UNIVERSITY OF MANITOBA

DATE: December 10 , 2010

FINAL EXAMINATION

TITLE PAGE

EXAMINATION: Multivariable Calculus

TIME: 3 hours

COURSE: MATH 2720

EXAMINER: G.I. Moghaddam

NAME: (Print in ink) ______________________________

STUDENT NUMBER: ______________

SEAT NUMBER: ______________

SIGNATURE: (in ink) ______________________________

(I understand that cheating is a serious offense)

INSTRUCTIONS TO STUDENTS:

This is a 3 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 11 pages of questions and also 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staple.

The value of each question is indicated in the left hand margin beside the statement of the question. The total value of all questions is 120 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	14	
2	8	
3	10	
4	8	
5	10	
6	12	
7	8	
8	8	
9	10	
10	10	
11	10	
12	12	
Total:	120	

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[14] 1. Let C be the curve with vector equation $\mathbf{r}(t) = < \frac{1}{3}t^3,\ t^2,\ 2t\ >$; and let $P(\frac{1}{3}, 1, 2)$ be a point on the curve C.

(a) Find the unit tangent vector of the curve C at the point P.

(b) Find the unit normal vector of the curve C at the point P.

(c) Find the arc length of the curve C from the origin to the point P.

(d) Find equation of the normal plane of the curve C at the point P.

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[8] 2. Show that $f(x, y) = x\,e^{2x-y}$ is differentiable at $(1, 2)$ and find its linearization. Then use it to approximate $f(1.01,\, 2.03)$.

[10] 3. Evaluate each of the following limit or explain why it does not exist. Show your work.

(a) $$\lim_{(x,y)\to(1,0)} \frac{2\,x^4 + x^2y^2 - 2\,x^2 - y^2}{2\,x^2y^2 - x^2 - 2\,y^2 + 1}$$

(b) $$\lim_{(x,y)\to(0,0)} (\sqrt{x^2+y^2}\,)\ \ln(x^2+y^2)$$

[8] 4. Let $u = r^3 + 1$ and $r = \sqrt{x^2 + y^2 + z^2}$. Show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2.$$

[10] 5. If $f(u)$ and $g(v)$ are differentiable functions, find the value of

$$\nabla f(x^2 - 2y^2) \bullet \nabla g(x^2 y).$$

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[12] 6. Find the absolute maximum and the absolute minimum of the function

$$f(x, y) = x^2 + 2xy + 3y^2$$

over the closed triangular region with vertices $(-1, 1)$, $(2, 1)$ and $(-1, -2)$.

[8] 7. Evaluate the following double integral.

$$\int_0^2 \int_0^{\sqrt{2-y}} e^{(2x-\frac{1}{3}x^3)}\, dx\, dy$$

[8] 8. Evaluate the following triple integral.

$$\int_0^1 \int_1^{x^2} \int_0^{x+3y} (2x^2y)\, dz\, dy\, dx$$

[10] 9. Find the volume of the solid bounded by the circular paraboloids $z = 2 - x^2 - y^2$ and $z = x^2 + y^2$.

[10] 10. Consider a thin plate with mass per unit area $\rho(x, y) = x^2 + y$ such that the edges of the plate are defined by the parabola $y = (x-2)^2$ and the line $y = x$. **Set up but do not evaluate** double integrals for each of the following :

(a) Moment of inertia of the plate about the origin.

(b) Center of mass of the plate.

[10] 11. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y, z) = \sin x\,\mathbf{i} + \cos y\,\mathbf{j} + xz\,\mathbf{k}$ and C is the curve with vector representation

$$\mathbf{r}(t) = t^3\,\mathbf{i} - t^2\,\mathbf{j} + t\,\mathbf{k}, \qquad 0 \le t \le 1.$$

[12] 12. Let C be the counterclockwise boundary of the region

$$D = \{(x, y) \mid 0 \leq y \leq \sqrt{4 - x^2}\,\}\,.$$

(a) Sketch C. Is C a simple closed curve ? Why?

(b) Using **Green's Theorem** , evaluate the line integral

$$\oint_C (\,1 - 3x^4 y\,)\,dx \;+\; (\,3xy^4 + 2x^3 y^2\,)\,dy\,.$$

Answers:

Q1) (a) $\hat{T}(1) = < \frac{1}{3}, \frac{2}{3}, \frac{2}{3} >$, (b) $\hat{N}(1) = < \frac{2}{3}, \frac{2}{3}, -\frac{1}{3} >$
(c) $\frac{7}{3}$, (d) $3x + 6y + 6z = 19$.

Q2) $L(x, y) = 3x - y$ and $f(1.01, 2.03) \approx 1$.

Q3) (a) limit is -2 , (b) limit is 0.

Q4)

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = (3r^2)^2 = \left(\frac{\partial u}{\partial r}\right)^2.$$

Q5) $\nabla f(x^2 - 2y^2) \bullet \nabla g(x^2 y) = 0$.

Q6) Maximum of f is 11 which occurs at $(2, 1)$.
Minimum of f is 0 which occurs at $(0, 0)$.

Q7) $\int_0^2 \int_0^{\sqrt{2-y}} e^{(2x - \frac{1}{3}x^3)} \, dx \, dy = e^{\frac{4\sqrt{2}}{3}} - 1$.

Q8) $\int_0^1 \int_1^{x^2} \int_0^{x+3y} (2x^2 y) \, dz \, dy \, dx = -\frac{41}{72}$

Q9) $V = \pi$

Q10) (a) $I_O = \displaystyle\int_1^4 \int_{(x-2)^2}^{x} (x^2 + y^2)(x^2 + y) \, dy \, dx$,
(b) $m = \displaystyle\int_1^4 \int_{(x-2)^2}^{x} (x^2 + y) \, dy \, dx$ and $\bar{x} = \displaystyle\frac{1}{m} \int_1^4 \int_{(x-2)^2}^{x} x(x^2 + y) \, dy \, dx$ and $\bar{y} = \displaystyle\frac{1}{m} \int_1^4 \int_{(x-2)^2}^{x} y(x^2 + y) \, dy \, dx$

Q11) $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r} = \frac{6}{5} - \cos 1 - \sin 1$

Q12) (a) It is closed and simple (it does not intersect itself).
(b) $\displaystyle\oint_C (1 - 3x^4 y) \, dx + (3xy^4 + 2x^3 y^2) \, dy = 32\pi$