**Term Test 1**



DATE: February 12, 2013 TIME: 70 minutes
COURSE: MATH 2132 EXAMINER: G.I. Moghaddam & M. Virgilio

NAME:_____

STUDENT # : _____

❑ A01 G.I. Moghaddam ❑ A02 M. Virgilio

| Q1 | Q2 | Q3 | Q4 | Q5 | Total (out of 50) |
|---|---|---|---|---|---|
| | | | | | |

[9] 1. Find all values of $x$ for which the sequence of functions

$$\left\{\frac{(x^2-1)n^2 + (x^2+5x+6)n}{(x+1)n^2 + (x-1)n + 1}\right\}_{n=1}^{\infty}$$

is convergent. Find the limit function for those values of $x$.

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[8] 2. Find the radius of convergence and the open interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n \, 3^n \, n^3}{\ln(n+2)} \, x^{2n} \, .$$

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[12] 3. Let $f(x) = e^{3x}$ for $-\infty < x < \infty$. Then:

[4] (a) Find the first 4 terms of the Taylor series of $f(x)$ about $-2$.

[4] (b) Find $R_n(-2, x)$ ( i.e. the $n^{th}$-remainder with $c = -2$ ).

[4] (c) Show that $\lim_{n\to\infty} R_n(-2, x) = 0$ *only* for the case $x > -2$.

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[6] 4. Find the value of **a** for which the sum of the series

$$\frac{2\mathbf{a}}{\sqrt{3}}\,x^2 - \frac{4\mathbf{a}}{3}\,x^4 + \frac{8\mathbf{a}}{3\sqrt{3}}\,x^6 - \frac{16\mathbf{a}}{9}\,x^8 + \cdots$$

is $\dfrac{20x^2}{\sqrt{3} + 2x^2}$.

[15] 5. Find the Taylor series about 2 for the function

$$f(x) = \frac{x-3}{x^2}\,.$$

Express your answer in sigma notation and simplify as much as possible. Determine the open interval of convergence.

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# ANSWERS

Q1 $f(x) = -1$ if $x = -1$, and $f(x) = x - 1$ if $x \neq -1$.

Q2 $R_x = \dfrac{1}{\sqrt{3}}, \quad -\dfrac{1}{\sqrt{3}} < x < \dfrac{1}{\sqrt{3}}.$

Q3-a $e^{-6} + 3e^{-6}(x+2) + \dfrac{9}{2}e^{-6}(x+2)^2 + \dfrac{9}{2}e^{-6}(x+2)^3$

Q3-b $R_n(-2, x) = \dfrac{3^{n+1}e^{3z_n}}{(n+1)!}(x+2)^{n+1}$ where $z_n$ is between $-2$ and $x$.

Q3-c First show that $e^{3z_n} < e^{3x}$ and then use squeeze theorem.

Q4 $a = 10$.

Q5 $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(3n+1)}{2^{n+2}}(x-2)^n, \quad 0 < x < 4.$