MATH 1210 Assignment 1 Fall 2017 **Due date: October 4**

Attempt all questions and show all your work. Attach to Honesty Declaration Form.

1. **Use mathematical induction on integer n to prove each of the following:**
   - **(a)** $1(2) + 2(2^2) + 3(2^3) + \cdots + (n-3)(2^{n-3}) = 2[1 + (n-4)2^{n-3}]$ **for** $n \geq 4$ **;**
   - **(b)** $9^n + (49)^n - 2$ **is divisible by** 8 **for** $n \geq 1$ **;**
   - **(c)** $n^3 > (n+1)^2$**, for** $n \geq 3$ **;**
   - **(d)** $n! > n^3$**, for** $n \geq 6$ **.**

2. **Identities** $\sum_{k=1}^{m} k = \frac{1}{2}[m(m+1)]$ **and** $\sum_{k=1}^{m} k^2 = \frac{1}{6}[m(m+1)(2m+1)]$ **are given.**
   - **(a) First write the sum** $1^2 + 3^2 + 5^2 + \cdots + (4n+1)^2$ **in sigma notation and then use the identities to prove that**

     $$1^2 + 3^2 + 5^2 + \cdots + (4n+1)^2 = \frac{1}{3}(2n+1)(16n^2 + 16n + 3)\,.$$

   - **(b) Use the identities to evaluate the sum** $\sum_{\ell=19}^{29} [\,\frac{1}{20}(\ell - 18)^2 - \frac{1}{5}\,]$ **.**

3. **Express each of the following in simplified Cartesian form.**
   - **(a)** $6\Big(\frac{1}{\sqrt{6}} + \frac{\sqrt{5}\,i}{\sqrt{6}}\Big)^{10}\Big(\frac{1}{\sqrt{6}} - \frac{\sqrt{5}\,i}{\sqrt{6}}\Big)^{8}$ **;**
   - **(b)** $\dfrac{i^{82}(\sqrt{2}+i)^8}{6 + 3\sqrt{5}\,i}$ **;**
   - **(c)** $\Big(\dfrac{-\sqrt{3}+3i}{\overline{3i - \sqrt{3}}}\Big)^{20}$ **.**

4. **Find a formula for the sigma** $\sum_{k=1}^{3n} \frac{2}{(2k+1)(2k+3)}$**.**

   Hint: First write the general term as a subtraction of two terms.

5. **Find all solutions of each of the following equations. Express your answers in polar form.**
   - **(a)** $x^6 + 6x^4 + 5x^2 = 0$ **;**
   - **(b)** $x^5 + 4x^3 - x^2 - 4 = 0$ **.**