Term Test 1

DATE: February 14, 2014
COURSE: MATH 2300

TIME: 60 minutes
EXAMINER: G.I. Moghaddam

NAME:______________________________

STUDENT # : ______________________

Q1	Q2	Q3	Q4	Q5	Total (out of 35)

[8] 1. For each of the following statements determine whether it is True or False.

(a) There is a vector space which contains only two vectors.

(b) Every subset of a vector space $\mathbf{V}$ that contains the zero vector in $\mathbf{V}$ is a subspace of $\mathbf{V}$.

(c) The set of 2×2 matrices, that each matrix contains exactly two 1's and two 0's, is a linearly dependent set in $\mathbf{M}_{22}$.

(d) Every linearly independent subset of a vector space $\mathbf{V}$ is a basis for $\mathbf{V}$.

(e) The coordinate vector of a vector $\mathbf{x}$ in $\mathbf{R}^n$ relative to the standard basis for $\mathbf{R}^n$ is $\mathbf{x}$.

(f) Every set of vectors that spans $\mathbf{R}^n$ contains a basis for $\mathbf{R}^n$.

(g) There is a set of 18 linearly independent vectors in $\mathbf{R}^{17}$.

(h) If $\mathbf{W}$ is a subspace of $\mathbf{R}^n$, then $\mathbf{W} \cap \mathbf{W}^{\perp} = \{\mathbf{0}\}$.

[5] 2. Let $\mathbf{U}$ and $\mathbf{W}$ be subspaces of a vector space $\mathbf{V}$. Prove that $\mathbf{U}+\mathbf{W}$ is also a subspace of $\mathbf{V}$ where

$$\mathbf{U}+\mathbf{W}=\left\{\mathbf{u}+\mathbf{w} \,\middle|\, \mathbf{u}\in\mathbf{U}, \mathbf{w}\in\mathbf{W}\right\}.$$

[8] 3. Let $B_1 = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ and $B_2 = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be the bases for $\mathbf{R}^3$ in which $\mathbf{u}_1 = (1,0,0)$, $\mathbf{u}_2 = (0,1,0)$, $\mathbf{u}_3 = (2,5,1)$, and $\mathbf{v}_1 = (1,2,0)$, $\mathbf{v}_2 = (3,7,0)$, $\mathbf{v}_3 = (0,0,1)$.

(a) Find the transition matrix from B_1 to B_2.

(b) Let $\mathbf{w} = (3,6,1)$; first find $[\mathbf{w}]_{B_1}$ and then use the transition matrix obtained in part (a) to compute $[\mathbf{w}]_{B_2}$ by matrix multiplication.

[5] 4. Find a basis and the dimension of W as a subspace of the vector space $\mathbf{P}_3$ where

$$W = \{a + ax^2 + bx^3 \,|\, a, b \in \mathbf{R}\}$$

[9] 5. For the matrix $A = \begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & -1 & -3 & 1 & 3 \\ -2 & -1 & 1 & -1 & 3 \\ 0 & 3 & 9 & 0 & -12 \end{bmatrix}$, it is given that the reduced row echelon form of A is $R = \begin{bmatrix} 1 & 0 & -2 & 0 & 1 \\ 0 & 1 & 3 & 0 & -4 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Find a basis for the column space of A.

(b) Find a basis for the null space of A.

(c) Find the rank and the nullity of A and also the nullity of A^T.

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Answers

Q1: F F T F T T F T

Q2: One must prove that the set $\mathbf{U}+\mathbf{W}$ is not empty and it is closed under addition and scalar multiplication.

Q3:

(a) $P_{B_1 \to B_2} = \begin{bmatrix} 7 & -3 & -1 \\ -2 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

(b) $[\mathbf{w}]_{B_1} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $[\mathbf{w}]_{B_2} = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$.

Q4: $\{1+x^2\,, x^3\}$ is a basis for W and $dim(W) = 2$.

Q5:

(a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix} \right\}$ is a basis for the column space of A.

(b) $\{(2,-3,1,0,0)\ , (-1,4,0,1,1)\}$ is a basis for the null space of A.

(c) $rank(A) = 3\,,\ nullity(A) = 2\,,\ nullity(A^T) = 1$.