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Biologically-Inspired Adaptive Learning: A Near Set Approach

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***Abstract*—The problem considered in this paper is how learning by machines can be influenced beneficially by various forms of learning by biological organisms. The solution to this problem is partially solved by considering considering a model of perception that is at the level of classes in a partition defined by a particular equivalence relation in an approximation space. This form of perception provides a basis for adaptive learning that has surprising acuity. Viewing approximation spaces as the formal counterpart of perception was suggested by Ewa Orłowska in 1982. This view of perception grew out the discovery of rough sets by Zdzisław Pawlak during the early 1980s. The particular model of perception that underlies biologically-inspired learning is based on a near set approach, which considers classes of organisms with similar behaviours. In this paper, the focus is on learning by tropical fish called glowlight tetra (Hemigarmmus erythrozonus). Ethology (study of the comparative behaviour of organisms), in particular, provides a basis for the design of an artificial ecosystem useful in simulating the behaviour of fish. The contribution of this paper is a complete framework for an ethology-based study of adaptive learning defined in the context of nearness approximation spaces.**

***Index Terms*— Approximate adaptive learning, behaviour, ethology, machine learning, near set, observation, perception.**

An approximation space ... serves as a formal counterpart of perception ability or observation.
– Ewa Orłowska, March, 1982.

I. Introduction

The problem considered in this paper is how learning by machines can be influenced beneficially by various forms of learning by biological organisms. The solution to this problem hearkens back to work done during the early 1980s by Oliver Selfridge [34] on primitive adaptive learning control strategies by biological organisms and work on delayed rewards during learning during the late 1980s by Chris J.C.H. Watkins [44]. The form of biologically-inspired machine learning introduced in this article also calls attention to the importance of work on classification of objects by Zdzisław Pawlak [15], [16], [17] and by Ewa Orłowska [12] on approximation spaces as formal counterparts of perception and observation. The proposed form of adaptive learning is carried within the framework of what is known as a nearness approximation space and a near set approach to approximating sets of perceptual objects. The basic idea underlying near sets is perceptual synthesis, which is closely related to observations made during the 1930s by the Bengali Nobel Prize winner Rabindranath Tagore and Albert Einstein about perceptible combinations of unperceptible minute elements [38]. It is rough ethology [30], [32] based on the pioneering work on ethology by Niko Tinbergen [39], [40], [41], [42] starting in 1940s that provides a basis for a biologically-inspired approach to approximate adaptive learning. Organism behaviour descriptions are stored in a form of short term memory called an ethogram. An *ethogram* is a set of comprehensive descriptions of the characteristic behaviour patterns of a species [3], [6].

Ethograms are one of the basic tools used in Ethology (comparative study of animal behaviour [10]) introduced by Niko Tinbergen [42]. This paper includes the results of a recent study of swarm behavior by the tropical fish Hemigarmmus erythrozonus commonly known as Glowlight tetra in design of an artificial ecosystem useful in studying the behaviour of robot societies. This ecosystem makes it possible to observe and explain the behavior of biological organisms that carries over into the study of what is known as approximate adaptive learning by interacting robotic devices. This work is a continuation of the study of approximate adaptive learning introduced in [26], elaborated in [18] and applied in a number of studies of biologically-inspired reinforcement learning [7], [8], [19], [20], [21], [28]. The form of adaptive learning reported in this paper has grown out of the study of nearness in approximation spaces [21], [24], and the near set approach to set approximation [4], [22], [25], [23]. The contribution of this paper is a complete framework an ethology-based study of adaptive learning defined in the context of nearness approximation spaces.

This paper is organized as follows. Sect. II briefly describes the physiology and behaviour of glowlight tetra freshwater fish. The actions included in the ecosystem simulation are described in Sect. III. A brief overview of near objects, near sets and nearness approximation spaces is given in Sect. IV. Adaptive learning algorithms and the results of learning experiments are given in Sections V and VI.

II. Glowlight Tetra Freshwater Fish

Hemigarmmus erythrozonus commonly known as Glowlight tetra, is a freshwater species living in South America. This fish has a silver color with iridescent orange to red stripe that extends from the snout to the base of its tail. The front part of the dorsal fins are the same color as the stripe and other fins are silver to transparent. Adult glowlight tetra are 4cm (max. 5 cm) in length. Among all small tetras, Glowlight tetra is the most active. They like to be in shoals (groups of four to eight or more) to feel secure. This small fish prefers a well-planted tank for hiding as well as some open water for free swimming. When a potential predator is around, these fish tend to swim in smaller groups. The female Glowlight tetra is usually longer than the male and can be 4.5 to 5 cm in length.

Glowlight tetra are very sensitive to the amount of acid in the water, so an acidic pH of 6.8 in the range of 6.0 - 7.5 is suitable survival. The water should be soft to slightly hard (d^o GH of 6^o to 15^o). The ecosystem for this study consists of a collection of glowlight tetras which are kept in an aquarium filled with fresh water (see Fig 1).

1.1: Sample Glowlight Tetra Population

1.2: The fish

Fig. 1. Glowlight tetra

III. Actions, States, and Rewards

This section gives an overview of the observed actions, states and rewards for Glowllight tetra that provided a basis for the study of learning reported in this paper.

A. Actions

The following actions have been included in this study.

- *Exploring*. Fish have the habit of exploring their environment in terms of finding and memorizing the location of different objects and forming a mental map of those objects based on spatial orientation [37]. In fact, some fish like gold fish are able to encode and use different kinds of geometry and spatial information to orient and navigate themselves [43]. Moreover, it is natural for fish to swim freely to the different places in their environment in the absence of any potential sources of danger or stress such as predators.

In this simulation, we have taken into account these two abilities. We use a random number generator so that fish move in a random fashion. In this way, we represent the ability of fish to explore their environment freely and more naturally. Also, geometric information such as places to hide and borders of the tank, has been saved in the memory of the simulated fish. In other words, our simulation takes into account the ability of the fish to recall the information contained in its mental geometric map.

- *Searching for food*. Whenever a fish feels hungry, it starts to search for food; and to do so, it starts looking for food using its vision and other senses, which is in keeping with Endler's sensory drive hypothesis, *i.e.*, fish tune to their environment to drive receiver perception, signal production and preference for a signal [9]. As soon as a fish finds the location of food, it goes toward it and starts to eat the food. In our simulation, we tried to mimic this behaviour by letting fish know about various positions of food. We use an Euclidean distance function to calculate and find the nearest place of food. After the fish finds the nearest place of food, this position is set as a goal and the fish then moves toward the set position.

However, it is also possible for fish to *search* for food even if it they are not hungry and do not need to eat. In our simulation, it is possible for the fish to choose the actions 'searching for food' and 'eating' at any time that it wants. • *Eating*. If the fish finds food, it opens its mouth and eats food and gets more energy. But sometimes it happens that a fish opens its mouth just a little bit sooner than the instant when it reaches the food. In that case, a fish misses the food and stays hungry. To avoid this in our simulation, we only reward the finding-food action if the fish is at the exact position of the food.

- *Resting (Being idle)*. When fish is idle, it has a lower metabolism than when it does other activities and also it has no movement. This action is almost equal to sleeping or resting. However, the sleep process in organisms such as fish is completely different from sleeping or resting in humans. This is, since fish have only a rudimentary neocortex, it is unlikely that a fish neocortex can generate brain waves that are characteristic in mammalian, slow-wave sleep [37].
- *Searching for a place to hide*. Whenever a fish does not feel safe, it needs to find a place to hide. The process of searching for a place to hide is similar to searching for food.
- *Hiding*. Once a fish has found a suitable place to hide, it needs to stay there and be quiet until the predator goes away.
- *Freezing*. Sometimes it happens that a fish does not have enough time to go to a place and hide, since the predator is quite near. So, the fish pretends that it is not moving and freezes so that predator is unable to distinguish it from the other objects in the environment.

B. States

States are based on the energy level of fish at each time step. *Energy level* is the amount of energy (Calorie) per 1 gram of fish body weight at each instant of time. Let $e \in \Re$ be the energy level of fish and $s \in S$ be the states. In our study, a hypothetical fish starts with an energy level of 10. Each action decreases its energy level by 0.1 with one exception, namely, eating which increases the its energy level by 2. It should be observed that there is an alternative way to estimate energy level based on the estimated energy present combined with different factors of environment such as light, temperature and absence or presence of predator, *i.e.*, a weighted sum can be used to estimate energy level. This more refined view of energy level estimation is outside the scope of this paper. In the study reported in this paper, three states are considered.

- No Hunger: If $e \geq 7$, then the current state $s = No\ Hunger$. This means that the fish is not hungry.
- Medium Hunger: If 3$\leq e \leq 7$ then the current state $s = Medium\ Hunger$. This means that the fish is hungry but still can wait to eat food. This is useful when a predator is around.
- Critical Hunger: If $e < 3$ then the current state $s = Critical\ Hunger$. This means that the fish needs to eat immediately.

C. Reward function

Let A ={1,2,3,4,5,6,7} be the set of actions that the fish performs. Each action $a \in A$ denotes exploring, eating, searching for food, resting, hiding, freezing and searching for a place to hide, respectively. Let u, e, th denote energy measure function, energy level, and threshold, respectively defined as follows:

$$u_{th}(e) = \begin{cases} 1, & \text{if } e \geq th, \\ 0, & \text{if } e < th, \end{cases}$$

and

$$\delta_{ij} = \begin{cases} 1, & \text{if } i = j, \\ 0, & \text{if } j \neq j, \end{cases}$$

where $u_t(e)$ is the step function and δ_{ij} is the Kronecker delta function [2], [5]. Also, given probe functions $\phi_0, \phi_1, \phi_2 : \Re \longrightarrow [0, 1]$, assume that weight m_i varies randomly so that

$$m_i = \begin{cases} 0 \leq m_i < 0.5, \text{if } i = 0, 3, 8, 9, 15, 16, \\ 0.5 \leq m_i \leq 1, \text{if } i = 1, 2, 7, 10, 14, 17, \\ 0, \quad \text{otherwise.} \end{cases}$$

The functions ϕ_0, ϕ_1, ϕ_2 are defined as

$$\begin{aligned} \phi_0(a, e) &= [m_0\delta_{1a}, m_1\delta_{2a}, \ldots, m_5\delta_{6a}, m_6\delta_{7a}], \\ \phi_1(a, e) &= [m_7\delta_{1a}, m_8\delta_{2a}, \ldots, m_{12}\delta_{6a}, m_{13}\delta_{7a}], \\ \phi_2(a, e) &= [m_{14}\delta_{1a}, m_{15}\delta_{2a}, \ldots, m_{19}\delta_{6a}, m_{20}\delta_{7a}]. \end{aligned}$$

Weight $m_i \in (0, 0.5)$ results in a lower reward for a particular action and $m_i \in [0.5, 1]$ results in a higher reward for an action. We introduce a variable F used to condition the *eating* action so that a fish be rewarded only if it is at the exact position of food. Assume $F = 1$ when a fish is at the exact position of food and zero otherwise.

$$F = \begin{cases} 1, & \text{if } (x, y, z) = (x_p, y_p, z_p), \\ 0, & \text{otherwise,} \end{cases}$$

where (x, y, z) is the current position of a fish and (x_p, y_p, z_p) is the position of food in a Cartesian coordinate system. Let $r : U \times \Phi \times \Delta \longrightarrow [0, 1]$, where, for example, r is defined by the product

$$[u_7(e)\ u_3(e) - u_7(e)\ u_{-3}(-e)] \cdot \begin{bmatrix} \phi_0(a, e) \\ \phi_1(a, e) \\ \phi_2(a, e) \end{bmatrix}.$$

$$[\delta_{1a}\delta_{2a} \cdot F\delta_{3a}\delta_{4a}\delta_{5a}\delta_{6a}\delta_{7a}]$$

The reward function forms a basis for the learning algorithms explained in Sect.V. We now give a brief introduction to near sets and nearness approximation spaces.

IV. Near Sets

TABLE I
Set, Relation, Probe Functions, Object Description

Symbol	Interpretation
$\Re$	Set of real numbers,
$\mathcal{O}$	Set of perceptual objects,
X	$X \subseteq \mathcal{O}$, set of sample objects,
B	$B \subseteq \mathcal{F}$, set of functions representing object features,
x	$x \in \mathcal{O}$, sample perceptual object,
$\sim_B$	$\{(x, x') \mid f(x) = f(x')\ \forall f \in B\}$, indiscernibility relation,
$[x]_B$	$[x]_B = \{x' \in X \mid x' \sim_B x\}$, elementary granule (class),
$\mathcal{O}/\sim_B$	$\mathcal{O}/\sim_B = \{[x]_B \mid x \in \mathcal{O}\}$, quotient set,
ξ_B	Partion $\xi_B = \mathcal{O}/\sim_B$,
ϕ	$\phi : X \longrightarrow \Re^L$, object description,
ϕ_i	$\phi_i : X \longrightarrow \Re$, probe function representing an object feature,
$\phi(x)$	$\phi(x) = (\phi_1(x), \phi_2(x), \phi_3(x), \ldots, \phi_i(x), \ldots, \phi_L(x))$.

Object recognition problems, especially in images [1], [4], and the problem of the nearness of objects have motivated the introduction of near sets (see, *e.g.*, [22], [24]).

A. Object Description

Perceptual as well as conceptual objects are known by their descriptions. An *object description* is defined by means of a tuple of function values associated with an object. The important thing to notice is the paramount important of the choice functions used to describe an object of interest. In defining what is meant by the description of an object, the focus here is on real-valued functions that provide a basis for an object description. This can only be done by understanding the objects associated with a problem domain such as sampling organisms by a biologist or collecting sample signals from an electronic device or sample observations from a medical clinical study.

Assume that $B \in \mathcal{O}$ is a given set of probe functions representing features of objects $x \in \mathcal{O}$. Let $\phi_i \in B$, where $\phi_i : \mathcal{O} \longrightarrow \Re$. In combination, the functions representing object features provide a basis for an *object description* ϕ in the form of a vector $\phi : \mathcal{O} \rightarrow \Re^L$ containing measurements (returned values) associated with each functional value $\phi_i(x),\ x \in \mathcal{O}$ in $\phi(x) = (\phi_1(x), \ldots, \phi_i(x), \ldots, \phi_L(x))$, where the description length $|\phi| = L$.

Definition 1: Nearness Description Principle (NDP)
Let $B \subseteq \mathcal{F}$ be a set of functions representing features of objects $x, x' \in \mathcal{O}$. Objects x, x' are minimally near each other if, and only if there exists $\phi_i \in B$ such that $x \sim_{\{\phi_i\}} x'$, *i.e.*, $\Delta\phi_i = 0$.

In effect, objects x, x' are considered *minimally near* each other whenever there is at least one probe function $\phi_i \in B$ so that $\phi_i(x) = \phi_i(x')$. Then ϕ_i constitutes a minimum description of the objects x, x' that makes it possible for us to assert that x, x' are near each other. Ultimately, there is interest in identifying the probe functions that lead to partitions with the smallest number of classes. This is an essential idea in the near set approach, and differs markedly from the minimum description length (MDL) proposed by Jorma Rissanen [33]. MDL deals with a set $X = \{x_i \mid \ i = 1, \ldots\}$ of possible data models and a set Θ of possible probability models. By contrast, the nearness description principle (NDP) deals with a set X that is the domain of a description $\phi : X \longrightarrow \Re^L$ and the discovery of at least one probe function $\phi_i(x)$ in a particular description $\phi(x)$ used to identify similar objects in X. The term *similar* is used here to denote the presence of objects $x, x' \in X$ and at least one ϕ_i in object description ϕ, where $x \sim_{\phi_i} x'$. In that case, objects x, x' are said to be similar.

Observation 1: Near Physical Objects
Let X denotes a set of wood furniture pieces. For example, an oak table x in Burma and an oak chair x' in Winnipeg are qualitatively near each other in the case where $\phi_i : X \longrightarrow$ type of wood and $\phi_i(x) = \phi_i(x') = oak$.

Observation 2: Near Behaviours
In combination, tuples of behaviour function values form the following description of an object x relative to its observed behaviour $\phi(x) = (s(x), a(x), r(x), V(s(x)))$. So, for example, objects x_1, x_2 are near each other if $V(s(x_1)) = V(s(x_2))$. In the sequel, this is important in the case where single feature neighbourhoods are considered.

B. Near Sets: Basic Concepts

The basic idea in the near set approach to object recognition is to compare object descriptions. Sets of objects X, X' are considered near each other if the sets contain objects with at least partial matching descriptions.

Definition 2: Near Sets
Let $X, X' \subseteq \mathcal{O}, B \subseteq \mathcal{F}$. Set X is near X' if, and only if there exists $x \in X, x' \in X', \phi_i \in B$ such that $x \sim_{\{\phi_i\}} x'$.

Remark 1: If X is near X', then X is a near set relative to X' and X' is a near set relative to X.

Definition 3: Reflexive Nearness
If $x, x' \in X$ and x is near x', then by Def. 2 X is a near set relative to itself. In fact, X is a near set.

Observation 3: Class as a Near Set
By definition, a class $[x]_B$ in a partition ξ_B is a set of objects having matching descriptions (set Table I), i.e., if $x, x' \in [x]_B$, then $x \sim_B x'$.

Theorem 1: A class in a partition ξ_B is a near set.
Proof: From Obs. 3 and from Def. 3, we know that a class $[x]_B \in \xi_B$ is a near set. ■

Affinities between objects of interest in the set $X \subseteq \mathcal{O}$ can be discovered by considering the relation between X and objects in elementary sets in partition $X/\sim_B$. Approximation of the set X begins by determining which elementary sets $[x]_B \subseteq \mathcal{O}/\sim_B$ are subsets of X.

Observation 4: Near Set with an Empty Boundary
It should also be observed that whenever $Bnd_B X = \emptyset$, this means that $|Bnd_B X| = 0$, $B_*X = B^*X$ and $B_*X \subseteq X$. From this, we know that B_*X and X share objects that have matching descriptions. Hence, X is a near set (see Theorem 2, case 2).

Theorem 2: Fundamental Near Set Theorem
A set X with an approximation boundary $|Bnd_B X| \geq 0$ is a near set.

C. Nearness Approximation Spaces

TABLE II
NEARNESS APPROXIMATION SPACE SYMBOLS

Symbol	Interpretation
B_r	$r \leq \lvert B\rvert$ probe functions in B,
$\sim_{B_r}$	Indiscernibility relation on $\mathcal{O}$ defined using B_r,
$[x]_{B_r}$	$[x]_{B_r} = \{x' \in \mathcal{O} \mid \ x \sim_{B_r} x'\}$, equivalence class,
$\mathcal{O}/\sim_{B_r}$	$\mathcal{O}/\sim_{B_r} = \left\{[x]_{B_r} \mid \ x \in \mathcal{O}\right\}$, quotient set,
$\xi_{\mathcal{O},B_r}$	Partition $\xi_{\mathcal{O},B_r} = \mathcal{O}/\sim_{B_r}$,
ϕ_i	Probe function $\phi_i \in \mathcal{F}$,
r	$\binom{\lvert B\rvert}{r}$, *i.e.*, $\lvert B\rvert$ functions $\phi_i \in \mathcal{F}$ taken r at a time,
$N_r(B)$	$N_r(B) = \left\{\xi_{\mathcal{O},B_r} \mid \ B_r \subseteq B\right\}$, set of partitions,
ν_{N_r}	$\nu_{N_r} : \mathcal{P}(\mathcal{O}) \times \mathcal{P}(\mathcal{O}) \longrightarrow [0, 1]$, overlap function,
$N_r(B)_*X$	$\bigcup_{x:[x]_{B_r} \subseteq X} [x]_{B_r}$, lower approximation,
$N_r(B)^*X$	$\bigcup_{x:[x]_{B_r} \cap X} [x]_{B_r} \neq \emptyset$, upper approximation,
$Bnd_{N_r(B)}(X)$	$N_r(B)^*X \backslash N_r(B)_*X$.

The original generalized approximation space (GAS) model [35] has been extended as a result of recent work on nearness of objects (see, *e.g.*, [4], [22], [25], [23], [24], [21], [36]). A nearness approximation space (NAS) is a tuple

$$NAS = (\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r, \nu_{N_r}),$$

where defined using $\mathcal{O}$ set of perceived objects, set of probe functions $\mathcal{F}$ representing object features, indiscernibility relation $\sim_{B_r}$ defined relative to $B_r \subseteq B \subseteq \mathcal{F}$, family of neighbourhoods $N_r(B)$, and neighborhood overlap function ν_{N_r}. The relation $\sim_{B_r}$ is the usual indiscernibility relation from rough set theory restricted to a subset $B_r \subseteq B$. The subscript r denotes the cardinality of the restricted subset B_r, where we consider $\binom{|B|}{r}$, *i.e.*, $|B|$ functions $\phi_i \in \mathcal{F}$ taken r at a time to define the relation $\sim_{B_r}$. This relation defines a partition of $\mathcal{O}$ into non-empty, pairwise disjoint subsets that are equivalence classes denoted by $[x]_{B_r}$, where

$$[x]_{B_r} = \{x' \in \mathcal{O} \mid \ x \sim_{B_r} x'\}.$$

These classes form a new set called the quotient set $\mathcal{O}/\sim_{B_r}$, where

$$\mathcal{O}/\sim_{B_r} = \left\{[x]_{B_r} \mid \ x \in \mathcal{O}\right\}.$$

In effect, each choice of probe functions B_r defines a partition $\xi_{\mathcal{O},B_r}$ on a set of objects $\mathcal{O}$, namely,

$$\xi_{\mathcal{O},B_r} = \mathcal{O}/\sim_{B_r}.$$

Every choice of the set B_r leads to a new partition of $\mathcal{O}$. Let $\mathcal{F}$ denote a set of features for objects in a set X, where each $\phi_i \in \mathcal{F}$ that maps X to some *value set* V_{ϕ_i} (range of ϕ_i). The value of $\phi_i(x)$ is a measurement associated with a feature of an object $x \in X$. The function ϕ_i is called a $probe$ [14]. The overlap function ν_{N_r} is defined by

$$\nu_{N_r} : \mathcal{P}(\mathcal{O}) \times \mathcal{P}(\mathcal{O}) \longrightarrow [0, 1],$$

where $\mathcal{P}(\mathcal{O})$ is the powerset of $\mathcal{O}$. The overlap function ν_{N_r} maps a pair of sets to a number in $[0, 1]$ representing the degree of overlap between sets of objects with features defined by probe functions $B_r \subseteq B$. For each subset $B_r \subseteq B$ of probe functions, define the binary relation $\sim_{B_r} = \{(x, x') \in \mathcal{O} \times \mathcal{O} : \forall \phi_i \in B_r, \phi_i(x) = \phi_i(x')\}$. Since each $\sim_{B_r}$ is, in fact, the usual indiscernibility relation [15], for $B_r \subseteq B$ and $x \in \mathcal{O}$, let $[x]_B$ denote the equivalence class containing x, *i.e.*,

$$[x]_{B_r} = \{x' \in \mathcal{O} \mid \ \forall f \in B_r, f(x') = f(x)\}.$$

If $(x, x') \in \sim_{B_r}$ (also written $x \sim_{B_r} x'$), then x and x' are said to be *B-indiscernible* with respect to all feature probe functions in B_r. Then define a family of neighborhoods $N_r(B)$, where

$$N_r(B) = \{\xi_{\mathcal{O},B_r} \mid \ B_r \subseteq B\}.$$

Families of neighborhoods are constructed for each combination of probe functions in B using $\binom{|B|}{r}$, *i.e.*, $|B|$ probe functions taken r at a time.

The family of neighbourhoods $N_r(B)$ contains a set of percepts. A *percept* is a byproduct of perception, *i.e.*, something that has been observed [13]. For example, a class in $N_r(B)$ represents *what has been perceived about objects belonging to a neighbourhood*, *i.e.*, observed objects with matching probe function values.

Theorem 3: A family of neigbourhoods $N_r(B)$ is a near set.

A sample $X \subseteq \mathcal{O}$ can be approximated relative $B \subseteq \mathcal{F}$ by constructing a *family of neighbourhoods* $N_r(B)$*-lower approximation* $N_r(B)_*X$, where

$$N_r(B)_*X = \bigcup_{x:[x]_{B_r} \subseteq X} [x]_{B_r},$$

and a *family of neighbourhoods* $N_r(B)$*-upper approximation* $N_r(B)^*X$, where

$$N_r(B)^*X = \bigcup_{x:[x]_{B_r} \cap X \neq \emptyset} [x]_{B_r}.$$

Theorem 4: A family of neigbourhoods lower approximation $N_r(B)_*X$ of a set X is a near set.

Theorem 5: A family of neigbourhoods upper approximation $N_r(B)^*X$ of a set X is a near set.

Then $N_r(B)_*X \subseteq N_r(B)^*X$ and the boundary region $Bnd_{N_r(B)}(X)$ between upper and lower approximations of a set X is defined using set difference, *i.e.*

$$Bnd_{N_r(B)}(X) = N_r(B)^*X \backslash N_r(B)_*X.$$

Observation 5: Near Set with an Empty Boundary It should also be observed that whenever $Bnd_{N_r(B)}(X) = \emptyset$, this means that $\left|Bnd_{N_r(B)}(X)\right| = 0$, $N_r(B)_*X = N_r(B)^*X$ and $N_r(B)_*X \subseteq X$. From this, we know that $N_r(B)_*X$ and X share objects that have matching descriptions, *i.e.*, objects in each class in $N_r(B)_*X$ are also objects contained in X. Recall from Theorem 1, also, that every class is a near set. By definition, all classes in $N_r(B)_*X$ are also subsets of X. Then it follows that X is a near set (see Theorem 6).

Theorem 6: A set X with an approximation boundary $\left|Bnd_{N_r(B)}(X)\right| \geq 0$ is a near set.

From Theorem 6, set X is termed a *near set* relative to a chosen family of neighborhoods $N_r(B)$ *iff* $|Bnd_{N_r(B)}(X)| \geq 0$. In the case where $\left|Bnd_{N_r(B)}(X)\right| > 0$, the set X has been *roughly* approximated, *i.e.*, X is a rough set as well as a near set. In the case where $\left|Bnd_{N_r(B)}(X)\right| = 0$, the set X is considered a near set but not a rough set. In effect, every rough set is a near set but not every near set is a rough set.

D. Average Rough Coverage

It is now possible to formulate a basis for measuring average the degree of overlap between each class in $N_r(B)$. Assume lower approximation $N_r(B)_*X$ defines a standard for classifying perceived objects. The notation $B_j(x)$ denotes a class in the family of neighborhoods in $N_r(B)$, where $a \in B_r$. Put

$$\nu_a([x]_{B_r}, N_r(B)_*X) = \frac{|\,[x]_{B_r} \cap N_r(B)_*X|}{|N_r(B)_*X|},$$

(called *lower rough coverage*) where ν_j is defined to be 1, if $N_r(B)_*X = \emptyset$.

Put $\mathcal{B} = \{[x]_{B_r} : a(x) = j, x \in U\}$, a set of blocks that "represent" action $a(x) = j$. Let D denote a decision class, *e.g.*, $D = \{x \mid d(x) = 1\}$, a set of objects having acceptable behaviours. Define $\bar{\nu}_a(t)$ (average rough coverage)[1] with respect to an action $a(x) = j$ at time t in (1).

$$\bar{\nu}_a(t) = \frac{1}{|\mathcal{B}|} \sum_{[x]_{B_r} \in \mathcal{B}} \nu\left([x]_{B_r}, N_r(B)_*D\right). \tag{1}$$

V. Learning Algorithms

The adaptive learning algorithms in this study use Monte Carlo methods to estimate the value of a state $V(s) \approx \frac{1}{n}\sum_{i=1}^{n} r_i$, *i.e.*, average reward received up to the current state. The simplest of these algorithms is Alg. 1.

Algorithm 1: Adaptive Learning Method

Input : States $s \in S$, Actions $a \in A(s)$, $V(s)$
Output: Policy $\pi(s, a)$
while *True* **do**
  Begin episode;
  Initialize policy $\pi(s, a), s, V(s) \longleftarrow 0$;
  episode = true;
  Estimate $V(s') = E[R_a]$ for all a in state s;
  while $(V(s) \leq V(s'))$ **do**
    Take action a, observe $r(t)$ signal;
    Choose new a from new s using $\pi(s, a)$;
    Estimate $V(s), V(s')$;
    if $(V(s) > V(s') for\, all\, a)$ **then**
      episode = false;
    else
      episode continues;
    end
  end
end

Alg. 1 partially implements the Selfridge-Watkins approach to learning with delayed rewards. This is a partial implementation because ethograms are not used (*i.e.*, an organism starts from scratch at the beginning of each episode and does not take into account what it has learned in previous episodes). Alg. 2 fully implements the Selfridge-Watkins approach using a traditional rough set approach to set approximation, but without considering near sets. Algorithm 3 is a variation of Alg. 2 that uses the near set approach limited to $r = 1$ (single feature neighbourhoods). Algorithm 3 has two forms of implementation based on a *short term memory model* and a *long term memory model*. In the short term memory model, average coverage($\bar{\nu}_{a_v}$) is derived from only the previous episode and passed to the next episode, which exactly matches the idea of short term memory in fish. In long term memory model, instead of using the average coverage from only one previous episode(short term memory model), average of average coverages($avg\ \bar{\nu}_{a_v}$) is derived from all of the the previous episodes and passed to the next episode. This matches suspected long term memory in fish.

[1] $\bar{\nu}_a(t)$ is computed at the end of each episode using an ethogram that is part of the adaptive learning cycle.

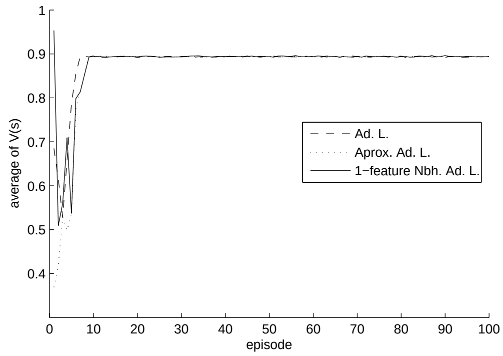
Algorithm 2: Approximate Adaptive Learning Method

Input : States $s \in S$, Actions $a \in A$, $V(s)$
Output: Policy $\pi(s, a)$
while *True* **do**
  Begin episode;
  Initialize $\bar{\nu}_a$ for each action a, policy $\pi(s, a)$, s, $V(s) \leftarrow 0$;
  Estimate $V(s') = E[R_a]$;
  while $(V(s) \leq V(s'))$ **do**
    Take action a, observe $r(t)$ signal, compute γ ;
    Choose new a from new s using $\pi(s, a)$;
    Estimate $V(s), V(s')$;
    $V(s) \leftarrow V(s) + (\bar{\nu}_a)[r + \gamma max_a V(s') - V(s)]$;
    if $(V(s) > V(s') for\, all\, a)$ **then**
      episode = false;
      Compute $\bar{\nu}_a$ for each action a;
      Clear ethogram;
    else
      episode continues;
    end
  end
end

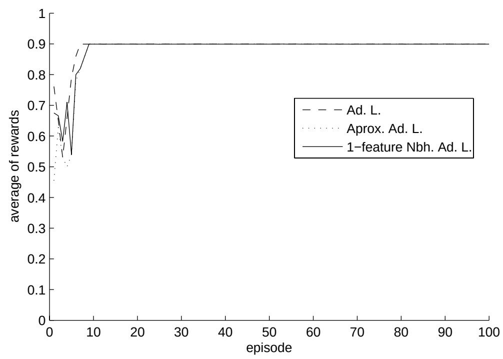
Algorithm 3: Approximate Adaptive Learning Method, Single Feature Neighbourhood, Near Set Approach

Input : States $s \in S$, Actions $a \in A$.
Output: Ethogram resulting from Policy $\pi(s, a)$.
Initialize $\overline{\nu}_{a_v}$ wrt 1-feature nbds, $\pi(s, a), s, V(s) \leftarrow 0$;
while *True* **do**
  Begin episode;
  episode = true;
  Estimate $V(s') = E[R_{a_v}]$;
  while $(V(s) \leq V(s'))$ **do**
    Take action a_v, observe $r(t)$ signal, compute γ ;
    Choose new a_v from new s using $\pi(s, a)$;
    Estimate $V(s), V(s')$;
    Pick action a_v using policy $\pi(s, a)$;
    $V(s) \leftarrow V(s) + (\overline{\nu}_{a_v})[r + max_a V(s') - V(s)]$;
    if $(V(s) > V(s') for\, all\, a)$ **then**
      episode = false;
      Compute new $\overline{\nu}_{a_v}$ for 1-feature nbds;
      Clear ethogram;
    else
      episode continues;
    end
  end
end

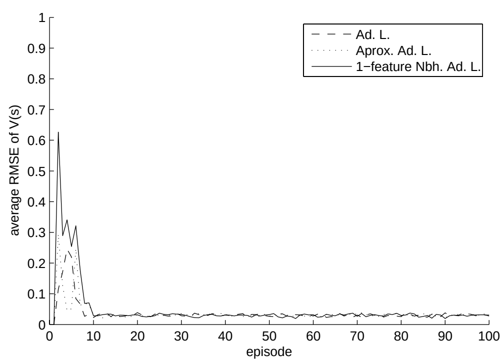
Remark 2: **Possible Learning Rate Measures**
In addition to average rough coverage measure used in Alg. 2



2.1: Avg. V(s)



2.2: Avg. reward



2.3: Avg. RMS error

Fig. 2. Results for Three Algorithms

and Alg. 3, other measures such as rough inclusion [17], [31], upper rough coverage (see,*e.g.*, [31]), accuracy [17], and behaviour growth [30] are possible. Lower rough coverage has been beneficially used in a number of reinforcement learning algorithms in [31], [18], [20], [26], [28] in ecosystem studies as well as in controllers for vision system cameras. The lower rough coverage measure has been favoured over the other measures because the lower approximation B_*D of the set D contains all of the classes in the partition $\xi_{\mathcal{O},B_r}$ that are subsets of D. This is important for us because the classes in B_*D represent all of the observed behaviours that have been accepted during an episode. By measuring the average overlap between classes $[x]_{B_r} \in \xi_{\mathcal{O},B_r}$, it is possible to gauge the learning rate of an organism during an episode. We have ruled out upper rough coverage because B^*D fails to provide a benchmark (standard) in assessing the episodic learning rate, since it is possible for B^*D to contain classes that contain objects that are not in D. The accuracy and growth measures are definitely interesting for us, but we inclusion of them in the reported ecosystem study is outside the scope of this article. This is part of our future work. It should also be mentioned that a differential learning rate based on average rough coverage has been recently introduced [18] but also has not been included in the current study. This is also part of our future work. We have not considered functions defined over subsets of functions representing object features because our work concentrated on classes in partitions. Definitely, in future work, we want to consider nearness measures that take into account the nearness description principle and the selection of features used to define partitions.

VI. Results

In this section, the simulation results of modeling fish behaviour using all three above algorithms are shown and compared together. Fig. 2 show sample results for the three algorithms. All three algorithms do well. It can be seen in Fig. 2.1 that Alg. 1 (Simple adaptive Learning) and Alg. 3 (Near Set, single feature neighbourhood learning) have a markedly higher average $V(s)$ than Alg. 2 during the initial episodes. After that, all 3 algorithms do well. Similarly, in Fig. 2.2, one can observe that all 3 algorithms have good performance after the initial 10 episodes. Initially, the root mean square error (RMSE) is higher for Alg. 3 than the RMSE for the other two algorithms. After the initial episodes, the RMSE for Alg. 3 is slightly better than the RMSE for the other two algorithms.

The sample RMSE values for short term memory version of Alg. 3 show high oscillation of the value of state $V(s)$. By contrast, sample RMSE values for long term memory version of Alg. 3 show very low oscillation of the value of state $V(s)$ after the initial episodes. Due to space constraints, the plots for RMSE for both forms of memory are not given. The results found so far reflect the benefits of taking into account past experience (*i.e.*, averaging the coverage values over all of the previous episodes rather than restrict memory to what is remembered about the previous episode). These results of our simulations confirm the fact that learning and memory in fish are inextricably linked together and have important influence on each other [37].

VII. Conclusion

This paper reports results from a recent study of adaptive learning algorithms with states, actions, and rewards defined relative to the observed behaviour of a species of freshwater fish commonly known as Glowlight tetra. These results are promising but are considered preliminary. Three classes of Selfridge-Watkins forms of adaptive learning algorithms have been considered, namely, adaptive learning without memory, ethogram-based adaptive learning, and short term as well as long term memory in a near set approach to approximate adaptive learning. It has also been observed that the long term memory model does better than the short term memory model in the near set approach to approximate adaptive learning, which tends to corroborate the observation that long term memory is influential in learning by fish. Future work will include the use of the near set approach in adaptive learning systems designed for task-specific, interactive exercise gaming in rehabilitation of persons benefiting from constraint-induced movement therapy.

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