PAGE: 1 of 5 TIME: <u>70 minutes</u> EXAMINER: G.I. Moghaddam

NAME:

STUDENT # : _____

Q1 [10]	Q2 [10]	Q3 [10]	Q4 [10]	Q5 [10]	Total [50]

1. Evaluate each of the following limits or explain why it does not exist.

[5] (a)
$$\lim_{(x,y)\to(1,0)} \frac{(x-1)y}{y^2+(x-1)^2}$$

[5] (b)
$$\lim_{(x,y)\to(0,0)} \frac{x^2 y^4}{4x^2 + y^2}$$

2. Let $f(x, y) = 1 + x \ln(xy - 1)$.

[3] (a) Show that the function f is differentiable at the point (2,1).

[4] (b) Find the linearization L(x, y) of the function f at the point (2, 1).

[3] (c) Use part (b) to approximate f(2.01, 0.99).

 $\begin{array}{c} \text{DATE: November 12, 2015} \\ \text{COURSE: } \underline{\text{MATH 2720}} \end{array}$

PAGE: 3 of 5 TIME: <u>70 minutes</u> EXAMINER: <u>G.I. Moghaddam</u>

[10] 3. If $g(s,t) = f(2t^3 - s^3, s^3 - 2t^3)$ and f is differentiable, show that g satisfies the equation

$$2t^2g_s \,+\, s^2g_t = 0 \;\;.$$

PAGE: 4 of 5 TIME: <u>70 minutes</u> EXAMINER: G.I. Moghaddam

- 4. Let $P(\sqrt{2}, 0, \sqrt{2})$ be a point and suppose that the temperature at a point (x, y, z) in space is given by $T(x, y, z) = e^{x^2 + y^2 z^2}$.
- [6] (a) Find the rate of change of the temperature at the point P in the direction of the vector $\mathbf{u} = (1, -4, -1)$.

[2] (b) In which direction does the temperature increases fastest at P?

[2] (c) Find the maximum rate of increase at P.

PAGE: 5 of 5 TIME: <u>70 minutes</u> EXAMINER: G.I. Moghaddam

[10] 5. Find and classify all critical points of $f(x,y) = 3xy - x^2y + xy^2$.