MATH 1210 Techniques of Classical and Linear Algebra

Assignment 2

Due on Monday, February 24

Attempt all questions and **show all your work.**

Completed assignment should be attached to the Honesty Declaration Form.

1. Let $P(x) = 2x^5 - 9x^4 + 12x^3 - 4x^2 - 8x + 4$.
   (a) Show that $(1 + i)$ is a zero of $P(x)$.
   (b) Find all zeros of $P(x)$.

2. Consider the equation $5x^7 - 9x^3 + 3x^2 + 4 = 4x^6 + 5x^4 - 4x^3 - 2$.
   (a) Find the possible number of positive and the possible number of negative real solutions of this equation.
   (b) Prove that the above equation has at least four non-real solutions.
   (c) Show that this equation has no solutions in the interval $[-7, -3]$.

(Hint: First rewrite the equation in the form $P(x) = 0$.)

3. Let $P(x) = 10x^4 - 9x^3 + 7x^2 + 3x - 2$.
   (a) Use the Rational Roots Theorem to find all possible rational roots of $P(x)$.
   (b) Find all roots of $P(x)$.

4. Consider the matrices

$$A = \begin{bmatrix} -1 & 1 & 4 \\ 3 & 2 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & -3 \\ 0 & 5 \\ 2 & 4 \end{bmatrix}, \qquad C = \begin{bmatrix} 2 & 0 & 1 \\ 1 & -2 & 3 \\ 0 & 1 & 2 \end{bmatrix}, \qquad D = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}.$$

In parts (a)-(e) find the specified matrix when possible. If not possible, explain why.

(a) $3A - 4B$ (b) $AB + 3D$ (c) BAC (d) CAB (e) $2DA - DB^T$

(f) Find a matrix X that satisfies the equation $2X^T + I_2 = D^3$.

(g) Find the dimensions of a matrix Y that would allow for the product YCA^TY to be defined.

5. Let $\mathbf{u} = \langle 2, 1, 3 \rangle$ and $\mathbf{v} = \langle 2, -5, -3 \rangle$. Find each of the following.
   (a) $|2\mathbf{u} + \mathbf{v}|$
   (b) the angle between $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$
   (c) the vector of length 3 in the direction opposite to $\mathbf{v}$

6. Consider the plane $\pi : 2x + 3y - z = -5$, the line $\ell : x = -1 - t$, $y = 6 + 4t$, $z = 1$, and the point $P(4, -2, 3)$.
   (a) Determine whether the plane π intersects with the line ℓ and in case it does, find the point(s) of intersection.
   (b) Find parametric and, if possible, symmetric equations of the line that is perpendicular to the plane π and passes through the point P.
   (c) Find an equation of the plane that is perpendicular to the plane π, parallel to the line ℓ, and passes through the point P.