MATH 1210 Assignment 1 Winter 2020 **Due date: February 3**

Attempt all questions and show all your work. Attach to Honesty Declaration Form.

1. **Use mathematical induction on integer n to prove each of the following:**

   (a) $1(4) + 2(5) + 3(6) + \cdots + n(n+3) \;=\; \frac{1}{3}(n)(n+1)(n+5)$ **for** $n \geq 1$ **;**

   (b) $3^{n+1}(n+2)! \geq 2^n(n+3)!$ **for** $n \geq 0$ **;**

   (c) $\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\left(1 - \frac{1}{5^2}\right) \cdots \left(1 - \frac{1}{(n+1)^2}\right) \;=\; \frac{2(n+2)}{3(n+1)}$ **for** $n \geq 2$ **;**

   (d) $2^{3n+2} + 3^{6n+1}$ **is divisible by** 7 **for** $n \geq 1$ **.**

2. **Simplify as much as possible using properties of sigma notation.**

$$\sum_{n=0}^{1000}(n+1)^6 \;-\; \sum_{n=2}^{1000} 2(n+1)^6 \;+\; \sum_{n=1}^{999}[(n+2)^6 + 1]\,.$$

3. **Identities** $\sum_{k=1}^{m} k = \frac{1}{2}\,[m(m+1)]$ **,** $\sum_{k=1}^{m} k^2 = \frac{1}{6}\,[m(m+1)(2m+1)]$ **and** $\sum_{k=1}^{m} k^3 = \frac{1}{4}\,[m^2(m+1)^2]$ **are given. Use the identities to evaluate the sum** $\sum_{\ell=100}^{110}\left[\,(\ell-100)^3 \;+\; (\ell-99)^2 \;-\; 4\,\right]$**.**

4. **Find all solutions of the following equation. Express your answers in polar form.**

$$(x^4 + 6x^2 + 9)(x^4 + x^3 + 5x^2 + 4x + 4) = 0\,.$$

   Hint: In the right bracket consider $5x^2$ **as** $x^2 + 4x^2$ **and then solve it by factoring.**

5. **Express each of the following in simplified Cartesian form.**

   (a) $\left(\frac{\sqrt[4]{2}}{2} - \frac{\sqrt[4]{18}}{2}i\right)^{10}$ **;**

   (b) $\frac{1}{81}\left(\frac{1}{2i}\right)^{11}(1-i)^8(-\sqrt{3}-3i)^8$ **.**

6. **Find all solutions of the equation** $2x^5 + \frac{1}{16} \;=\; 0\,.$ **Express all solutions in polar form, simplified as much as possible.**