UNIVERSITY OF MANITOBA

DATE: April 18 2009 FINAL EXAMINATION
PAPER # 439 TITLE PAGE
DEPARTMENT & COURSE NO: 1520 TIME: 2 hours
EXAMINATION: Intro. Calc. for Mgmt. and Soc. Sci. EXAMINER: Various

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NAME: (Print in ink) _____

STUDENT NUMBER: _____

SEAT NUMBER: _____

SIGNATURE: (in ink) _____
(I understand that cheating is a serious offense)

❑ A01
D. Kalajdzievska

❑ A02
R. Craigen

❑ A03
G. Lukács

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 10 pages of questions and also 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staple.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 120 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1-15	30	
16	9	
17	9	
18	18	
19	13	
20	17	
21	24	
Total:	120	

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Multiple Choice Questions

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Response																

For questions 1-16, answer each part by placing the letter of the answer which is *most* correct into the corresponding box in the table above.

[2] 1. The expression $\lim_{x \to 0^+} f(x) = 0$ means that:

A. the slope of the tangent line to the graph of $y = f(x)$ at $x = 0$ is 0

B. the derivative of $f(x)$ is not defined at 0

C. the limit of $f(x)$, as $x \to 0$, is not defined

D. the limit of $f(x)$, as x approaches 0 from the right is 0

E. the antiderivative of $f(x)$ at 0 is 0

[2] 2. The domain of the function $f(x) = \dfrac{x^4}{x^2 + x - 20}$ consists of all $x \neq$

A. 0 B. 20 C. 0, 5, 4 D. -5, 4 E. -4, 5

[2] 3. The domain of the function $f(x) = \dfrac{\sqrt{x-3}}{\ln(x+2)}$ is:

A. $[-3, 2)$ B. $(-3, 2]$ C. $[2, 3]$ D. $(-2, 3]$ E. None of these

[2] 4. Find the indefinite integral $\displaystyle\int \left(\frac{3}{t^4} + t^4\right) dt$

A. $\dfrac{-1}{t^3} + \dfrac{1}{3}t^3 + C$ B. $\dfrac{-1}{t^3} + \dfrac{1}{5}t^5 + C$ C. $\dfrac{4}{x^3} + x^5 + C$ D. $-\dfrac{3}{5}t^{-5} + \dfrac{1}{5}t^5 + C$

E. None of these

[2] 5. A critical number for the function f is defined as

A. An endpoint of the domain of f or point of discontinuity of f

B. A number c at which $f''(x)$ changes sign

C. A number c such that $f(c) = 0$ or $f(c)$ does not exist

D. A number c such that $f'(c) = 0$ or $f'(c)$does not exist

E. None of these

[2] 6. A company selling x items of its product has cost $C(x) = 15x + 1000$ (\$) and revenue function $R(x) = 20x$ (\$). Then the break-even quantity is

A. $R(x) - C(x)$ B. $R'(0) = C'(0)$ C. \$200 D. 200 items E. $\lim_{h \to 0} \dfrac{R(x+h) - C(x)}{h}$.

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[2] 7. At what points is the function $f(x) = \dfrac{(x-1)(x-2)}{e^x(x-3)(x-4)}$ continuous?

A. For all x B. For all $x \neq 0$ C. For all $x \neq 0, 1, 2, 3, 4$

D. For all $x \neq 0, 1, 2$ E. For all $x \neq 3, 4$

[2] 8. What are all the asymptotes of the function $f(x) = \dfrac{(x-1)(x-2)}{(x-3)(x-4)}$?

A. Vertical: $y = 3$, $y = 4$

B. Vertical: $x = 3$, $x = 4$; Horizontal: $y = 1$

C. Vertical: $x = 1$, $x = 2$; Horizontal: $y = 1$, $y = 3$, $y = 4$

D. Vertical: $x = 0$; Horizontal: $y = 1$, $y = 2$

E. It has no asymptotes

[2] 9. The expression $\log_5(24)$ can be written differently as

A. $\log_5 3 + 3\log_5 2$

B. $\ln(120)$

C. $(\log_5 3)\,(\log_5 8)$

D. 5^{24}

E. $\log_{10}(12)$

[2] 10. Let $f(x) = x^{10} - 1000x^9 + e^x$. What is the value of $f^{(50)}(3)$? (Recall: $f^{(k)}(x)$ is a notation for the kth derivative of $f(x)$)

A. $3^{10} - 1000 \cdot 3^9 + e^3$ B. 3^{11} C. 0 D. e^3 E. None of these

[2] 11. If the revenue function for selling x items is $R(x) = 30x^2 - 10x$ and the cost function is $C(x) = 20x + 30$ then the marginal profit, $P'(x)$, is

A. $30x^2 + 10x + 30$

B. $30(1 - 2x)$

C. $(60x - 10x)'(20x + 30) + (60x - 10x)(20x + 30)'$

D. $60x + 10$

E. None of these

[2] 12. Let $f(x) = \dfrac{|x|}{x}$. Which one of the following statements is true?

A. f is continuous on the whole real line

B. $\lim\limits_{x \to 0} f(x)$ exists, but $f(0)$ does not

C. $y = f(x)$ has a vertical asymptote at 0

D. $\lim\limits_{x \to 0^+} f(x)$ and $\lim\limits_{x \to 0^-} f(x)$ both exist, but they are not equal

E. $\lim\limits_{x \to 0} f(x)$ and $f(0)$ both exist, but they are not equal.

Questions 13, 14 and 15 all refer to the following situation: The manufacturer of inexpensive MP3 players known as Earpods is researching its market and finds that its demand function is

$$p = 300 - q - \frac{q^2}{15000},$$

where q is the quantity that can be sold, and p is the price, in dollars. The cost for manufacturing q items for sale is

$$C(q) = 250q - q^2.$$

[2] 13. Find the marginal revenue for Earpods.

A. $300q - q^2 - \frac{q^3}{15000}$ B. $300 - 2q - \frac{q^2}{5000}$ C. $-1 - \frac{q}{7500}$ D. \$300

E. None of these

[2] 14. Find the profit function for Earpods.

A. $P(q) = 50 - \frac{q^2}{5000}$ B. $50 - \frac{q^2}{5000}$ C. $\frac{q^3}{15000} - 50q$ D. $50q - \frac{q^3}{15000}$

E. None of these

[2] 15. What number of Earpods should be manufactured in order to maximize the company's profit?

A. \$500 B. 500 Earpods C. 15000 Earpods D. \$250000

E. None of these

Short Answers

In this part, only your answers will be marked. No partial marks will be given for work. You may work in this space but make sure that it is clear what your answer is!

16. Given the following information about the values of functions f, g, f' and g':

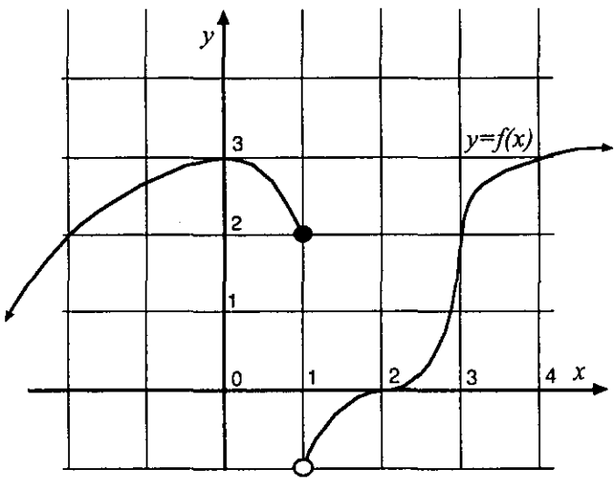
$x =$	0	1	2	3	4
$f(x) =$	4	3	2	0	1
$f'(x) =$	3	1	2	4	3
$g(x) =$	3	3	1	1	2
$g'(x) =$	4	3	2	1	0

[3] (a) If $h(x) = f(g(x))$, then $h'(2) =$

[3] (b) If $k(x) = f(x)g(x)$, then $k'(2) =$

[3] (c) If $m(x) = \dfrac{g(x)}{f(x)}$, then $m'(2) =$

[9] 17. The graph of $y = f(x)$ is shown to the right. Answer each part in the space given (Possible answers may include "not defined", "∞", "$-\infty$" or "none exist").



(a) $\lim_{x \to 1^+} f(x) =$

(b) $f(1) =$

(c) $f'(2) =$

(d) The sign ("+" or "−") of $f'(3)$ is

(e) The sign ("+" or "−") of $f''(4)$ is

(f) One inflection point for the graph $y = f(x)$ is

(g) $f(x)$ attains a relative maximum at $x =$

Full Answers

In this section show your work. Marks will be based on both your answer and the work used to arrive at that answer.

18. Find:

[4] (a) $\lim_{x \to 3} \dfrac{2^x(x^2 - 5x + 6)}{x^2 - 3x}$

[4] (b) The derivative of $x\,3^x - \dfrac{\ln(x)}{x}$

[5] (c) The second derivative of $\dfrac{x+1}{x-2}$ (do any obvious simplifications)

[5] (d) The function $f(x)$ such that $f'(x) = x^2 + 5$ and such that $f(1) = 11$

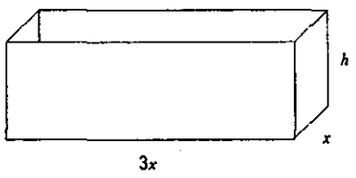
19. Ahmed deposited \$1000 into a savings account whose interest is compounded quarterly. After 4 years, he had \$1500 in his account.

[5] (a) Find an expression for the annual interest rate of the savings account.

[3] (b) Using the result of (a), write the function $A(t)$ that gives the accumulated value of the account at time $t \geq 0$.

[5] (c) Find the instanteneous rate of change of the accumulated value when $t = 5$.

20. A recycling box is to be made with 1 square meter of plastic material. Its shape is to be rectangular, three times as long as it is wide, and it is to have an open top (see diagram).



[3] (a) Express the area of material used in terms of the variables x and h.

[4] (b) Express the volume of the box in terms of a single variable.

[6] (c) Find the dimensions of the box that give it maximum volume.

[4] (d) What is that maximum volume?

[24] 21. Here is a function and its first two derivatives, in factored form:

$$f(x) = \frac{(x-3)(x+2)}{(x-1)^2}, \qquad f'(x) = \frac{13-x}{(x-1)^3}, \qquad f''(x) = \frac{2(x-19)}{(x-1)^4}.$$

(a) Give the domain:

(b) Give any intercepts :

(c) Give any asymptotes:

(d) For relevant intervals, organize the following information below:

- The sign of f' and the sign of f'';
- intervals of increase and decrease;
- intervals of constant concavity;
- Coordinates of relative maxima/minima and inflection points.

(e) Sketch the graph of $y = f(x)$ on the axes given below, clearly labelling all features of interest.

Two points you may choose to plot: $f(13) = \frac{25}{24} \approx 1, \quad f(19) = \frac{28}{27} \approx 1.$