MATH 1210 Assignment 1 Winter 2019 **Due date: February 4**

Attempt **all** questions and show all your work. Selected number of questions will be marked.

Attach to Honesty Declaration Form

1. Use mathematical induction on integer n to prove each of the following:

   (a) $\dfrac{1}{1\cdot 2\cdot 3}+\dfrac{1}{2\cdot 3\cdot 4}+\dfrac{1}{3\cdot 4\cdot 5}+\cdots+\dfrac{1}{n(n^2-1)}=\dfrac{(n-1)(n+2)}{4n(n+1)}$ for $n\geq 2$;

   (b) $(10)^n+(10)^{2n}-2$ is divisible by 9 for $n\geq 1$.

2. Identities $\displaystyle\sum_{k=1}^{m} k=\frac{1}{2}\,[m(m+1)]$ and $\displaystyle\sum_{k=1}^{m} k^2=\frac{1}{6}\,[m(m+1)(2m+1)]$ are given.

   (a) First write the sum $2^2+5^2+8^2+11^2+\cdots+(6n+2)^2$ in sigma notation and then use the identities to prove that

   $$2^2+5^2+8^2+11^2+\cdots+(6n+2)^2=(2n+1)(12n^2+15n+4)\,.$$

   (b) Use the identities to evaluate the sum $\displaystyle\sum_{j=14}^{25}[\,(5j-65)^2-6\,]$.

3. Express each of the following in simplified Cartesian form.

   (a) $\dfrac{(\sqrt{2}-\sqrt{6}\,i)^{30}}{(\sqrt{2}+\sqrt{6}\,i)^{28}}$;

   (b) $\dfrac{1}{2^{1010}}\left(i^{13}(-1+i)^{2020}+(\sqrt{3}+i)^{1010}\right)$.

4. Find the complex number z, in Cartesian form, such that it satisfies the equation

   $$(3-\sqrt{3}\,i)^5+(\overline{\sqrt{3}-3i}\,)^5 z=288\sqrt{3}\,(-\frac{\sqrt{3}}{2}+i)\,.$$

5. Find all solutions of the equation

   $$x^4+\frac{1}{16}=0\,.$$

   Express any complex solutions in Cartesian form, simplified as much as possible.