Term Test 1

DATE: February 27, 2017
COURSE: MATH 2020

TIME: 50 minutes
EXAMINER: G.I. Moghaddam

NAME:____

STUDENT # : ____________________________

Q1 [9]	Q2 [11]	Q3 [6]	Q4 [10]	Q5 [14]	Total [50]

1. Let $\mathbb{N} = \{1, 2, 3, \cdots\}$ be the set of all natural numbers. Define $\sim$ on $\mathbb{N}$ such that for each $a, b \in \mathbb{N}$, $a \sim b$ if $gcd(a, 2) = gcd(b, 2)$.

[6] (a) Is $\sim$ an equivalence relation on $\mathbb{N}$? Why?

[2] (b) List all the equivalence classes.

2. Let $G = \{1, 2, 3, 4, 5\}$ be a set. A binary operation $*$ is defined on G with the table:

$*$	1	2	3	4	5
1	1	2	3	4	5
2	2	1	4	5	3
3	3	5	1	2	4
4	4	3	5	1	2
5	5	4	2	3	1

[3] (a) Is G associative under $*$? Why?

[5] (b) Find the identity element, order of each element, and inverse of each element.

[3] (c) Is $(G, *)$ a group? Why?

[6] 3. Let G be a group and H be a nonempty subset of G. Prove that if $ab^{-1} \in H$ (for every $a, b \in H$), then H is a subgroup of G.

4. Consider $U(\mathbb{Z}_{10})$, that is the group of all units of $\mathbb{Z}_{10}$ with multiplication modulo 10.

[3] (a) List all elements of $U(\mathbb{Z}_{10})$.

[3] (b) Find $< \bar{9} >$ in $U(\mathbb{Z}_{10})$.

[4] (c) Is $U(\mathbb{Z}_{10})$ a cyclic group? Why?

[14] 5. For each of the following statements determine if it is *true* or *false*. (You do not need to explain)

- If $f : \mathbb{Q} \to \mathbb{Z}$ such that $f(\frac{p}{q}) = p + q$ where $p, q \in \mathbb{Z}, q \neq 0$, then f is a well defined function.

- For the nonzero elements $\bar{a}, \bar{b}, \bar{c}$ in $(\mathbb{Z}_6, \times)$ if $\bar{a}\bar{b} = \bar{a}\bar{c}$ then $\bar{b} = \bar{c}$.

- If H_1 and H_2 are both subgroup of a group G then $H_1 \cup H_2$ is a subgroup of G.

- If H and G are groups and $H \subset G$, then H is a subgroup of G.

- The only elements of finite order of the group $(\mathbb{C}^*, \times)$ are 1 and -1.

- If $H = \left\{ \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} \,\middle|\, n \in \mathbb{Z} \right\}$ is a subgroup of the group $GL_2(\mathbb{R})$, then H is a cyclic subgroup generated by $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

- Every subgroup of the group $(\mathbb{Z}, +)$ is cyclic.

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Answers:

Q1 (b) $[1] = \{1, 3, 5, 7, \cdots\}$, $[2] = \{2, 4, 6, 8, \cdots\}$ and $\mathbb{N} = [1] \cup [2]$.

Q2 (a) No it is not. For instance $2 \star (3 \star 3) = 2$ but $(2 \star 3) \star 3 = 5$.

(b) Identity is 1 which is of order 1. For any $a \neq 1$ in G, since $a^2 = 1$ so a is of order 2 and $1^{-1} = 1\,, 2^{-1} = 2\,, 3^{-1} = 3\,, 4^{-1} = 4\,, 5^{-1} = 5$.

(c) No G is not a group because it is not associative under $\star$.

Q3 Read the textbook.

Q4 (a) $U(\mathbb{Z}_{10}) = \{\bar{1}, \bar{3}, \bar{7}, \bar{9}\}$

(b) Since $\bar{9}^2 = \bar{1}$ so $\bar{9}$ is of order 2 and $< \bar{9} >= \{\bar{1}, \bar{9}\}$.

(c) $(\bar{3})^0 = \bar{1}\,, (\bar{3})^1 = \bar{3}\,,\ (\bar{3})^2 = \bar{9}\,,\ (\bar{3})^3 = \bar{7}$ so $< \bar{3} >= \{\bar{1}, \bar{3}, \bar{7}, \bar{9}\} = U(\mathbb{Z}_{10})$ which means it is a cyclic group and $\bar{3}$ is a generator. Note that $\bar{7}$ is also a generator.

Q5 F , F , F , F , F , T , T .