MATH 1210 Assignment 3, Winter 2020   Due date : March 23rd
Attempt all questions and show all your work. Attach to Honesty Declaration Form.

1. Consider the system

$$a+b-2c-d=0, \quad b-d=2, \quad 4a+2b-9c+d=1, \quad 2b+d=-2.$$

   (a) Solve the system using Gauss-Jordan Elimination.

   (b) Use Cramer's Rule to solve for a and d **only**.

2. Let a_0, a_1, and a_2 be real numbers and let $p(x) = a_0 + a_1 x + a_2 x^2$. Suppose

$$p(1) = -1, \quad p(2) = 0, \quad p(3) = 2.$$

   What are the values of a_0, a_1, and a_2?
   (**Hint** : Write out what the above 3 equations say about the coefficients a_0, a_1, a_2.)

3. Find all values of a and b such that the system

$$ax + 3y + z = 1, \quad bx + y - z = 0, \quad x - y + z = 1$$

   has exactly one solution.

4. Find the basic solutions of the following homogeneous system:

$$\begin{aligned} x + 2y + z - w + 2v + 4u &= 0 \\ 2x + y - z + w + v - u &= 0 \\ x - 2z - 2w + 3u &= 0 \end{aligned}$$

5. Use properties of determiants to compute $\det(F)$, where

$$F = \begin{bmatrix} 0 & 0 & a & 0 & 0 \\ 0 & b & b & 0 & 0 \\ 0 & 0 & c & c & 0 \\ d & d & d & 0 & 0 \\ 0 & 0 & e & e & e \end{bmatrix}$$

6. Consider the vectors

$$\vec{u}_1 = \langle 1,3,2 \rangle, \quad \vec{u}_2 = \langle 1,1,1 \rangle, \quad \vec{u}_3 = \langle 0,-1,2 \rangle, \quad \vec{u}_4 = \langle 1,1,6 \rangle, \quad \vec{u}_5 = \langle -3,1,1 \rangle.$$

   (a) Without any computation, explain why $\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4$, and $\vec{u}_5$ are linearly dependent.

   (b) Express $\vec{u}_4$ as a linear combination of $\vec{u}_1, \vec{u}_2$, and $\vec{u}_3$.

   (c) Show that $\vec{u}_1, \vec{u}_3$, and $\vec{u}_5$ are linearly independent.

   (d) Prove that any vector $\vec{v} = \langle a,b,c \rangle$ is a linear combination of $\vec{u}_1, \vec{u}_3$, and $\vec{u}_5$.
   (**Hint** : Write $\langle a,b,c \rangle = x\vec{u}_1 + y\vec{u}_3 + z\vec{u}_5$ as a linear system. Does it have a solution?)