
Approach Merotopy (APm) Toolset

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Submitted 7 May 2011

UM CI Laboratory Technical Report
Number TR-CI-7-May-2011-902-v1
7 May 2011

University of Manitoba APm

Computational Intelligence Laboratory

URL: <http://wren.ee.umanitoba.ca/>

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Project no. CI-7-May-2011-902

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Abstract

This CI Lab 900 report series includes a toolset called APm (APproach merotopy). This toolset facilitates measurement of the degree of nearness between regions-of-interest (ROIs) in pairs of digital images (either colour or greyscale images). Such nearness measurements are computed in terms of ε -neighbourhoods contained in selected ROIs. Each neighbourhood contains points y that are descriptively near a selected centre point x_0 in a particular ROI, *i.e.*, the distance between the feature vector of x_0 and the feature vector for each y is less than some $\varepsilon \in (0, \infty]$. A feature vector constitutes a description of a point. In the case of a digital image, a point is defined to be an image pixel.

Key Words: Digital image, distance, feature vector, merotopy, nearness, neighbourhood, region-of-interest (ROI).

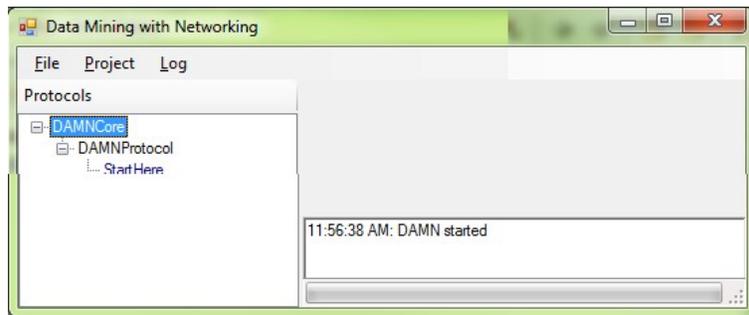


Figure 1: Main APm window

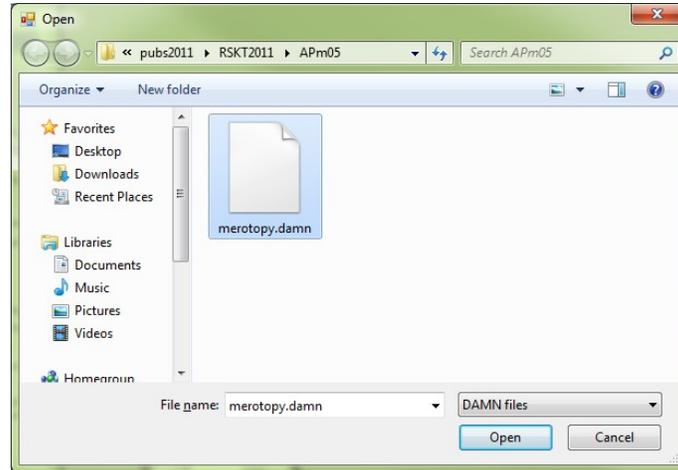


Figure 2: APm Project window

1 Introduction

This report briefly explains how approach merotopic spaces can be used in the study of ε -near collections of subsets in pairs of digital images. The solution to this problem arises from M. Katětov's observation [5] that merotopic spaces are obtained by topologising certain parts of a nonempty set, recent work by S. Tiwari on approach nearness and merotopological spaces [15] and the introduction of an approach merotopy derived

from E. Čech's suggestion [16] that the distance between sets be measured in terms of the greatest lower bound of the Hausdorff lower distance [4] between points and sets. The details concerning the underlying theory for the APm toolset are given in the Appendix of this report. For a more detailed explanation of merotopies with a number of examples (including an application of the APm toolset), see [12].

2 Steps to Use APm Toolset

To use the APm toolset to measure the nearness of selected ROIs in a pair of digital images, do the following.

- (Step.1) Use the APm zip file to set up a directory containing the AMm toolset files.
- (Step.2) Double click on APm.exe and the main APm window will be displayed (see Fig. 1).
- (Step.3) Click on the Project tab in the upper lefthand corner of the APm display,
- (Step.4) Click on the **Load** option in the Project menu. At this point, a Project window will appear (see Fig. 2) showing one file, namely, **merotopy.damn**. Click on **merotopy.damn** and then click the **Open** button. This loads the merotopy project.
- (Step.5) Click on the File tab in the upper lefthand corner of the APm display,

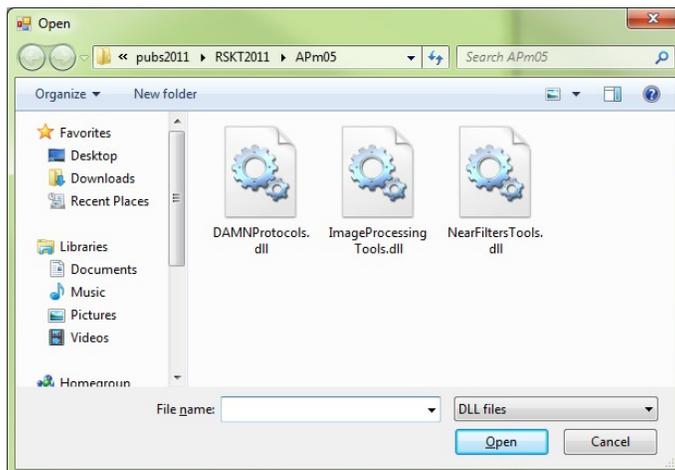


Figure 3: APm Device Driver window

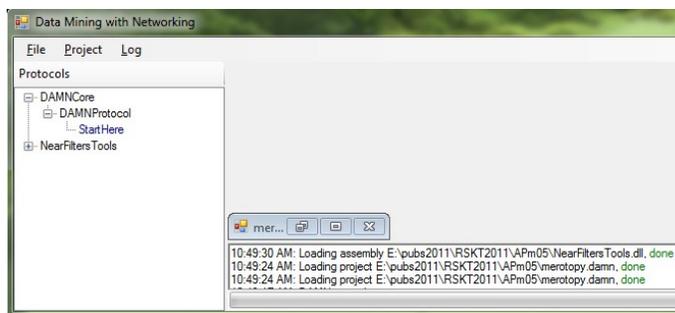


Figure 4: APm Assembly window

- (Step.6) Click on the **Load assembly** option in the File menu. At this point, an **Device driver** window will appear showing several dll files (see Fig. 3). Click on **NearFiltersTools.dll** and then click

the **Open** button. This loads the merotopy assembly and an APM assembly window will appear (see Fig. 4).

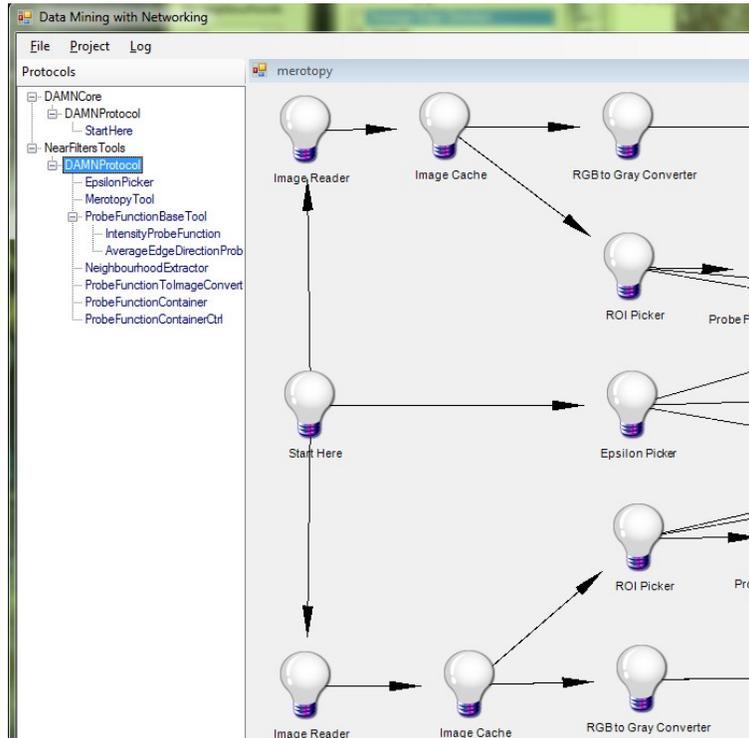


Figure 5: APM Action Choices window

- (Step.7) Click on the first of the 3 buttons (nested boxes) in the tiny menu in the lower lefthand corner of bottom of the APM display. As a result, a directed graph (dg) is displayed. Each node in the dg is represented by a light bulb.
- (Step.8) Right-click on the light bulb labelled **Start here** (see Fig.5). This will display three windows
 - Window.1** **Current directory** window showing digital images that can be selected. Select first image and then Click the **Open** button. Select second image and then Click the **Open** button. The pair of selected images will then be displayed in separate windows. See, for example, camouflaged man in Fig 6.1 and Fig. 6.2.
 - Window.2** **Epsilon picker** window with a slider that makes it possible for you to select a ε value, starting a $\varepsilon = 1$. Pick a ε value. See, for example, sample ε -picker window in Fig. 6.3.
 - Window.3** **Merotopy** window that starts working after you select a pair of ROIs. To see this, use the cursor to select a small rectangular region (ROI) in each of the displayed images. The default features that can be selected in the current version of APM are **Average Edge Direction** and **Intensity**. Try changing the selected features to obtain a merotopy value. Each increase in the selected ε value leads to a longer time to compute a merotopy value. See sample merotopy value shown in Fig.6.4.

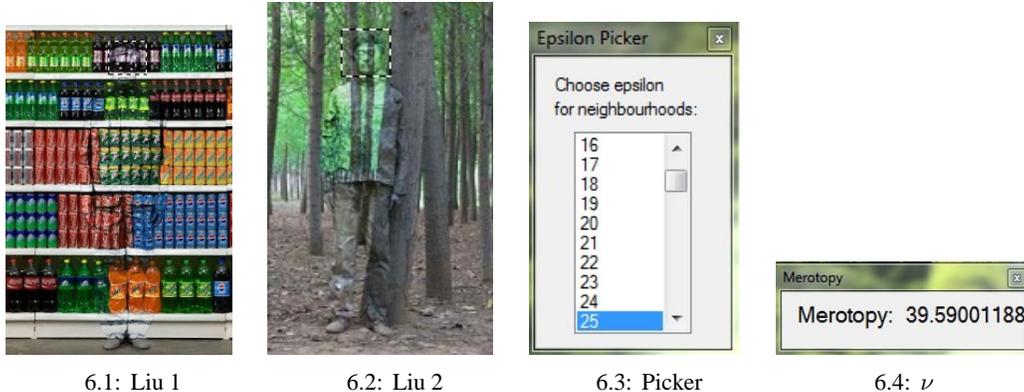


Figure 6: Sample Use of APm

3 About the APm Toolset

The APm framework facilitates easy creation of image analysis as well as data mining applications from smaller tasks. The system consists of a main application that allows for creation of projects consisting of tasks. A task is built by a researcher or developer with favourite tools as long as each task compiles to .NET libraries.

These libraries are then loaded by the APm framework and executed. The communication between tasks within the APm framework is via messages. Each task is built as a separate library that can be reused by many APm projects. The project used for this article consists of 12 tasks. After images are loaded and a user selects ROIs, probe function values are extracted. Currently two probe functions (pixel greylevel intensity and Prewitt gradient estimate) are used and the extension by additional probe functions can be done without recompiling the original project. Once probe functions are ready, they are passed to the merotopy, tool where a merotopy distance is calculated and displayed (see Alg. 1 for details).

Only sample results using APm are given in this section. After loading a pair of images, the cursor is used to select an ROI in each image. Then it is possible to move the ε -picker in Fig. 6.3 up or down to find a case so that the distance $\nu_{D, \rho_{\|\cdot\|}}$ approaches zero.

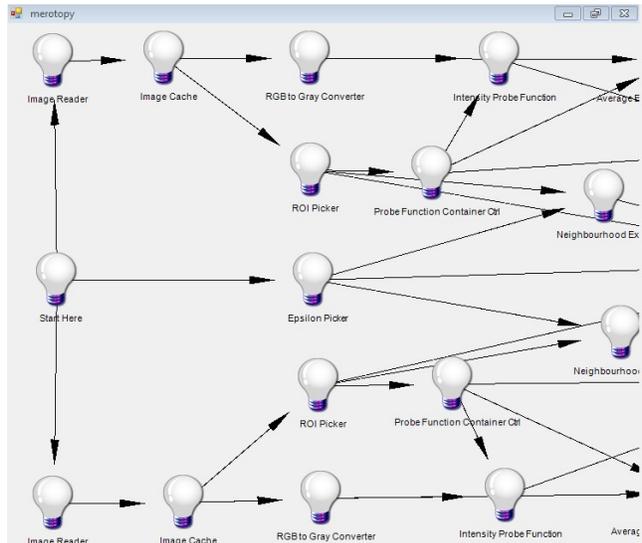


Figure 7: APm Snapshot

4 Appendix A: Approach Spaces and Čech Distance Between Sets

Approach spaces and ε -approach merotopic spaces are briefly explained in this Appendix. Let X be a nonempty set. The family of all subsets of X is denoted by $\mathcal{P}(X)$ and the collections of subsets in $\mathcal{P}(X)$ is denoted by $\mathcal{P}^2(X)$. For collections $\mathcal{A}, \mathcal{B} \in \mathcal{P}^2(X)$, $\mathcal{A} \vee \mathcal{B} \equiv \{A \cup B : A \in \mathcal{A}, B \in \mathcal{B}\}$ and $\mathcal{A} \wedge \mathcal{B} \equiv \{A \cap B :$

Algorithm 1: APm Merotopy Algorithm

Input : Images A, B and $ROIA, ROIB$.

Output: Merotopy distance between ROIs.

Convert ROIs to greyscale images;

Use probe functions to extract values in pixel feature vectors;

Select ε value;

Define ROIs as collections of neighbourhoods $N_{D_{\rho_{\|\cdot\|}}}(x_A, \varepsilon), N_{D_{\rho_{\|\cdot\|}}}(x_B, \varepsilon)$;

more := true;

while (more) **do**

 Select $x_A \in ROIA, x_B \in ROIB$;

 Extract $N_{D_{\rho_{\|\cdot\|}}}(x_A, \varepsilon) \subset ROIA, N_{D_{\rho_{\|\cdot\|}}}(x_B, \varepsilon) \subset ROIB$;

$\mathcal{A} \doteq \{A : A \doteq N_{D_{\rho_{\|\cdot\|}}}(x_A, \varepsilon) \subset ROIA\} \supset N_{D_{\rho_{\|\cdot\|}}}(x_A, \varepsilon)$;

$\mathcal{B} \doteq \{B : B \doteq N_{D_{\rho_{\|\cdot\|}}}(x_B, \varepsilon) \subset ROIB\} \supset N_{D_{\rho_{\|\cdot\|}}}(x_B, \varepsilon)$;

if (no additional points remain to be selected in $ROIA, ROIB$) **then**

 more := false;

 Compute $\nu_{D_{\rho_{\|\cdot\|}}}(\mathcal{A}, \mathcal{B}) \doteq \min_{B \in \mathcal{B}, A \in \mathcal{A}} D_{\rho_{\|\cdot\|}}(B, A)$;

$A \in \mathcal{A}, B \in \mathcal{B}$. \mathcal{A} *corefines* \mathcal{B} (written as $\mathcal{A} < \mathcal{B}$) if, and only if, for all $A \in \mathcal{A}$, there exists $B \in \mathcal{B}$ such that $B \subseteq A$.

Definition 1. An approach space (X, ρ) [10, 11] is a nonempty set X equipped with a distance function $\rho : \mathcal{P}(X) \times \mathcal{P}(X) \rightarrow [0, \infty]$ if and only if for all nonempty subsets $A, B, C \in \mathcal{P}(X)$, ρ satisfies properties (A.1)-(A.5), i.e.,

(A.1) $\rho(A, A) = 0$,

(A.2) $\rho(A, \emptyset) = \infty$,

(A.3) $\rho(A, B \cup C) = \min\{\rho(A, B), \rho(A, C)\}$,

(A.4) $\rho(A, B) = \rho(B, A)$,

(A.5) $\rho(A, B) \leq \rho(A, B^{(\alpha)}) + \alpha$, for every $\alpha \in [0, \infty]$, where $B^{(\alpha)} = \{x \in X : \rho(\{x\}, B) \leq \alpha\}$.

It has been observed that the notion of distance in an approach space is closely related to the notion of nearness [7, 6].

Definition 2. Čech Distance Between Sets

For a nonempty subset $A \in \mathcal{P}(X)$ and a nonempty set $B \in \mathcal{P}(X)$, define a gap functional, a variation of the distance functions introduced by E.Čech in his 1936–1939 seminar on topology [16] (see, also, [2, 3, 9]), where $D_\rho : \mathcal{P}(X) \times \mathcal{P}(X) \rightarrow [0, \infty]$ is defined by

$$D_\rho(A, B) = \begin{cases} \inf \{\rho(a, b) : a \in A, b \in B\}, & \text{if } A \text{ and } B \text{ are not empty,} \\ \infty, & \text{if } A \text{ or } B \text{ is empty.} \end{cases}$$

For simplicity, we also write $D_\rho(A, B)$ to denote the Čech distance between sets.

Observe that (X, D_ρ) is an approach space.

5 Appendix B: Merotopy and ε -Approach Nearness

Merotopic distance is briefly explained in this section. Let $\mathcal{A}, \mathcal{B} \in \mathcal{P}^2(X)$. Suppose that \mathcal{A} and \mathcal{B} are two distinct images. In an approach merotopic space (X, ν) , \mathcal{A} is said to be near (similar) to \mathcal{B} if and only if there exists subimages $A \in \mathcal{A}$ and $B \in \mathcal{B}$ such that $\nu(\{A\}, \{B\}) = 0$ (cf. [11]). In practice, this seldom occurs. The sets A and B can be close enough and not far apart. Therefore, a function that measures the almost nearness (ε -nearness, $\varepsilon \in (0, \infty]$) of collections of sets is required, i.e. we are considering the more general nearness measurement $\nu(\{A\}, \{B\}) < \varepsilon$. This mainly motivates the introduction of an ε -approach merotopy on X [13]. In this section, we axiomatize ε -approach nearness spaces, where $\varepsilon \in (0, \infty]$, to study the topological properties of ε -approach merotopic spaces. Examples are constructed in support of the existence of such spaces.

Definition 3. *Let $\varepsilon \in (0, \infty]$. Then, in a manner similar to [15], a function $\nu : \mathcal{P}^2 X \times \mathcal{P}^2 X \rightarrow [0, \infty]$ is an ε -approach merotopy on X if and only if for any collections $\mathcal{A}, \mathcal{B}, \mathcal{C} \in \mathcal{P}^2(X)$, the properties (AN.1)-(AN.5) are satisfied:*

- (AN.1) $\mathcal{A} < \mathcal{B} \implies \nu(\mathcal{C}, \mathcal{A}) \leq \nu(\mathcal{C}, \mathcal{B})$,
- (AN.2) $(\cap \mathcal{A}) \cap (\cap \mathcal{B}) \neq \emptyset \implies \nu(\mathcal{A}, \mathcal{B}) < \varepsilon$,
- (AN.3) $\nu(\mathcal{A}, \mathcal{B}) = \nu(\mathcal{B}, \mathcal{A})$ and $\nu(\mathcal{A}, \mathcal{A}) = 0$,
- (AN.4) $\emptyset \in \mathcal{A} \implies \nu(\mathcal{C}, \mathcal{A}) = \infty$,
- (AN.5) $\nu(\mathcal{C}, \mathcal{A} \vee \mathcal{B}) \geq \nu(\mathcal{C}, \mathcal{A}) \wedge \nu(\mathcal{C}, \mathcal{B})$.

The pair (X, ν) is termed as an ε -approach merotopic space.

Def. 3 gives the properties required for a distance function with two arguments to be an ε -approach merotopy. Approach merotopies with two arguments were introduced in [13]. Such merotopies are useful in measuring the distance between nonempty collections. For an ε -approach merotopic space (X, ν) , we define: $cl_\nu(A) = \{x \in X : \nu(\{\{x\}\}, \{A\}) < \varepsilon\}$, for all $A \subseteq X$. Then cl_ν is a Čech closure operator on X .

Let $cl_\nu(\mathcal{A}) = \{cl_\nu(A) : A \in \mathcal{A}\}$. Then an ε -approach merotopy ν on X is called an ε -approach nearness on X , if the following condition is satisfied:

- (AN.6) $\nu(cl_\nu(\mathcal{A}), cl_\nu(\mathcal{B})) \geq \nu(\mathcal{A}, \mathcal{B})$.

In this case, cl_ν is a Kuratowski closure operator on X [8, p. 20]. This was recently pointed out by S. Tiwari [15, p. 16] in a review of the Čech closure operator, merotopy on a set, nearness and farness structures.

Definition 4. *For any ε -approach nearness spaces (X, ν) and (Y, ν') , a map $f : X \rightarrow Y$ is called a contraction if $\nu'(f(\mathcal{A}), f(\mathcal{B})) \leq \nu(\mathcal{A}, \mathcal{B})$, for all $\mathcal{A}, \mathcal{B} \in \mathcal{P}^2(X)$.*

6 Appendix C: Descriptive Nearness of Collections

In this appendix, we briefly consider the perceived distance between descriptions of collections of pixels in digital images such as those shown in Fig. 8. The distance function $\rho_{\|\cdot\|}$ is defined in the context of a normed space. Let X be a linear space over the reals with origin 0.

A **norm** on X is a function $\|\cdot\| : X \rightarrow [0, \infty]$ satisfying several properties for a normed space [14]. Each norm on X induces a metric d on X defined by $d(x, y) = \|x - y\|$ for $x, y \in \mathbb{R}$ [1]. For example, let \vec{a}, \vec{b} denote a pair of n -dimensional vectors of numbers representing object feature values (e.g., positive real values representing intensities of light reflected from objects in a visual field), i.e., $\vec{a} = (a_1, \dots, a_i, \dots, a_n)$, $\vec{b} = (b_1, \dots, b_i, \dots, b_n)$ such that $a_i, b_i \in \mathbb{R}^{0+}$. Then, the distance function $\rho_{\|\cdot\|} : \mathbb{R}^n \times \mathbb{R}^n \rightarrow [0, \infty]$ is defined,

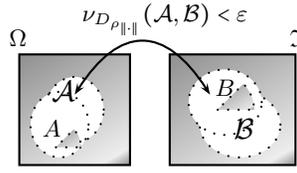


Figure 8: Near Images

e.g., by the $\|\cdot\|_1$ norm called the taxicab distance, i.e., $\rho_{\|\cdot\|}(\vec{a}, \vec{b}) = \|\vec{a} - \vec{b}\|_1 = \sum_{i=1}^n |a_i - b_i|$. For example, let \vec{a}, \vec{b} denote a pair of n -dimensional vectors of numbers that are positive real values representing intensities of light reflected from objects in a visual field, i.e., $\vec{a} = (a_1, \dots, a_i, \dots, a_n), \vec{b} = (b_1, \dots, b_i, \dots, b_n)$ such that $a_i, b_i \in \mathbb{R}^+$. For this reason, we introduce $D_{\rho_{\|\cdot\|}} : \mathcal{P}(X) \times \mathcal{P}(X) \rightarrow [0, \infty]$ defined by

$$D_{\rho_{\|\cdot\|}}(A, B) = \begin{cases} \inf \{ \rho_{\|\cdot\|}(\vec{a}, \vec{b}) : a \in A, b \in B \}, & \text{if } A \text{ and } B \text{ are not empty,} \\ \infty, & \text{if } A \text{ or } B \text{ is empty.} \end{cases}$$

Then $D_{\rho_{\|\cdot\|}}(A, B)$ is a norm gap functional that is defined in terms of $\rho_{\|\cdot\|}$ to measure the lower distance between the descriptions of objects in a pair of non-empty sets A, B .

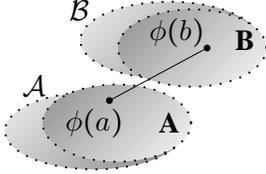


Fig. 3 Sample Nbds A and B

7 Appendix D: Neighbourhood Interpretation of Nearness

The intuitive notion of *nearness* translates into the more manageable notion of a neighbourhood (nbd). For a point $x \in X$, an open nbd $N_d(x, \varepsilon)$ with centre $x \in X$ and radius $\varepsilon \in (0, \infty]$ is always a subset of X , if we consider

$$N_d(x, \varepsilon) = \{y \in X : d(x, y) < \varepsilon\}.$$

For simplicity, we will use the notation $N(x)$ to denote a neighbourhood (nbd) and think of a nbd intuitively by associating with each point $x \in X$ an open set $N(x)$, where x belongs to the interior of a nbd $N(x)$.

The nearness of collections of neighbourhoods of points contained in regions of interest (ROIs) in digital images is considered, next.

Definition 5. ε -Near Collections [13]. A nonempty collection \mathcal{A} is ε -near a collection \mathcal{B} if, and only if there are $B \in \mathcal{B}, A \in \mathcal{A}$ such that $\nu(\mathcal{A}, \mathcal{B}) < \varepsilon$. A nonempty collection \mathcal{A} is **descriptively ε -near** a nonempty collection \mathcal{B} (denoted by $\mathcal{A} \bowtie_{\Phi} \mathcal{B}$) if, and only if a description merotomy

$$\nu_{D_{\rho_{\|\cdot\|}}}(\mathcal{A}, \mathcal{B}) \doteq \min_{B \in \mathcal{B}, A \in \mathcal{A}} D_{\rho_{\|\cdot\|}}(B, A) < \varepsilon,$$

for at least one $B \in \mathcal{B}$ that is near at least one $A \in \mathcal{A}$. When $\mathcal{A} \bowtie_{\Phi} \mathcal{B}$ applies, then one or more probe functions are used to define pairs of feature vectors, one for each collection.

Example 1. The pair of \bullet bullets in Fig. 3 each has a feature value extracted with probe ϕ . The egg-shaped regions labelled A, B in the foreground represent descriptively similar neighbourhoods. If we consider only greylevel intensity of the pixels in A, B , then, by definition, for some choices of ε , $\min_{B \in \mathcal{B}, A \in \mathcal{A}} D_{\rho_{\|\cdot\|}}(B, A) < \varepsilon$.

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