MATH 1310 Assignment 3 Winter 2009

1. Three government departments, **A**, **B** and **C** control all data processing. They want to charge fees for the use of their services by the other departments. Department **A** uses $\frac{1}{2}$ of its own processing capacity, $\frac{1}{2}$ of **B**'s capacity, and $\frac{3}{8}$ of **C**'s. Department **B** uses $\frac{5}{16}$ of capacity of **A**, $\frac{1}{16}$ of its own, and $\frac{3}{8}$ of **C**'s. Department **C** uses $\frac{3}{16}$ of capacity of **A**, $\frac{7}{16}$ of **B**'s, and $\frac{1}{4}$ of its own.

   (a) Find the exchange matrix for this system.

   (b) What price should each department charge to keep this system in equilibrium?

   (c) What prices should departments **A** and **B** charge if department **C** charges \$4000.

2. An economy consists of two sectors, electricity and petroleum. The production of \$1 of electricity requires 40 cents in electricity and 10 cents in petroleum. Whereas the production of \$1 of petroleum requires 20 cents in electricity and 30 cents in petroleum.

   (a) Find the consumption matrix C of this economy.

   (b) Is this economy productive? Justify your answer.

   (c) If there is an outside demand for \$320 of electricity and \$280 of petroleum, find the production vector.

3. Let $\mathbf{u} = (5,\, 8,\, 3,\, \sqrt{2})$, $\mathbf{v} = (-1,\, 0,\, -2,\, 0)$ and $\mathbf{w} = (4,\, 2,\, -2,\, 1)$. Evaluate each of the following expressions or explain why it is not defined.

   (a) $\frac{\|\mathbf{u}\|}{\|\mathbf{w}\|}\,(\mathbf{u}\,.\,\mathbf{w})$

   (b) $(\|\mathbf{v}\|\,\|\mathbf{w}\|)\,\mathbf{u} + \|\mathbf{v}+\mathbf{w}\|^2$

   (c) $(\mathbf{w}.\mathbf{v})\,\mathbf{u} - 2(\mathbf{v}.\mathbf{u})\,\mathbf{v}$

4. Let $A(2,\, 0,\, 2), B(2,\, 1,\, 2),$ and $C(3,\, 0,\, 2)$ be three points in $\mathbf{R}^3$; and let $\mathbf{u} = \overrightarrow{BC}$, $\mathbf{v} = \overrightarrow{BA}$ and $\mathbf{w} = \overrightarrow{CA}$.

   (a) Find the components of the vectors $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$.
   (b) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.
   (c) Find the angle between $\mathbf{u}$ and $\mathbf{w}$.
   (d) Find the angle between $\mathbf{v}$ and $\mathbf{w}$.

5. Find all values of $\mathbf{a}$ such that $\|\,(\sqrt{3},\, a,\, 2\sqrt{a}\,)\,\| = 2\sqrt{2}$.

6. Let $\mathbf{u} = (-\sqrt{5},\, \sqrt{2}\,)$ and $\mathbf{v} = (2,\, 3,\, -2)$.

   (a) Find a unit vector in the direction of the vector $\mathbf{u}$.
   (b) Find a unit vector in the direction of the vector $\mathbf{v}$.

7. Determine the ***tail*** of the vector $\mathbf{u} = (-1,\, 3,\, -3,\, 4)$ whose ***head*** is the point $P(1,\, 6,\, -1,\, -2)$.

8. Let W be the set of all 2×2 matrices. For any two matrices A and B in W and c in $\mathbf{R}$, it is defined that:

$$A \oplus B = A + B + I \qquad \text{and} \qquad c \odot A = cA$$

   Determine if the property $c \odot (A \oplus B) = (c \odot A) \oplus (c \odot B)$ fails or holds in W.