Term Test 1

DATE: February 14, 2014
TIME: 60 minutes
COURSE: MATH 2300
EXAMINER: G.I. Moghaddam

NAME:____

STUDENT # : ____________________________

Q1	Q2	Q3	Q4	Q5	Total (out of 35)

[8] 1. For each of the following statements determine whether it is True or False.

(a) There is a vector space which contains only two vectors.

(b) Every subset of a vector space $\mathbf{V}$ that contains the zero vector in $\mathbf{V}$ is a subspace of $\mathbf{V}$.

(c) The set of 2×2 matrices, that each matrix contains exactly two 1's and two 0's, is a linearly dependent set in $\mathbf{M}_{22}$.

(d) Every linearly independent subset of a vector space $\mathbf{V}$ is a basis for $\mathbf{V}$.

(e) The coordinate vector of a vector $\mathbf{x}$ in $\mathbf{R}^n$ relative to the standard basis for $\mathbf{R}^n$ is $\mathbf{x}$.

(f) Every set of vectors that spans $\mathbf{R}^n$ contains a basis for $\mathbf{R}^n$.

(g) There is a set of 18 linearly independent vectors in $\mathbf{R}^{17}$.

(h) If $\mathbf{W}$ is a subspace of $\mathbf{R}^n$, then $\mathbf{W} \cap \mathbf{W}^{\perp} = \{\mathbf{0}\}$.

[5] 2. Let $\mathbf{U}$ and $\mathbf{W}$ be subspaces of a vector space $\mathbf{V}$. Prove that $\mathbf{U}+\mathbf{W}$ is also a subspace of $\mathbf{V}$ where

$$\mathbf{U}+\mathbf{W} = \{\mathbf{u}+\mathbf{w} \mid \mathbf{u}\in\mathbf{U}, \mathbf{w}\in\mathbf{W}\}\,.$$

[8] 3. Let $B_1 = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ and $B_2 = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be the bases for $\mathbf{R}^3$ in which $\mathbf{u}_1 = (1, 0, 0)$, $\mathbf{u}_2 = (0, 1, 0)$, $\mathbf{u}_3 = (2, 5, 1)$, and $\mathbf{v}_1 = (1, 2, 0)$, $\mathbf{v}_2 = (3, 7, 0)$, $\mathbf{v}_3 = (0, 0, 1)$.

(a) Find the transition matrix from B_1 to B_2.

(b) Let $\mathbf{w} = (3, 6, 1)$; first find $[\mathbf{w}]_{B_1}$ and then use the transition matrix obtained in part (a) to compute $[\mathbf{w}]_{B_2}$ by matrix multiplication.

[5] 4. Find a basis and the dimension of W as a subspace of the vector space $\mathbf{P}_3$ where

$$W = \{a + ax^2 + bx^3 \,|\, a, b \in \mathbf{R}\}$$

[9] 5. For the matrix $A = \begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & -1 & -3 & 1 & 3 \\ -2 & -1 & 1 & -1 & 3 \\ 0 & 3 & 9 & 0 & -12 \end{bmatrix}$, it is given that

the reduced row echelon form of A is $R = \begin{bmatrix} 1 & 0 & -2 & 0 & 1 \\ 0 & 1 & 3 & 0 & -4 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Find a basis for the column space of A.

(b) Find a basis for the null space of A.

(c) Find the rank and the nullity of A and also the nullity of A^T.