MATH 1310 Assignment 2 Winter 2009

1. The augmented matrix of a linear system is $\left(\begin{array}{ccc|c} 1 & 2 & a+2 & b \\ 0 & 1 & b-1 & a \\ 0 & 0 & a & b \\ 0 & 0 & 0 & 0 \end{array}\right)$.
   (a) Find **all** values (if any) of a and b for which the system is inconsistent.
   (b) Find **all** values (if any) of a and b for which the system has exactly one solution.
   (c) Find **all** values (if any) of a and b for which the system has infinitely many solutions.

2. Determine whether each of the following matrices is invertible. If yes find the inverse and if no explain why.

$$(a)\quad A=\begin{pmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{pmatrix} \qquad (b)\quad B=\begin{pmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{pmatrix} \qquad (c)\quad C=\begin{pmatrix} 3 & 1 & 2 \\ 1 & -2 & -4 \\ -5 & 3 & 6 \end{pmatrix}$$

3. Let $A=\begin{pmatrix} 1 & 1 & 2 \\ 0 & 3 & -1 \\ 2 & 1 & 4 \end{pmatrix}$. First find A^{-1} then find all solutions of each of the following systems:

   (a) $A\mathbf{x}=\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$, (b) $(-3A)\mathbf{x}=\begin{pmatrix} -3 \\ 0 \\ 6 \end{pmatrix}$, (c) $A^{-1}\mathbf{x}=\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$, (d) $A^{T}\mathbf{x}=\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$

4. Find the determinant of each of the following matrices.

$$(a)\quad A=\begin{pmatrix} 3 & 4 & 7 \\ 5 & 6 & 2 \\ 1 & 8 & 9 \end{pmatrix} \qquad (b)\quad B=\begin{pmatrix} 2 & 7 & -3 & 0 \\ 0 & 2 & 6 & 7 \\ 0 & 1 & 0 & 3 \\ 4 & 15 & -6 & 0 \end{pmatrix} \qquad (c)\quad C=\begin{pmatrix} 3 & 1 & 2 \\ 1 & -2 & -4 \\ -5 & 3 & 6 \end{pmatrix}$$

5. Find the value of x such that

$$\begin{vmatrix} x & 2 & 1 \\ -1 & 0 & 1 \\ 0 & 3 & x \end{vmatrix} = \begin{vmatrix} 0 & x & -1 \\ 2 & 3 & 4 \\ 0 & 1 & -2 \end{vmatrix}$$

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6. Find all values of x for which the matrix $A=\begin{pmatrix} x & 1-x & 3 \\ 1 & x & -1 \\ 2 & 1 & 1 \end{pmatrix}$ is singular.

7. Let A, B and C be 5×5 matrices such that $det(A)=3$, $det(B)=-2$ and $det(C)=10$. Evaluate each of the following:
   1. $det(AB^{T}C)$.
   2. $det(-2A^{2}B^{-1})$.
   3. $det(A^{-1}DB^{-3}D^{-1})$. (where D is another 5×5 matrix)

8. Which one of the following matrices is an elementary matrix ? Explain.

$$(a)\quad A=\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \qquad (b)\quad B=\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \qquad (c)\quad C=\begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

9. Let $A=\begin{pmatrix} 5 & -2 & 1 \\ 3 & 2 & 0 \\ 1 & 1 & -1 \end{pmatrix}$ and $B=\begin{pmatrix} 0 & 2 & 1 & -1 \\ 2 & 3 & 6 & 7 \\ 1 & -1 & 2 & 1 \\ 0 & 1 & 2 & 1 \end{pmatrix}$. Find $(3,2)$-cofactor and also $(2,3)$-cofactor for each of the two matrices.