Term Test 1

DATE: February 7, 2012
COURSE: MATH 2132

TIME: 70 minutes
EXAMINER: G.I. Moghaddam

NAME:____________________________________

STUDENT # : ________________________

Q1	Q2	Q3	Q4	Q5	Total (out of 50)

1. Let $\{f_n(x)\}$ be a sequence of functions whose first four terms are

$$f_1(x) = 2 + x^2 \cos 2x\,, \qquad f_2(x) = 2 + \frac{1}{2}\,x^4 \cos 4x\,,$$
$$f_3(x) = 2 + \frac{1}{6}\,x^6 \cos 6x\,, \qquad f_4(x) = 2 + \frac{1}{24}\,x^8 \cos 8x\,,$$

where $-\infty < x < \infty$.

[3] (a) Find the general term of the sequence that is $f_n(x)$.

[5] (b) Determine if the sequence converges. If yes, find the limit function.

[9] 2. Find the value of c such that the 3^{rd} - term of the Taylor series of $f(x) = \dfrac{1}{1+x}$ about c is $8x^2 + 8x + 2$. Show your work.

[9] 3. For $f(x) = \dfrac{1}{3+2x}$, it is given that $f^{(n)}(x) = \dfrac{(-1)^n\, n!\, 2^n}{(3+2x)^{n+1}}$ where $n \geq 1$. Find the n^{th} Taylor's remainder $R_n(0,x)$ and show that $\lim_{n\to\infty} R_n(0,x) = 0$ for $0 < x < \dfrac{3}{2}$.

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[9] 4. Find the radius of convergence and the open interval of convergence for the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n\,(3n)!}{2^n\,n!\,(2n+1)!}\;x^{3n}.$$

[15] 5. Find the Taylor series about 2 for the function

$$f(x) = \frac{x^2 - 4x - 2}{x^2 + x}\,.$$

Express your answer in sigma notation and simplify as much as possible. Determine the open interval of convergence.

(**Hint**: $\dfrac{x^2 - 4x - 2}{x^2 + x} = \dfrac{x-2}{1+x} - \dfrac{2}{x}\,.$)

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ANSWERS

Q1-a $f_n(x) = 2 + \frac{1}{n!} x^{2n} \cos 2nx$

Q1-b $\lim_{n\to\infty} f_n(x) = 2, \quad -\infty < x < \infty$

Q2 $c = -\frac{1}{2}$

Q3 First show that $0 < \dfrac{2x}{3+2z_n} < 1$ and then use squeeze theorem.

Q4 $R_x = \frac{2}{3}, \quad -\frac{2}{3} < x < \frac{2}{3}.$

Q5 $f(x) = 1 + \sum_{n=0}^{\infty} (-1)^{n-1} (\frac{1}{3^n} + \frac{1}{2^n})(x-2)^n, \quad 0 < x < 4.$