MATH 1210 Assignment 2 Fall 2016 Due date: October 21

Attempt all questions and show all your work. Attach to Honesty Declaration Form.

1. **Find all** 4^{th} **roots of** -8 **in Cartesian form. Simplify as much as possible.**

2. **For each of the following statements, if it is true prove it, and if it is false give a counter example.**

   (a) $z = \dfrac{|\sqrt{2}z|^2}{2\overline{z}}, \quad (z \neq 0)$;

   (b) $\overline{z}\,(z + z|z|) = |\overline{z}|^2\,(1 + |\overline{z}|)$;

   (c) $\dfrac{e^{4\theta^2 i}\, e^{i^5}}{(e^{\theta i})^4} = \cos(2\theta - 1)^2 + i\sin(2\theta - 1)^2$.

3. **Let** $P(x) = 8x^4 - 2kx^3 + 2k^2x^2 + \frac{k}{2}$, **where** k **is a complex number. Find all values of** k **such that the remainder of** $P(x)$ **divided by** $2x - 1$ **is** $\frac{15}{32} + \frac{1}{32}i$.

4. **Let** $P(x) = 1 + \sum_{i=1}^{6}(-1)^i x^{2i} + \sum_{i=0}^{5}(-1)^i x^{2i+1}$.

   Use Descartes' Rule of Signs to determine

   (a) The number of positive real roots.

   (b) The number of negative real roots.

   (c) The total number of real roots. How many real linear factors does the total number of real roots imply, are their roots positive or negative, how many irreducible quadratics divide $P(x)$ **(i.e. what configuration of real linear and irreducible quadratics does each number of total real roots imply)?**

5. **For each of the following polynomials either use the Rational Root Theorem to make a list of possible rational roots or explain why the Rational Root Theorem cannot be used.**

   (a) $P(x) = 6x^5 + 3x^3 + 2x + 12$.

   (b) $Q(x) = 7x^4 + 10x^2 + 2x$.

   (c) $R(x) = 15x^4 + (8 - 6i)x^3 + 3x + x^2 + 1$.

6. **For the following polynomials use the Bounds Theorem to determine an upper bound for the modulus of their roots.**

   (a) $P(x) = 6x^5 + 3x^3 + 2x + 12$.

   (b) $Q(x) = 10.5x^3 + 12x + 3$.

   (c) $R(x) = 15x^4 + (8 - 6i)x^3 + 3x + x^2 + 1$.

7. **Let** $P(x) = x^5 + 6x^4 + 8x^3 - 4x^2 - 9x - 2$

   (a) Using Descartes' Rules of Signs determine the number of real positive roots of $P(x)$ **and the number of real negative roots of** $P(x)$.

   (b) Use the Rational Root Theorem to determine all possible rational roots of $P(x)$.

   (c) Evaluate $P(x)$ **at possible rational roots and use the Factor Theorem to find one or more linear factor(s) which divide** $P(x)$.

   (d) Show that $P(x)$ **has no roots in the interval** $[2, 5]$.

   (e) Find all roots of $P(x)$.

8. **Let** $P(x) = x^6 - 6x^5 + \frac{17}{2}x^4 - 7x^3 + \frac{21}{2}x^2 + 3x$

(a) Using Descartes' Rules of Signs determine the number of real positive roots of $P(x)$ and the number of real negative roots of $P(x)$. What is the minimum number of real roots? What is the maximum?

(b) Use the Rational Root Theorem to determine all possible rational roots of $P(x)$. (Hint: If $P(x) = Q(x) \cdot R(x)$ where $Q(x)$ and $R(x)$ are polynomials then rational roots of $Q(x)$ and $R(x)$ are rational roots of $P(x)$.)

(c) Use the Bounds Theorem to determine an upper bound for the modulus of roots of $P(x)$. Does this eliminate any possible rational roots? Which ones?

(d) Evaluate $P(x)$ at possible rational roots and use the Factor Theorem to find one or more linear factor(s) which divide $P(x)$.

(e) Given that $(\sqrt{2}x - i\sqrt{3})$ divides $P(x)$ find all roots of $P(x)$.

9. Consider the following matrices

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 13 \end{bmatrix}, \qquad B = \begin{bmatrix} 3 & 5 \\ 1 & 9 \\ -1 & 13 \end{bmatrix}, \qquad C = \begin{bmatrix} 5 & 23 \\ 1 & k \end{bmatrix}.$$

(i) Find all values of k such that $AB = C$.

(ii) Compute each of the following, or explain why it is undefined

(a) $(B^T + A)C$.

(b) $A + 3B^T$.

(d) BA.

(e) $A^T B$.