MATH 1310 Assignment 1 Winter 2009

1. Determine whether each of the following statements is **True** or **False**. Explain your reason.
   1. If $A = \begin{pmatrix} a & a \\ a & a \end{pmatrix}$ such that $a \neq 0$ then $A^2 = \begin{pmatrix} a^2 & a^2 \\ a^2 & a^2 \end{pmatrix}$
   2. The matrix $\begin{pmatrix} 12 & -8 \\ -9 & 18 \end{pmatrix}$ is a linear combination of the two matrices $\begin{pmatrix} 3 & -4 \\ 6 & 0 \end{pmatrix}$ and $\begin{pmatrix} 2 & 0 \\ -7 & 6 \end{pmatrix}$.
   3. For matrices A and B, if $A^2 = B^2$ then $A = B$.
2. Let $A = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix}$; first find **all** values of $\boldsymbol{a}$ and $\boldsymbol{b}$ such that $A^2 - 3A = B$, then list all possible answers for the matrix A.
3. Let $A = \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix}$; find **all** values of $\boldsymbol{a}$ and $\boldsymbol{b}$ such that $A^2 + 4A = 0$.
4. Let $A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 0 \\ 1 & 3 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 4 & 1 \\ 0 & -1 \\ 5 & 0 \end{pmatrix}$;

   evaluate each of the following expressions or explain why it is not defined.
   1. $A^2 - 2A + I_2$
   2. $-2A(B + C^T)$
   3. $(CB)^2$
5. Let $A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \\ 3 & 1 \\ -4 & 5 \\ 0 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 & 0 & 1 & 3 & -6 & -1 \\ 1 & 3 & 1 & -4 & -1 & 5 & -6 \end{pmatrix}$; find each of the following:

   (a) The (5, 4)-entry of $B^T A^T$.

   (b) The (1, 2)-entry of $A^T A + BB^T$.

   [**Hint**: In parts (a) and (b), you do not need to find all entries.]
6. Let $A = \begin{pmatrix} 1 & x & 2 \\ y & 3 & -1 \\ 0 & 1 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ -1 \\ z \end{pmatrix}$. If $AB = \begin{pmatrix} -3 \\ 5 \\ 7 \end{pmatrix}$ find x, y and z.
7. Find all solutions of each linear system using Gauss-Jordan elimination. Show your work and explain what elementary operations you are using.

(a)
$$\begin{aligned} -x - y + z &= -1 \\ -3x - y - z &= -9 \\ 3x - y + 5z &= 15 \\ -2y + 4z &= 6 \end{aligned}$$

(b)
$$\begin{aligned} -2x_1 \qquad\quad -4x_3 - 2x_4 &= -20 \\ -x_1 + x_2 \quad + x_3 - x_4 &= -5 \\ -x_1 - x_2 \quad -5x_3 - 2x_4 &= -21 \\ x_2 \quad +3x_3 &= 5 \end{aligned}$$

(c)
$$\begin{aligned} -3x_1 - x_2 \quad +x_3 \qquad &= -2 \\ 2x_1 + x_2 \quad +2x_3 - x_4 &= 1 \\ -7x_1 - x_2 \quad +13x_3 - 4x_4 &= -2 \end{aligned}$$