UNIVERSITY OF MANITOBA

DATE: December 16, 2009, 6:00pm   FINAL EXAMINATION
PAPER # 411   TITLE PAGE
DEPARTMENT & COURSE NO: MATH 1500   TIME: 2 hours
EXAMINATION: Introduction to Calculus   EXAMINER: Math 1500 Committee

NAME: (Print in ink) _____

STUDENT NUMBER: _____

SEAT NUMBER: _____

SIGNATURE: (in ink) _____
(I understand that cheating is a serious offense)

Please indicate your lecture section by checking the correct box below:

☐	A01	Slot 3, and 5T	MWF 10:30 and T 10:00	D. Kalajdzievska
☐	A02	Slot 2	MWF 9:30	E. Schippers
☐	A03	Slot 5	TTh 10:00	R. Craigen
☐	A04	Slot 6	MWF 11:30	T. Kucera
☐	A05	Slot 7	MWF 12:30	T. Mohammed
☐	A06	Slot 12	MWF 3:30	C. Podder
☐	A07	Slot E2	T 7 pm	J. Sichler
☐	A92	Challenge for Credit	Dakota ☐	Sisler ☐
☐	Deferred			

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam.
Please show your work clearly.

No texts, notes, or other printed aids are permitted. You are not allowed to have calculators, cellphones, electronic translators, personal music devices or any other electronic devices or mechanical aids at the exam table with you.

This exam has a title page and 11 numbered pages including 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staple.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 120 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	10	
2	20	
3	9	
4	12	
5	8	
6	24	
7	12	
8	10	
9	15	
Total:	120	

This exam counts for 60% of your final grade.

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[10] 1. Find the following limits. Fully justify your answer.

(a) $\lim\limits_{x \to \infty} \sqrt{x^2 + 3x} - x$

(b) $\lim\limits_{x \to 0} x^2 \cos\left(\dfrac{1}{x^2}\right)$

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[20]   2. Find $\dfrac{dy}{dx}$ in each case (DO NOT SIMPLIFY YOUR ANSWERS):

(a) $y = \dfrac{e^{-x}}{x^3 + 1}$

(b) $y = 3^{2x} x^5$

(c) $y = \left(x^2 + 1\right)^{\sin x}$

(d) $y = \displaystyle\int_0^{x^2} \ln\left(2 + t^2\right) dt.$

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[9] 3. (a) State the Mean Value Theorem.

(b) Let f be differentiable on an open interval (a, b). Prove that if $f'(x) = 0$ for all x in (a, b) then $f(x)$ is constant on (a, b).

[12] 4. Find the absolute maximum and minimum values of the function

$$f(x) = 3x^4 + 4x^3 - 12x^2$$

on $[-1, 2]$.

[8] 5. A particle moves in a straight line so that its velocity at time t is

$$v(t) = 2t - \frac{1}{2}t^2 \quad \text{cm/s}.$$

(a) What is its acceleration at time $t = 6\,\text{s}$?

(b) If its position at time $t = 0\,\text{s}$ was $x = 3\,\text{cm}$, what is its position at time
     $t = 6\,\text{s}$?

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[24]   6. You are given the following information:

$$f(x) = \frac{2(x+1)}{3x^2}, \quad f'(x) = \frac{-2(x+2)}{3x^3}, \quad \text{and } f''(x) = \frac{4(x+3)}{3x^4}.$$

(a) Find the domain of f.

(b) Find the location of any x and y intercepts of f.

(c) Find the location of any vertical or horizontal asymptotes of f. Justify your answer with a limit.

(d) Find the intervals on which f is increasing or decreasing, and the coordinates of all of the local maxima and minima of f.

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(e) Find the intervals on which f is concave up or concave down, and the coordinates of any inflection points of f.

(f) Sketch the graph of f.

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[12] 7. A rectangular box with square base is constructed from material costing 1¢ per
square centimetre for the sides and base; the top costs 2¢ per square centimetre.
If the total cost of the box is to be 900¢, what is the maximum volume possible?

[10] 8. Find the most general antiderivative of the following functions.

(a) $f(x) = x^{1/3} - \dfrac{2}{x^2} + 2$

(b) $f(x) = e^{3t} - \dfrac{3}{t}$

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[15] 9. (a) Evaluate the following definite integral: $\int_0^{\pi/4} \left(2\sec^2(x) + \sin(x)\right)\, dx$

(b) Find the area of the region of the plane above the x-axis and below the graph of $y = -x^2 - x + 6$.