## MATH 1500 A01 Assignment 4 Winter 2010 (Due date is Monday April 5)

[12] 1. A window is in the shape of a rectangle surmounted on both sides by a semicircles as in the picture below. If the perimeter of the window is 6 m find the dimensions x and y ( as shown in the figure) so that the greatest amount of light is admitted through the window.



- [12] 2. Sara is going to enclose a rectangular field of corn whose area wile 60,000 square meters. One side of here field is parallel to a high way where expensive fencing is required. The fencing along the highway costs \$40 per meter, while the remaining fencing costs \$20 per meter. What should the dimensions of her field be to make the total cost a minimum? justify your answer.
- [12] 3. Postal regulations require that for a rectangular package with a square base, the perimeter of the square base plus the height should be 108 cm. Find the dimensions that result in the maximum possible volume.
- [12] 4. A farmer has 200 meters of fence with which to enclose a rectangular field with a fence down the middle, creating two adjacent pastures of identical size. (see diagram below). What dimensions should be used to obtain a maximum enclosed area?



[12] 5. A book store has 160 copies of a certain calculus book each \$70 with the limit of one book per customer. The owner of the book store decides to increase the price but he knows that for every one dollar added to the price of each book he will lose two customers. What price should he put for each book in order to maximize his revenue ? What is the maximum revenue? [24] 6. Find the most general antiderivative of each of the following functions:

(a) 
$$f(x) = 4\sqrt{x} - \frac{1}{x^7} + \frac{3}{2x}$$
  
(b)  $g(x) = \frac{3}{\sqrt{x}} + \frac{1}{e^{3x}} - 2$   
(c)  $h(x) = \frac{5}{2x+1} + \cos\left(\frac{x}{2}\right) - \sec^2 x$   
(d)  $k(x) = (x-1)^2 + 6e^{9x} - \frac{1}{\sin^2 x}$ 

[8] 7. A particle is moving along the x-axis. Its velocity is given by  $v(t) = 3t^2 + 4t - 5$ . The particle is at x = 2 at time t = 1. Where is the particle when t = 5?

[8] 8. Find 
$$f(x)$$
 if  $f''(x) = 12x + \frac{1}{x^2}$ ,  $f'(1) = 2$  and  $f(1) = -2$ .