Term Test 2

DATE: March 14, 2014
TIME: 60 minutes
COURSE: MATH 2300
EXAMINER: G.I. Moghaddam

NAME:____

STUDENT # : _________________________

Q1 (6 marks)	Q2 (6 marks)	Q3 (8 marks)	Q4 (5 marks)	Q5 (10 marks)	Total (out of 35)

[6] 1. For each of the following statements determine whether it is True or False.

(a) The eigenvalues of a matrix A are the same as the eigenvalues of the reduced row echelon form of A .

(b) Every orthogonal set of nonzero vectors in an inner product space is linearly independent.

(c) In an inner product space, there exist two distinct vectors $\mathbf{u}$ and $\mathbf{v}$ such that $d(\mathbf{u}, \mathbf{v}) = 0$

(d) If $\mathbf{u}$ and $\mathbf{v}$ are vectors in an inner product space, then $\langle \mathbf{u}, \mathbf{v} \rangle = 0$ if and only if $\mathbf{u} = 0$ or $\mathbf{v} = 0$.

(e) Let $\mathbf{W}$ be a subspace of an inner product space $\mathbf{V}$. If $\mathbf{u}$ and $\mathbf{v}$ are in $\mathbf{W}$ and $\mathbf{w}$ is in $\mathbf{W}^{\perp}$ then $\langle \mathbf{u} + \mathbf{v}, \mathbf{w} \rangle = 0$.

(f) If the characteristic polynomial of a square A is $P(\lambda) = (\lambda^2+1)(\lambda^4-1)$, then A is of size 6×6 and $det(A) = 1$.

2. In the vector space $\mathbf{P}_3$, define the inner product of $\mathbf{p}$ and $\mathbf{q}$ by

$$\langle \mathbf{p}, \mathbf{q} \rangle = a_0 b_0 + a_1 b_1 + a_2 b_2 + a_3 b_3 ,$$

where $\mathbf{p} = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ and $\mathbf{q} = b_0 + b_1 x + b_2 x^2 + b_3 x^3$.

If $\mathbf{p} = 1 + x^3$ and $\mathbf{q} = -1 + x$ then:

[2] (a) Is $\mathbf{p}$ orthogonal to $\mathbf{q}$? Explain.

[2] (b) Find $d(\mathbf{p}\,,\,\mathbf{q})$.

[2] (c) Find the angle between $\mathbf{p}$ and $\mathbf{q}$.

3. Let $A = \begin{bmatrix} -7 & 0 & 3 \\ 9 & 2 & -3 \\ -18 & 0 & 8 \end{bmatrix}$. It is given that

$$\det(\lambda I - A) = (\lambda + 1)\,(\lambda - 2)^2\,;$$

also it is given that the eigenspaces of A are

$W_1 = \{\, t(1,-1,2) \mid t \in \mathbf{R} \,\}$ and $W_2 = \{\, t(1,0,3)+s(0,1,0) \mid t, s \in \mathbf{R} \,\}$.

[3] (a) Find a matrix P that diagonalizes A. Explain your work.

[5] (b) Find $A^6 P$ where P is the matrix obtained in part (a).

[5] 4. Let V be an inner product space. Show that if $\mathbf{u}$ and $\mathbf{v}$ are orthogonal unit vectors in V, then $\|3\mathbf{u} - 4\mathbf{v}\| = 5$.

[10] 5. Let $\mathbf{R}^3$ have the inner product $\langle \mathbf{u}, \mathbf{v} \rangle = a_1 b_1 + 2\, a_2 b_2 + a_3 b_3$, where $\mathbf{u} = (a_1, a_2, a_3)$ and $\mathbf{v} = (b_1, b_2, b_3)$. Use the Gram-Schmidt process to transform a basis

$$\{\, \mathbf{u}_1 = (0, -1, 0)\,,\ \mathbf{u}_2 = (1, 3, -1)\,,\ \mathbf{u}_3 = (1, 1, -2)\,\}$$

into an orthonormal basis.

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Answers

Q1: F T F F T F

Q2:

(a) No it is not.

(b) $d(\mathbf{p}\,,\,\mathbf{q}) = \sqrt{6}\,.$

(c) $\theta = \frac{2\pi}{3}\,.$

Q3:

(a) $P = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & 3 & 0 \end{bmatrix}.$

(b) $A^6 P = \begin{bmatrix} 1 & 64 & 0 \\ -1 & 0 & 64 \\ 2 & 192 & 0 \end{bmatrix}.$

Q4: Expand $\|3\mathbf{u} - 4\mathbf{v}\|^2 = \langle 3\mathbf{u} - 4\mathbf{v}, 3\mathbf{u} - 4\mathbf{v} \rangle\,.$

Q5: $\{\,(0, \frac{-1}{\sqrt{2}}, 0)\,,\,(\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}})\,,\,(\frac{-1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}})\,\}$ is an orthonormal basis.