Question. *Trends, a telephone survey company, has been hired to conduct a television viewing poll among urban and suburban families. The client has stiputlated that a maximum of 1500 families is to be interviewed. At least 500 urban famiies must be interviewed and at least half of the total number of families interviewed must be from the suburban area. For this service, Trends will be paid \$3000 plus \$4.00 for each completed interview. From previous experience, Trends has determined that it will incur an expense of \$2.20 each successful interview with an urban family and \$2.50 for each succesful interview with a suburban family. How many urban and suburban families should Trends interview in order to maiximize profits? What is the maximum profit?*

Solution. We are interested in the number of each type of family the company should interview, so these are our variables: let x be the number of urban families to interview, and let y be the number of suburban families to interview.

Objective Function

(1) Income: the company received \$3000 plus \$4 for each completed interview.

(2) Expenses: the company pays \$2.20 for each urban family interview, and \$2.50 for each suburban family interview.

(3) Profit = Income - Expenses. Therefore

$$P = 3000 + 4(x+y) - 2.2x - 2.5y.$$

We seek to maximize profit.

Constraints

(1) Max 1500 families interviewed $\Longrightarrow$

$$x + y \leq 1500.$$

(2) At least 500 urban families interviewed $\Longrightarrow$

$$x \geq 500.$$

(3) At least half of the total interviewed must be from the suburban area $\Longrightarrow$

$$y \geq \frac{x+y}{2}.$$

(4) Further, we have the trivial bounds

$$x \geq 0 \qquad\qquad y \geq 0.$$

Therefore, the linear programming problem becomes to maximize the function

$$P = 3000 + 4(x+y) - 2.2x - 2.5y$$

subject to the constraints:

$$x + y \leq 1500$$

$$x \geq 500$$

$$y \geq \frac{x+y}{2}$$

$$x \geq 0 \qquad\qquad y \geq 0.$$

This simplify as follows:

$$\begin{aligned} P &= 3000 + 4(x+y) - 2.2x - 2.5y \\ &= 3000 + 4x + 4y - 2.2x - 2.5y \\ &= 3000 + 1.8x + 1.5y \\ &= 3000 + \frac{18}{10}x + \frac{15}{10}y \\ &= 3000 + \frac{9}{5}x + \frac{3}{2}y. \end{aligned}$$

$$\begin{aligned} y &\geq \frac{x+y}{2} \\ y &\geq \frac{1}{2}x + \frac{1}{2}y \\ y - \frac{1}{2}y &\geq \frac{1}{2}x \\ \frac{1}{2}y &\geq \frac{1}{2}x \\ y &\geq x. \end{aligned}$$

So in simplified form, the problem says to maximize the function

$$P = 3000 + \frac{9}{5}x + \frac{3}{2}y$$

subject to the constraints:

$$x + y \leq 1500$$

$$x \geq 500$$

$$y \geq x$$

$$x \geq 0$$

$$y \geq 0.$$

For a first step, turn each inequality into an equality (a line):

$$x + y = 1500$$

$$x = 500$$

$$y = x$$

$$x = 0$$
$$y = 0.$$

Now graph each line. $x = 0$ and $y = 0$ are just the y and x-axis respectively. $x = 500$ is just a vertical line at the x value 500. For the others, use a table of values:

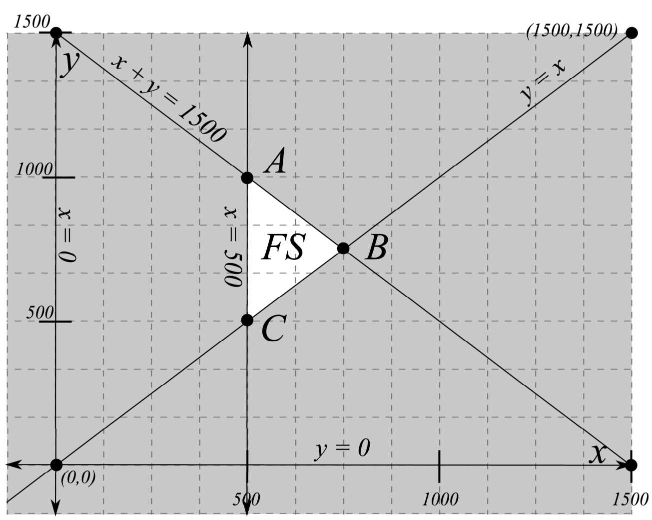
$$x + y = 1500$$

x	y
0	1500
1500	0

$$y = x$$

x	y
0	0
1500	1500

Plotting the lines, and shading correctly, I get the figure below:



The points A, B, and C are the corner points of the feasible set. Points A and C are on the line $x = 500$, and so have x coordinate equal to 500. A is on the line $x + y = 1500$, and so it must be that $y = 1000$, that is, A is the point $(500, 1000)$. C is also on the line $y = x$, therefore C is the point $(500, 500)$.

The point B is the intersection of $x+y = 1500$ and $y = x$. Its a quick process to solve and get that B is the point $(750, 750)$.

Now all that remains is to find the maximum profit:

x	y	$P = 3000 + \frac{9}{5}x + \frac{3}{2}y$
500	1000	$3000 + \frac{9}{5}(500) + \frac{3}{2}(1000) = 5400$
750	750	$3000 + \frac{9}{5}(750) + \frac{3}{2}(750) = 5475$
500	500	$3000 + \frac{9}{5}(500) + \frac{3}{2}(500) = 4650$

Therefore the max profit is achieved at the point $B = (750, 750)$, which is interpreted as interviewing 750 urban families and 750 suburban families, the profit at which point is \$5,475. This completes the question.