Term Test 2

DATE: March 27, 2017 TIME: 50 minutes
COURSE: MATH 2020 EXAMINER: G.I. Moghaddam

NAME:____________________________________

STUDENT # : ________________________

Q1 [9]	Q2 [10]	Q3 [10]	Q4 [21]	Total [50]

[9] 1. Consider the dihedral group D_4 and choose elements $s = (1\ 3)$ and $r = (1\ 2\ 3\ 4)$ in D_4 such that $s^2 = e$, $r^4 = e$ and $srs = r^3$. Complete the table of D_4. Explain.

$\circ$	e	s	r	r^2	r^3	sr	sr^2	sr^3
e	e	s	r	r^2	r^3	sr	sr^2	sr^3
s	s	e	sr	sr^2	sr^3	r	r^2	r^3
r	r	sr^3	r^2	r^3	e	s		sr^2
r^2	r^2	sr^2	r^3	e	r	sr^3		
r^3	r^3	sr	e	r	r^2	sr^2	sr^3	s
sr	sr	r^3	sr^2	sr^3	s	e	r	r^2
sr^2	sr^2	r^2	sr^3	s	sr			
sr^3	sr^3	r	s	sr	sr^2	r^2	r^3	e

2. Let G and H be two groups and let $\phi : G \to H$ be an isomorphism.

[5] (a) Prove that if G is abelian, then H is abelian.

[5] (b) Prove that if G is cyclic, then H is cyclic.

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[10] 3. Let $\mathbb{C}^*$ be the group all complex numbers except 0 with multiplication. Prove that $\mathbb{C}^*$ is isomorphic to the group H where $H = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \,\middle|\, a^2 + b^2 \neq 0\,,\ a, b \in \mathbb{R} \right\}$ is a subgroup of $GL_2(\mathbb{R})$.

4. Short answer questions:

[4] (a) Express the permutation $(1\,2\,3\,4)(4\,5)(1\,23)^{-1}$ as a single cycle.

[3] (b) List three elements of the alternating group A_4.

[3] (c) Let G be a group of order 18 and let H be a subgroup of G. What is the possible order of H?

[4] (d) Explain why $\mathbb{Z}_4$ and $\mathbb{Z}_2 \times \mathbb{Z}_2$ are not isomorphic.

[4] (e) Explain why for a subgroup H of a group G if $g_1 \in g_2H$ and $g_2 \in g_1H$ then $g_1H = g_2H$ (where $g_1, g_2 \in G$).

[3] (f) The number of elements of $U(\mathbb{Z}_{31})$ is $\cdots$ and a generator for this group is $\cdots$.

Answers:

Q1

$\circ$	e	s	r	r^2	r^3	sr	sr^2	sr^3
e	e	s	r	r^2	r^3	sr	sr^2	sr^3
s	s	e	sr	sr^2	sr^3	r	r^2	r^3
r	r	sr^3	r^2	r^3	e	s	sr	sr^2
r^2	r^2	sr^2	r^3	e	r	sr^3	s	sr
r^3	r^3	sr	e	r	r^2	sr^2	sr^3	s
sr	sr	r^3	sr^2	sr^3	s	e	r	r^2
sr^2	sr^2	r^2	sr^3	s	sr	r^3	e	r
sr^3	sr^3	r	s	sr	sr^2	r^2	r^3	e

Q2 (a) See the textbook page 115.

(b) Let $G = \langle a \rangle$ where $a \in G$ and let $\phi(a) = b$. Show that $H = \langle b \rangle$.

Q3 Define $\phi : \mathbb{C}^* \leftarrow H$ such that $\phi(a + bi) = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$. Show that ϕ is 1-1, onto and preserves the group operations.

Q4 (a) $(1\,4\,5)$.

(b) (1), $(1\,2\,3)$, $(1\,2\,4)$. In fact any cycle of length 3 in S_4 is also an answer.

(c) $1, 2, 3, 6, 9$ and 18.

(d) $\mathbb{Z}_4$ is cyclic but $\mathbb{Z}_2 \times \mathbb{Z}_2$ is not cyclic because all elements have order at most 2.

(e) If $g_1 = g_2$ then $g_1H = g_2H$. If $g_1 \neq g_2$ then since $g_1 \in g_2H$ so $g_1H \subset g_2H$ and since $g_2 \in g_1H$ so $g_2H \subset g_1H$. Therefore $g_1H = g_2H$.

(f) A generator is $\bar{3}$. Also any of $\bar{1}\bar{1}, \bar{1}\bar{2}, \bar{1}\bar{3}, \bar{1}\bar{7}, \bar{2}\bar{1}, \bar{2}\bar{2}$ and $\bar{2}\bar{4}$.