Question. *The Prairie Grains company sells two animal feeds, Suregro and Fulltime, both made from mixtures of wheat, oats, and barley. They sell for \$200 and \$250 per tonne, respectively. Suregro feed contains 4 parts wheat, 1 part oats, and 1 part barley. Fulltime feed contains 1 part wheat, 1 part oats, and 2 parts barley. The company currently has 8 tonnes of wheat, 3.5 tonnes of oats, and 6 tonnes of barley. Assuming that the company can sell all the feed it produces, how much of each type of feed should it produce in order to maximize income from its available stocks of grain?*

Solution.

First, one needs to choose our variables. The final answer should be to say "produce this much Suregro", and "produce this much Fulltime".

Variables

So, let x be the number of tonnes of Suregro to produce, and let y be the number of tonnes of Fulltime to produce.

Objective Function

We wish to maximize income. Income, call it I, is given by the formula:

$$I = 200x + 250y.$$

Constraints

Currently available are 8 tonnes of wheat. Each tonne of Suregro produced uses 4 parts wheat, 1 part oats, and 1 part barley. Therefore, $\frac{4}{6}$ of each tonne of Suregro is wheat. That is, each tonne of Suregro uses $\frac{4}{6}$ of a tonne of wheat.

Similarly, each tonne of Fulltime produced uses 1 part wheat, 1 part oats, and 2 parts barley. Therefore, $\frac{1}{4}$ of each tonne of Fulltime produced is wheat. That is, each tonne of Fulltime uses $\frac{1}{4}$ of a tonne of wheat.

Therefore, for wheat, the following constraint exists:

$$\frac{4}{6}x + \frac{1}{4}y \leq 8$$

Similarly, we have the following constraints for oats and barley respectively:

$$\frac{1}{6}x + \frac{1}{4}y \leq 3.5,$$
$$\frac{1}{6}x + \frac{2}{4}y \leq 6.$$

Also, we have the trivial constraints:

$$x \geq 0$$
$$y \geq 0$$

Therefore the linear programming problem can be rewritten as: Maximize

$$I = 200x + 250y$$

subject to the constraints

$$\frac{4}{6}x + \frac{1}{4}y \leq 8,$$
$$\frac{1}{6}x + \frac{1}{4}y \leq 3.5,$$
$$\frac{1}{6}x + \frac{2}{4}y \leq 6,$$
$$x \geq 0,$$
$$y \geq 0$$

Simplifying, this problem becomes:

Maximize

$$I = 200x + 250y$$

subject to the constraints

$$\frac{2}{3}x + \frac{1}{4}y \leq 8,$$
$$\frac{1}{6}x + \frac{1}{4}y \leq 3.5,$$
$$\frac{1}{6}x + \frac{1}{2}y \leq 6,$$
$$x \geq 0,$$
$$y \geq 0$$

In order to makes things easier, I will kill all the fractions in each inequality, turning the problem into the following problem:

Maximize

$$I = 200x + 250y$$

subject to the constraints

$$8x + 3y \leq 96,$$
$$2x + 3y \leq 42,$$
$$x + 3y \leq 36,$$
$$x \geq 0,$$
$$y \geq 0$$

For each of the first three lines I will use a table of values to find points in order to graph the lines:

$$8x + 3y \leq 96$$

x	y
0	32
12	0

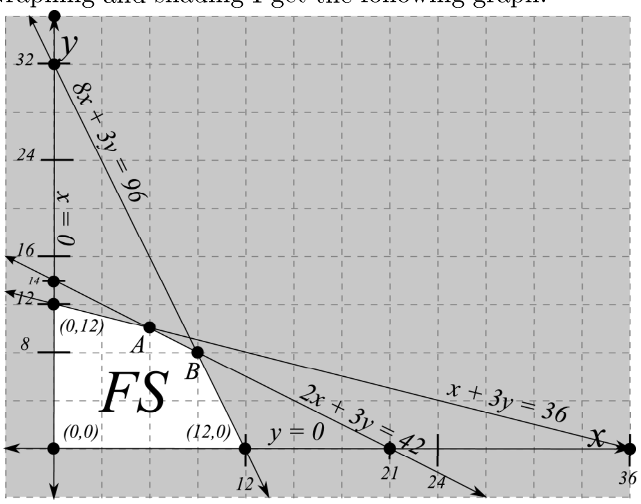
$$2x + 3y \leq 42$$

x	y
0	14
21	0

$$x + 3y \leq 36$$

x	y
0	12
36	0

Graphing and shading I get the following graph:



The point A is the intersection of the lines:

$$x + 3y = 36$$

$$2x + 3y = 42$$

This system can be solved by elimination. Subtracting straight down gives:

$$-x = -6$$

and therefore $x = 6$. Plugging this back into the first equation, I get:

$$\begin{aligned} x + 3y &= 36 \\ (6) + 3y &= 36 \\ 3y &= 36 - 6 \\ 3y &= 30 \\ y &= 10 \end{aligned}$$

Therefore A is the point $(6, 10)$.

The point B is the intersection of the lines:

$$8x + 3y = 96$$

$$2x + 3y = 42$$

Again, these can be solved by elimination, subtracting straight down, to get:

$$6x = 54,$$

and thus $x = 9$. Plugging $x = 9$ into the second equation, I get:

$$\begin{aligned} 2x + 3y &= 42 \\ 2(9) + 3y &= 42 \\ 18 + 3y &= 42 \\ 3y &= 24 \\ y &= 8 \end{aligned}$$

Therefore B is the point $(9, 8)$.

Now all that remains is to find the maximum income:

x	y	$I = 200x + 250y$
0	0	$I = 200(0) + 250(0) = \$0$
0	12	$I = 200(0) + 250(12) = \$3,000$
6	10	$I = 200(6) + 250(10) = \$3,700$
9	8	$I = 200(9) + 250(8) = \$3,800$
12	0	$I = 200(12) + 250(0) = \$2,400$

Therefore the max income is achieved at the point $B = (9, 8)$, which translates to producing 9 tonnes of Sure-gro and 8 tonnes of Fulltime, in which case the income is \$3,800. This completes the question.