following graph:

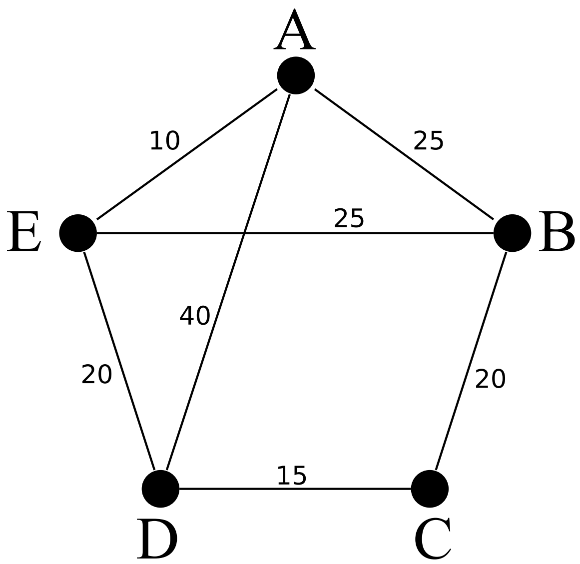


(a) Does this graph have an Euler circuit? If so, then find one. If not, then explain why not.

(b) Does this graph have an Euler path? If so, then find one. If not, then explain why not.

(c) If the vertex E is removed from the graph (together with the edge attached to it), does the graph then have an Euler circuit? If so, then find one. If not, explain why not.

10. The following graph represents available flights between cities A, B, C, D, and E (there is an edge from one city to another if one can travel either way between them). Each edge is assigned a value measuring how expensive the trip is.



(a) Find all hamilton circuits in this graph.

(b) Which hamilton circuits have the least total cost?

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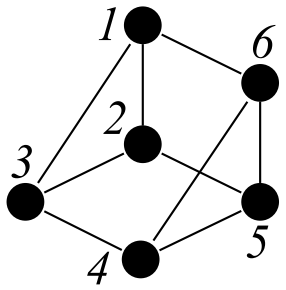
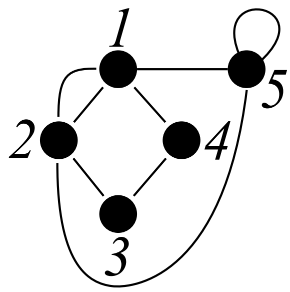
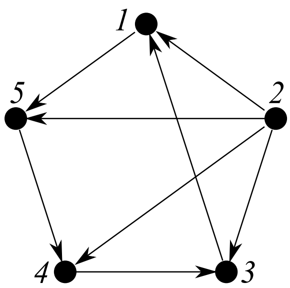
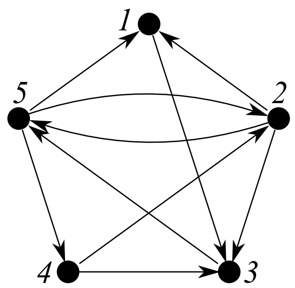
11. Find the adjacency matrices for each of the following graphs:

(a) 

(b) 

12. Find the adjacency matrices for each of the following digraphs:

(a) 

(b) 

13. For each of the following matrices, draw a graph that has the matrix as an adjacency matrix.

(a) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

14. For each of the following matrices, draw a digraph that has the matrix as an adjacency matrix.

(a) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

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15. Let

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}, \qquad A^2 = \begin{bmatrix} 3 & 2 & 1 & 2 & 1 \\ 2 & 4 & 2 & 1 & 1 \\ 1 & 2 & 3 & 1 & 2 \\ 2 & 1 & 1 & 2 & 1 \\ 1 & 1 & 2 & 1 & 2 \end{bmatrix}.$$

The matrix A is the adjacency matrix for a graph with nodes labelled 1, 2, 3, 4, and 5.

(a) How many routes are there from node 5 to node 2 using **exactly** 2 edges?

(b) How many routes are there from node 5 to node 2 using **exactly** 3 edges? (Hint: you do not have to find all of A^3).

(c) How many routes are there from node 5 to node 2 using **at most** 3 edges?