Math 1210 Assignment 2

Due: Friday, October 20, 2017

Problem 1. Find all 6^{th} roots of -64 in Cartesian form. Simplify your answer as much as possible.

Problem 2. Find the remainder when $f(x)$ is divided by $g(x)$.

(a) $f(x) = ix^6 + (1-2i)x^5 + 5ix^4 - x + 4$ and $g(x) = x - i$

(b) $f(x) = x^{72} + 2x^{31} - 1$ and $g(x) = x + \frac{1}{2} + \frac{\sqrt{3}}{2}i$

(c) $f(x) = 2x^4 + 5x^3 + 5x^2 + 3x + 1$ and $g(x) = x^2 + 2x + 1$

Problem 3. For each of the following polynomial equations either use the Rational Root Theorem to make a list of possible rational solutions or explain why the Rational Root Theorem cannot be used.

(a) $f(x) = 27x^3 - 54x^2 + 36x - 8$

(b) $g(x) = \frac{1}{6}x^4 - \frac{2}{3}x^3 + \frac{1}{3}x^2 + \frac{2}{3}x + \frac{1}{6}$

(c) $h(x) = \sqrt{3}x^4 - 8x^3 + 6\sqrt{3}x^2 - 3\sqrt{3}$

Problem 4. For the following polynomials use the Bounds Theorem to determine an upper bound for the modulus of their roots.

(a) $f(x) = 27x^3 - 54x^2 + 36x - 8$

(b) $g(x) = \frac{1}{6}x^4 - \frac{2}{3}x^3 + \frac{1}{3}x^2 + \frac{2}{3}x + \frac{1}{6}$

(c) $h(x) = ix^6 + (1-2i)x^5 + 5ix^4 - x + 4$

Problem 5. Let $P(x) = x^5 + 7x^4 + 9x^3 - 21x^2 - 52x - 28$.

(i) Using Descartes' Rules of Signs determine the number of real positive roots of $P(x)$ and the number of real negative roots of $P(x)$.

(ii) Use the Rational Root Theorem to determine all possible rational roots of $P(x)$.

(iii) Evaluate $P(x)$ at possible rational roots and use the Factor Theorem to find one or more linear factor(s) which divide $P(x)$.

(iv) Explain why $P(x)$ has no roots in the interval $[3, \infty)$.

(v) Find all roots of $P(x)$.

Problem 6. Let $P(x) = 4x^6 - 8x^5 + 8x^4 - 4x^3 + 8x^2 - 8x$.

(i) Using Descartes' Rules of Signs determine the number of real positive roots of $P(x)$ and the number of real negative roots of $P(x)$. What is the minimum number of real roots? What is the maximum?

(ii) Use the Rational Root Theorem to determine all possible rational roots of $P(x)$ (Hint: If $P(x) = Q(x) \cdot R(x)$ where $Q(x)$ and $R(x)$ are polynomials, then the rational roots of $Q(x)$ and $R(x)$ are also rational roots of $P(x)$).

(iii) Use the Bounds Theorem to determine an upper bound for the modulus of roots of $P(x)$. Does this eliminate any possible rational roots? Which ones?

(iv) Evaluate $P(x)$ at possible rational roots and use the Factor Theorem to find one or more linear factor(s) which divide $P(x)$.

(v) Given that $\frac{-1}{2} - \frac{\sqrt{3}}{2}i$ is a root of $P(x)$, find all roots of $P(x)$.

Problem 7. Consider the following matrices:

$$A = \begin{pmatrix} 4 & 0 & 5 \\ -1 & 3 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 3 \\ 1 & 5 \\ 1 & 7 \end{pmatrix}, \quad \text{and} \quad C = \begin{pmatrix} 9 & k \\ 4 & 26 \end{pmatrix}.$$

(a) Find the value(s) of k such that $AB = C$.

(b) Compute each of the following, or explain why it is undefined.

- **(i)** $(A - B^T)C^T$
- **(ii)** $2A^T + 3B$
- **(iii)** $A^T B^T$
- **(iv)** $B^T A$