MATH 1310 Assignment 4 Winter 2009

1. For each of the following, determine whether the given set is a **subspace** of the given vector space. If your answer is yes prove it, and if your answer is no explain why.

   (a) In the vector space $\mathbb{R}^3$, the set of all vectors of form (a, b, c) such that $a + b - c = 2$.

   (b) In the vector space M_{22}, the set of matrices of form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that $2a - c = 0$ and $3b - d = 0$.

   (c) In the vector space M_{22}, the set of matrices A such that $A + A^T = 0$

   (d) In the vector space P_3, the set of all polynomials of form $at^3 + bt^2 + ct$ where a , b and c are real numbers.

   (e) In the vector space P_2, the set of all polynomials of form $t^3 + a$ where a is a real number.

   (f) In the vector space M_{33}, the set of matrices of form $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ such that $a_{11}+a_{12}+a_{13} = 3$.

2. Let $S = \{(1, 0, 1), (1, 1, 3), (0, 1, 2)\}$; determine whether $(3, 4, 11)$ is in $\operatorname{Span} S$. Repeat it for $(1, -1, 1)$

3. For each of the following, determine whether the given set is linearly dependent or linearly independent in the given vector space.

   (a) $S = \{(1, -2, 1), (2, -1, 3), (0, 1, 2), (0, 1, 0)\}$, in $\mathbb{R}^3$.

   (b) $S = \{t^2 - 1,\ 3t + 2,\ t^2 + 1\}$, in P_2.

   (c) $S = \left\{ \begin{pmatrix} 1 & -2 \\ 0 & 6 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & -3 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & -2 \\ 0 & 7 \end{pmatrix} \right\}$, in M_{22}.

4. Let W be a subset of the vector space M_{22}, consisting of all matrices A such that $AA^T = I$.

   (a) Is W closed under addition?

   (b) Is W closed under scalar multiplication?

   (c) Is W a subspace of M_{22}? Explain.

5. Show that the set $S = \{t + 1,\ t - 1\}$ form a basis for the vector space P_1.

6. For each of the following, find a basis and the dimension of the given subspace.

   (a) In $\mathbb{R}^4$, subspace of all vectors of form (a, b, c, d) such that $c = 2a + b$ and $d = a - 3b$.

   (b) In M_{22}, subspace of all matrices of form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that $a = b + c$ and $d = b - 2c$.

   (c) In P_2, subspace of all polynomials of form $at^2 + bt + c$ such that $a - 2b = 0$ and $c + b = 0$.

   (d) In P_3, subspace of all polynomials of form $at^3 + bt^2 + ct + d$ such that $a - 4b = 0$ and $c - d = 0$.